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DNS AND MODELING OF TURBULENT CHANNEL FLOW WITH WALL-NORMAL ROTATION

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ABSTRACT

Turbulent channel flow affected by wall-normal rotation at different rotation rates has been investigated. A reference data base for Reynolds numbers 180 and 360 based on the friction velocity in non-rotating case at different rotation rates has been provided by means of direct numerical simulation. The ability of a relative simple two equation RANS model (Chien model) has been also studied.

INTRODUCTION

Rotating wall shear flows are encountered in many engineering relevant applications. In these flows rotation induces additional body forces (Coriolis and Centrifugal), acting on the turbulence structures, so that the momentum transfer mechanism becomes increasingly complex. The simplest flow mode in this category is a pressure driven turbulent channel flow with arbitrary rotating vector, which can be decomposed into componential rotation vectors in the streamwise, the spanwise and the wall-normal directions. Among them, the spanwise and the streamwise rotating channel flows have been investigated by many authors (Johanson, et al., 1972), (Grundestam, et al., 2008) and (Oberlack, et al., 2006) to name only a few. However turbulent channel flow with wall-normal rotation has been rarely studied. Because of the important role played by Ekman type boundary layer in this flow, we shall later refer to it as Poiseuille-Ekman flow. Since there is no possible experimental approach to investigate these flows, direct numerical simulation (DNS) is the only available method to examine them. In order to study the ability of simple turbulence models to predict the wall-normal rotating channel flow, a reference data base for turbulent channel flow with wall-normal rotation with different Reynolds number at different rotation rates has been established by means of DNS of Navier-Stokes equations. In the Figure.1 the flow geometry is shown. No slip boundary condition at both walls and periodic boundary conditions are employed in the streamwise and the spanwise direction. Note that all variables are non-dimensionalized by the friction velocity in the non-rotating case ($U_{\tau 0}$) and the channel half height (h). For the entire analysis, the only rotation rate, i.e. Ω_2 or alternatively $Ro = \frac{2\Omega_2 h}{U_{\tau 0}}$ is to be varied and $Re_{\tau 0} = \frac{U_{\tau 0} h}{\nu}$ is the Reynolds number based on the friction velocity in the non rotating case.

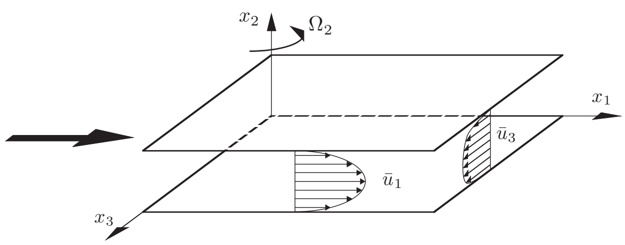

Figure 1: Sketch of the geometry of Poiseuille-Ekman flow

RESULTS AND DISCUSSION OF DNS

The numerical technique employed is a standard spectral method with Fourier decomposition in the streamwise and the spanwise directions and Chebyshev decomposition in the wall-normal direction. The numerical code for channel flow was developed at KTH/Stockholm (Lundbladh, et al 1992). Additional features such as wall-normal rotation have been added during the project. Series of DNS(s) have been conducted for two Reynolds numbers (180, 360). The numerical results indicate that the flow is very sensitive to the rotation. Even a very small rotation number can induce a strong secondary motion in the spanwise direction and reduce the streamwise mean velocity substantially. A linear region in $\overline{u_2' u_3'}$ profile can be observed, which is related to the existence of the mean flow in the spanwise direction. Due to an increase in the rotation rate, the streamwise mean velocity and turbulent kinetic energy decrease, but the spanwise mean velocity and $\overline{u_2'u_3'}$ first increase, reach their maxima and then decrease by further increase in the rotation rate, (Figure.$2-5$). It has also been established that by further increase of the rotation rate ($Ro > 0.091$ for $Re_{\tau 0} = 180$) and ($Ro > 0.072$ for $Re_{\tau 0} = 360$), the flow reaches another state, which is completely different compared to the fully turbulent state. This state is called quasi laminar and has not been discussed here.

Results from Chien turbulence model

To study the ability of relative a simple two-equation RANS model, which is based on the standard $k-\epsilon$, we have used the near wall model developed by Chien (Chien, 1982). In this model viscous corrections for $k-\epsilon$ model are devised to permit their integration through the viscous sublayer. The model for incompressible boundary layers is as following:

$$\frac{Dk}{Dt} = \nu_t \left(\frac{\partial \bar{U}_i}{\partial x_j}\right)^2 - \epsilon + \frac{\partial}{\partial x_j}\left[(\nu + \nu_t/\sigma_k)\frac{\partial k}{\partial x_j}\right] \quad , \quad (1)$$

$$\frac{D\widetilde{\epsilon}}{Dt} = C_{\epsilon 1} f_1 \frac{\widetilde{\epsilon}}{k}\nu_t \left(\frac{\partial \bar{U}_i}{\partial x_j}\right)^2 - C_{\epsilon 2} f_2 \frac{\widetilde{\epsilon}^2}{k} + E + \frac{\partial}{\partial x_j}\left[(\nu + \nu_t/\sigma_\epsilon)\frac{\partial \widetilde{\epsilon}}{\partial x_j}\right] \quad , \quad (2)$$

where the dissipation rate, ϵ, is related to quantity $\widetilde{\epsilon}$ by:

$$\epsilon = \epsilon_o + \widetilde{\epsilon} \quad .$$

The eddy viscosity is defined as:

$$\nu_t = \frac{C_\mu f_\mu k^2}{\widetilde{\epsilon}} \quad .$$

In Chien model five empirical damping functions, f_1, f_2, f_μ, ϵ_o and E, are defined as below:

$$f_\mu = 1 - e^{-0.0115 x_n^+} \quad ,$$
$$f_1 = 1 \quad ,$$
$$f_2 = 1 - 0.22 e^{-(Re_t/6)^2} \quad ,$$
$$\epsilon_o = 2\nu \frac{k}{x_n^2} \quad ,$$
$$E = -2\nu \frac{\widetilde{\epsilon}}{x_n^2} e^{-x_n^+/2} \quad ,$$
$$C_{\epsilon 1} = 1.35, C_{\epsilon 2} