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# MULTISLOT POLYMER INJECTION AND ASSOCIATED PROBLEMS OF DRAG REDUCTION

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#### **ABSTRACT**

This paper describes different aspects of experimental and theoretical investigation of drag reduction by means of multislot injection technique. The semiempirical model of a turbulent boundary layer of fluid with heterogeneous distribution of the polymer additive concentration across thickness is developed. The analysis of influence of several multislot relative factors on flow development is carried out.

Several variants of different combined methods using an injection of polymeric solutions and microbubble mixtures as possible elements of shear flow control have been worked out. The developed method of multilayer injection into turbulent boundary layer has been verified in experiments both on models and in natural investigations of a surface vessel in USA under supervision of K.J. Moore in Pennsylvania State University and on Hawaiian Islands.

The obtained results can be applied as a tool of optimization of a construction and regime parameters of equipment developed with the aim of flow control by injecting the systems of high molecular weight polymeric substances.

# PLACE OF THE COMBINED APPROACHES IN DRAG REDUCTION METHODOLOGY

Development of new technologies on the base of results of modern scientific researches is an actual problem for different practical applications and, in particular, for a wide range of transport means. Modern transport vehicles interact intensively with fluid and this interaction is based on fluiddynamics laws. It is a known fact that friction drag of modern configurations of airplanes and submarines exceeds 50-60% of the total amount of drag. So, drag reduction is a very actual field for skills and efforts applying due to necessity of transport characteristics improvement and flow consumption decreasing. During the last quarter of century various kinds of specific deterministic structural phenomena of turbulent shear flows - coherent vortical structures (CVS) and methods of their control have been the subject of detailed researches. An obtained experience shows that it is possible to act essentially on interaction processes between a body and fluid. There are several basic directions of drag reduction researches, but nowadays one of very perspective is a combination of different techniques that is effective for enhancement of the advantages of its elements together with simultaneous damping their weak sides. Here it is important to note the fact that in a living nature the majority of combined methods are applied. In frames of this research the various combined methods of drag reduction have been investigated. One of them is based on mutual interaction of longitudinal vortical structures together with injections of liquids with different properties through two cross cracks. In this case the gradient of density and viscosity across a

boundary layer was provided. Fig.1, illustrates the flow about longitudinal cylinder. The design of model allowed forming a system of longitudinal vortices inside the slothole chamber. Injection of water through slot demonstrates the presence of longitudinal systems of vortexes (Fig.1a) and injection of polymers solutions essentially increases length of these vortical systems (Fig.1b).

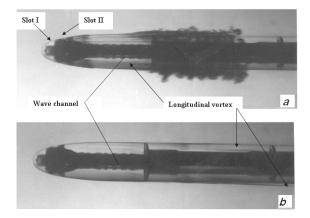


Fig. 1. Modeling of the gill apparatus work. Influence of liquid injection through slot on development of vortical structure system of: a water injection, b - polymers solutions injection

At injection of polymers solutions in the model, the feeding channel was executed in the form of a wavy surface with the certain shape of wave. It promoted both drag reduction in this channel and preliminary extension of polymers.

A model presented on Fig.1 has two slots in nasal part. Injection in a boundary layer of liquids through some slots provided a task to change a gradient of density and viscosity across boundary layer. Thus it was important to reduce also disturbances in a flow behind a slot and diffusion of solutions along a streamline body. Further the multislot design with independent supply of liquids has been developed (Babenko, 2000). Functioning prototype of mixer and injector has been designed. Its design and manufacturing was paid by Cortana Corporation. Discussion of the given project was repeatedly spent with the president of the Cortana Corporation K. J. Moore, who paid for researches and registrations of all patents in this field. Fig.2 shows the model of device for injecting liquids with different molecular properties into boundary layer. The injector functioning is based on the method of boundary layer receptivity to various disturbances. This method consists in creating a resonant interaction of the disturbances in the flow withh disturbances generated by solutions injected through a slot. The chamber under slot is designed flexible and controllable for this purpose. The problem of increasing efficiency of polymeric solutions injection is supposed to be solved by simultaneous use of several methods. The first solution is that two chambers are installed in under the slot. First chamber promotes reduction of the speed discontinuity on slot outlet. The second solution consists in installing longitudinal vortex generators on the internal surface of the second chamber. Thus a polymer solution flows through a slot with not smooth rigid walls, but with a system of longitudinal votices

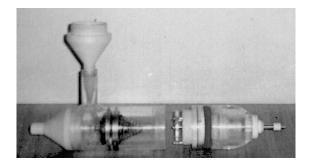


Fig. 2. Model of a mixer and injector

on one side. The third solution is that the first chamber under slot generates a system of transversal vortices in the boundary layer, and the second chamber creates a system of longitudinal vortices. The fourth solution is that the chambers under slot are controlled and work in a foreground of the monitoring system and automatic control. The fifth solution is that the chambers under slot are fabricated with coating of an elastic material. The sixth solution is that the system of longitudinal slots for polymer solutions injection is installed in front each transversal slot. The flow consists of a complex of vortical disturbances, the systems of longitudinal vortices and transversal (toroidal) ring, will approach the system of longitudinal vortexes originating from the slot II. The main purpose of the injector is to decrease diffusion of polymers across the boundary layer thickness and thereby decrease the consumption of polymers. Besides, the injector should continue preconditioning of a polymer solution with the purpose to increase its efficiency including further dissolving of the polymer. Thus, in addition to altering the rheological characteristics of the near-wall fluid, multiple ejectors can be employed to stratify additives, which are known to be effective in specific strata of the boundary layer. For example, some additives like microbubbles can effectively interact with polymers or can accelerate the polymer solutions activity. Fig.3 illustrates a set of three tandem ejectors, each scaled for the desired mass flow rate (Moore, 2002). This scheme is able to generate a threetiered strata of clear water (low viscosity), solution of polymer and microbubbles system.

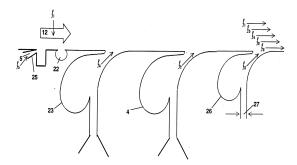


Fig.3. Principal scheme of multislot ejector

# **EXPERIMENTAL BACKGROUND OF MULTISLOT DRAG REDUCTION METHOD**

Similarly, multiple layers of appropriately scaled bubbles or multiple layers of different species of polymers can be ejected from tandem ejectors. Fig.3 designates details of the device with multislot injection, proposed in that patent.

Main

In the past, shipbuilders have concluded that high concentrations and high flow rates of additive from a single ejector system were more efficient than if the same amount of additive were ejected from multiple ejection sites distributed along the length of the hull. The increase in local skin friction produced by traditional ejectors and boundary layer blow-off, which leads to an increase in pressure drag, ware contributors to these phenomena. By avoiding those effects, the present invention makes it possible to employ sets of ejectors at multiple locations along a vehicle or propulsor, and thereby optimize the distribution of additive as a function of shape and length of the wall (vehicle). Thus, very long walls may be treated without a significant loss in efficiency. Various variants of the combined methods of drag reduction are experimentally investigated. For example, in (Babenko, 2001) the results of research on a longitudinal streamline cylinder in diameter 0.175m under influences of an elastic covering and injection of polymer solutions are presented. The total effect is about 40 %. According to (Moore, 200) in Cortana Corporation a plenty of researches (Moore, 2005) the combined method of drag reduction which consists in multilayered injection in a boundary layer of water polymer solutions and micro bubbles is executed. In (Moore, 2006) results of natural researches of the specified method are resulted: "Four two-to-three-week test periods were conducted during Summer 2005 and Spring 2006 on the ONR technology demonstrator vessel, SEA FLYER, to characterize the performance of an advanced polymer drag reduction system. Length Reynolds numbers varied between 1.0 x 10<sup>8</sup> and 1.5 x 10<sup>8</sup> when the ship was flying. Among the results demonstrated were: the ability to mix polymer powder on demand and thereby avoid the need to carry a large volume of polymer slurry; that polymer persistence appears to have temporal as well as spatial characteristics; that in sea water, the reduction in viscous drag for treated components can exceed 60 percent; and that the expenditure rate required at sea could be reduced relative to our best water tunnel performance.

During previous water tunnel tests, we had demonstrated over 60-percent friction reduction 6.2 meters downstream from a similar ejector on a 53.3 cm body of revolution at an expenditure rate of 20 Qs and 2,000 wppm of Polyethylene Oxide-309 (PEO-309). We therefore projected a 50-percent reduction in skin friction for the 10-meter LB and had programmed for the ability to accommodate about a 25-percent increase in expenditure rate (17 Qs at 3,000 wppm) to achieve that level of drag reduction. If the base speed were 28 knots, then about a one-knot increase could be expected. Since the value of towed (i.e., no hullpropeller interaction) resistance (R) and, of course, thrust reduction (t) were not known, these values were recognized as first-order approximations and served principally as guidelines for carrying out the tests. It was also recognized that large reductions in friction drag could impact other aspects of viscous drag, such as the component referred to as profile drag and possibly the level of interference or junction drag produced at the junction of the LB and its supporting struts. Hence, these predictions also could be modest."

# MATHEMATICAL MODELING OF MULTIPHASE INJECTION INTO TURBULENT BOUNDARY LAYER

The goal of this part of investigation is to expand several ideas formulated above with the help of mathematical methods of shear flow modeling. An advantage of this problem formulation is accounting the heterogeneity of concentration distribution of injected substance (or system of substances) across the thickness of shear flow.

## Influence of polymer components on the profile of longitudinal velocity component

The presence of the polymeric components in a stream initiates a deformation of velocity profile in near-wall area of boundary layer. In accordance with the law of a wall it is convenient to describe a character of this deformation by using transition to the non-dimentional form of a longitudinal component of a velocity u and normal coordinate y on a shear velocity and to a near-wall scale of length accordingly  $(u^+ = u/v_*, y^+ = yv_*/v)$  and by presenting distribution  $u^+ = f(y^+)$  in semilogarithmic coordinates, where  $U_*$  - is a shear velocity, u - longitudinal velocity component, V - kinematic viscosity coefficient.

The thickness of the viscous sublayer  $\delta^+ = \delta v_* / v$ is proportional to the coefficient of anisotropy of turbulent viscosity  $A = \mu_{tv}/\mu_{tx}$ , which is inversely proportional (up to a certain threshold value) to concentration of the polymer component.

The logarithmic part of velocity profile is shifting in the direction of higher velocity proportionally to polymer concentration. This influence can be accounted by the constant C of the logarithmic law  $u^+ = k^{-1} \ln v^+ + C$  (C = 4.9-5.5; k=0.4- for the flow over a smooth flat surface without pressure gradient, without polymeric components) that is increased on some value  $\Delta u^+ > 0$ , depending on properties and concentration of polymer and also on shift properties of a flow. The examples of empirical functions simulating the shift of logarithmic part of velocity  $\Delta u^{\dagger}$  as function of properties of a shift flow and properties of a polymer can be found, for example, in (Matvievsky S.A., 1984). Thus, the presence of polymeric additives can be modeled by changing of fullness of longitudinal component velocity profile. As to Karman's, Unluckily, it is not possible to formulate the general point of view about constant k. In some experiments the invariance of a conventional value k = 0.4 has been reached, whereas in results of other researches the slope of a logarithmic part has been bound as reciprocal to growth of polymer concentration (Khabakhpasheva E.M., Perepelitsa B.V., 1970). Nevertheless, for limit values of concentration, at which the condition of saturation is reached, there were reveled the limiting logarithmic distributions of velocity, which are characterized by the following pairs of "constants" offered by different authors: (/C = 17, k = 0.0855); /C = 18.2, k = 0.0885; /C = 18.220.2, k = 0.077). These small enough values of Karman's Sixth International Symposium on Turbulence and Shear Flow Phenomena Seoul, Korea, 22-24 June 2009

constant indicate the preference of approaching about the fact that the Karman's constant is rather a function of polymer concentration in a wide interval of concentration significances than really constant value.

The mentioned above data about the structure and peculiarities of boundary layer with presence of polymeric components allow constructing a semi-empirical model of the investigated flow.

#### Governing equations

In connection with the set of assumptions formulated above about validity of the concept of a boundary layer with reference to the investigated class of flows, we shall consider the following system of the non-dimensional equations of a boundary layer

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{v}} + \frac{1}{u_H} \frac{\partial u_H}{\partial \overline{x}} = 0; \tag{1}$$

$$\overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} + \overline{u}^2 \frac{1}{u_H} \frac{\partial u_H}{\partial \overline{y}} = -\frac{\partial \overline{p}}{\partial \overline{x}} + \frac{\partial \overline{\tau}}{\partial \overline{y}}; \quad (2)$$

$$\overline{u}\frac{\partial \overline{c}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{c}}{\partial \overline{y}} = \frac{\partial}{\partial \overline{y}} \left( \overline{D}_{eff} \frac{\partial \overline{c}}{\partial \overline{y}} \right), \tag{3}$$

where (1) is the continuity equation; (2) - the momentum equation for longitudinal component of velocity u; (3) describes transport of the injected component. Nondimensional coordinates x and y were constructed on the base of typical linear size of calculated area L. The dimensionless shear stress in correspondence with the supposition about flow of a medium is simulated by the Boussinesq' formula

$$\overline{\tau} = \overline{\nu}_{eff} \, \frac{\partial \overline{u}}{\partial \overline{y}} \,, \tag{4}$$

here  $\overline{V}_{eff} = (v + V_t)/(u_H L)$  is dimensionless kinematic factor of effective viscosity. Accordingly, the concentration of polymer c in the equation (4) is scaled by the value  $c_o$  in the initial crossection,  $\overline{c} = c/c_o$ , and effective simulated by the  $\overline{D}_{\rm eff} = \frac{(\nu/Sc + \nu_t/Sc_t)}{r}$  , where Sc and  $Sc_t$  are molecular

and turbulent Schmidt numbers, which are determined as ratios of kinematical factors of molecular and turbulent (V and  $V_t$ ) to the corresponding diffusion coefficients (D and  $D_t$ ), that is: Sc = v/D,  $Sc_t = v_t/D_t$ . The value Sc is determined mainly by physicochemical properties of polymer and solvent (water), whereas the value  $Sc_t$ , being a criterion of similarity, establishes a measure of analogy of a turbulent diffusion of a solvent in relation to a turbulent diffusion of additive, transporting by this solvent. As a first approach in the present research the turbulent Schmidt number Sct was taken as a

The system (1-4) is solved under the following bound-

$$\overline{y} = 0 \ \overline{u} = 0 \ \overline{v} = 0 \ \frac{\partial \overline{c}}{\partial \overline{y}} = 0 \text{ on the surface; (5)}$$

$$\overline{y} \to \infty \ \overline{u} \to u_H(\overline{x}) \ \frac{\partial \overline{c}}{\partial \overline{y}} \to 0 \ \ {\rm on \ the \ outer}$$

boundary of boundary layer;

$$\overline{x} = \overline{x}_o \ \overline{c}_o = \varphi(\overline{y})$$
 in the initial calculated

crossection (
$$x = x_o$$
); (7)

The function  $\overline{u} = f(\overline{y})$  determines an initial profile of a velocity; that can be given analytically or taken, for example, from a set of experimental data. The function  $\overline{c}_o = \varphi(\overline{y})$  is determined by known concentration  $c_o$  of a polymer solution injected into the stream through the ejector, and also by geometric parameters of the ejector, namely, height of its the lower edge from the surface h and width of the slot s. That function is

$$\overline{c} = \begin{cases} 0 & if & 0 \le \overline{y} < \overline{h}; \\ 1 & if & \overline{h} \le \overline{y} < \overline{h} + \overline{s}; \\ 0 & if & \overline{y} \ge \overline{h} + \overline{s}. \end{cases}$$

# Method of calculation

Two-step implicit noniterative method is used for solving the equations (1-4) with the specified boundary conditions (5-7). The equations are solved on a rectangular nonuniform in both directions grid. The amount of nodes in direction of a x ( $i_{max}$ ) coordinate has been found from the condition that the first calculated step is associated with two thicknesses of a boundary layer in an initial profile, and further steps were increased under the law of a geometrical progression  $\Delta x^{i+1} = \Delta x^i q_x$  with a ratio  $q_x$  = 1.05-1.1. In normal to the surface direction the grid is also clustered under the law of geometrical progression. The first step from the wall is determined by the condition  $\Delta y_1^+ = 0.8$ , and the last is limited as  $\Delta y_{j_{\mathrm{max}}} = 0.05 \delta$  . Thus, the grid is created in correspondence with modeled flow. Usually, the number of nodes in streamwise direction is equal to  $i_{max}$ = 40-70, and the normal direction is covered by  $j_{max} = 70$ -

For convenience of the calculation method applying, equations (2) and (3) are represented in the generalized form

$$A_{x} \frac{\partial \overline{\varphi}}{\partial \overline{x}} + A_{y} \frac{\partial \overline{\varphi}}{\partial \overline{y}} = S_{\varphi} + \frac{\partial}{\partial \overline{y}} \left( F \frac{\partial \overline{\varphi}}{\partial \overline{y}} \right), \quad (8)$$

where  $\varphi = \{u, c\}$  is a generalized variable;  $A_x$   $A_y$  are factors of convective transport along x and y respectively; F is a factor of diffusion;  $S_{\omega}$  is a source term.

# Model of turbulence

Main

In the present research with the purpose of modeling of turbulence the algebraic model of turbulence offered by Prof. Movchan V.T. has been applied as basic tool. This model has been modified by Shkvar E.A. for modeling of a series of peculiarities of near-wall flows, such as a roughness of a streamlined surface, near-wall jet flow, three-dimensionality, heat- and scalar additives transport etc (Movchan V.T., Shkvar E.A., 1997, 2005). The choice of an algebraic model is stipulated by the circumstance, that this class of turbulence models, at its relative simplicity, approximates well properties of turbulent mean flow in the vicinity of a wall.

The model of turbulent viscosity applied in the present work has the following structure

$$\mu_{t} = \chi \rho \delta^{*} u_{H} \chi \tanh \left( \frac{\ell \sqrt{\bar{\tau}}}{\chi \Delta} \right); \qquad (9)$$

$$\ell = ky_1 \tanh \frac{\sinh^2[\chi_1 y_1^+] \tanh[\sinh^2(\chi_2 y_1^+)]}{ky_1^+ \sqrt{\bar{\tau}}}$$
 (10)

where  $\chi = 0.0168 \div 0.0215$ 

 $\chi_1 = 0.068 \div 0.072 \,, \quad \chi_2 = 0.223 \,, k = f(c,\ldots)$  are the model coefficients; l is length scale of the near-

wall turbulent motion,  $\bar{\tau} = \tau(y)/\tau_w$  is dimensionless shear stress in the vicinity of the surface, which is determined in an association with the sign of the pressure gradi-

ent 
$$\frac{dp}{dx}$$
 by relations:  $\overline{\tau} = 1 + \frac{dp}{dx}y$  at  $\frac{dp}{dx} \ge 0$ ;

$$\overline{\tau} = \left(1 - \frac{dp}{dx}y\right)^{-1} \text{ at } \frac{dp}{dx} < 0 \; ; \; \Delta = \delta^* u_H^+ \quad \text{is Rotta-}$$

Klauser length parameter;  $y_1$  is the coordinate y shifted concerning the wall with the purpose of accounting the influence of surface roughness and polymer components, determined by the formula  $y_1 = y_1^+ v/v_*$ . The value  $y_1^+$  in "wall law" coordinates is determined by a relation

$$y_1^+ = \begin{cases} 0 & if \quad s \le 0 \\ s & if \quad s > 0 \end{cases}$$

where  $s=y^++\Delta y^+_{rhn}-\Delta y^+_{pol}$ ;  $\Delta y^+_{rhn}$  - parameter which is taking into account the influence of a streamlined surface roughness;  $\Delta y^+_{pol}$  is a parameter introduced here with the purpose of modeling of polymeric components

influence. The introduction of the shift function  $\Delta y_{rhn}^+ > 0$  allows reflecting within the framework of the turbulence model the effect of the surface roughness, known from experimental researches. This effect consists in downward displacement of the logarithmic part of the velocity profile in the semi-logarithmic coordinates, relative its location for case the flow on a smooth surface, by some magnitude  $\Delta u^+$ . This magnitude  $\Delta u^+$  is a function of parameters of the surface roughness. The structure of this function and its connection with  $\Delta y_{rhn}^+$  is described in (Movchan V.T., Shkvar E.A., 2005). Similarly, the shift function  $\Delta y_{pol}^+ > 0$  takes into account a displacement of the logarithmic part of the velocity profile in the opposite direction, which arises under the influence of the polymer components. Similarly to an influence function of roughness, within the framework of the given model there is a uniqueness and universal dependence between  $\Delta y_{nol}^+$  and

corresponding shift of velocity profile  $\Delta u^+$ . However, analysis and generalization of experimental data discover impossibility of establishing a universal and unequivocal dependence between  $\Delta u^+$  and parameters of the polymeric component. Therefore, in each considering case it is necessary to use the known experimental information about the influence of concentration and other performances of applied polymer on a turbulence and mean parameters of modeled boundary layer. The empirical dependences of such type are found in (Matvievsky S.A., 1984) and other works for various polymers. This information also was used as the first approximation within the framework of the present model of turbulence.

The following hypothesis is adopted for registration of non-uniformity of polymer concentration through the thickness of the flow: the shift function  $\Delta y_{pol}^+$ , unlike to the function of roughness  $\Delta y_{rhn}^+$ , is not a constant in particular cross-section  $x^{i+1} = const$ . It is determined in each node  $y_j^{i+1}$  by the value of local polymeric additives concentration  $c_j^{i+1}$ , which is known from the solution of (3) that is  $\Delta y_{pol}^+ = f(c, y, ...)$ .

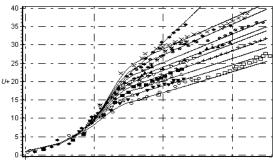
# Calculations results and discussion

The basic and very informative step of testing of the model is its checking in computations of profiles of longitudinal velocity component of the average turbulent flow. With this purpose the numerical integration of the following

transformed form of (4) was made 
$$\frac{\partial u^+}{\partial y^+} = \frac{1}{1 + v_t/v}$$
.

The integration was fulfilled along the y-coordinate inside the near-wall zone of the boundary layer thickness. Equations (9, 10) were applied for turbulent viscosity mod-

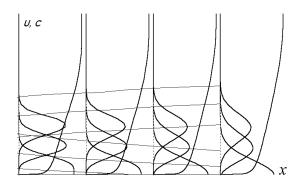
eling. Fig. 8 illustrates the comparison of the computed longitudinal velocity component  $u^+ = f(y^+)$  (lines) with experimental data (points) for flows of homogeneous solutions of different polymers with various concentrations, obtained by different authors. In process of modeling of turbulent viscosity the type of polymer and level of its concentration were accounted in agreement with known empirical information.



**Fig.4.** Comparison of predicted velocity profiles and different experimental data for different types of polymers with wide range of concentrations (c=0-7,4·10<sup>-4</sup> g/cm3)

Besides, the numerical experiment has shown that the dependence of Karman's coefficient on the concentration of polymer k=f(c) is not universal and, as well as the dependence  $\Delta y_{pol}^+=f(c)$ , it depends on physical and chemical properties of applied polymers.

Figure 10 illustrates a process of convection, diffusion and mutual interaction of a system of different phases of polymeric additives after their injecting into a boundary layer in different places.



**Fig.5.** Prediction of flow development: monotonic lines are velocity profiles, dome-shaped lines are profiles of polymer concentrations

#### **CONCLUSIONS**

Several effective variants of a combined drag reduction method have been described and compared. In particular, the results of experimental investigations of different types of methods of longitudinal vortical systems generating and different combined methods based on an injection of polymeric solutions and microbubble mixtures as possible elements of shear flow control have been worked out.

A semi-empirical approach based on an algebraic model of the turbulent viscosity for accounting the influence of a system of polymeric additives injected into liquid flow has been elaborated.

A method of calculation of turbulent boundary layer is developed to describe the flow, in which the local injection of a polymer solution has been carried out. The average velocity profiles of flow of homogeneous solutions of polymers have been computed and demonstrated satisfactorily enough level of correlations with experimental data.

The obtained preliminary results demonstrate the ability of the elaborated method to predict the boundary layer development under the influence of a system of injected soluteions of polymer additives.

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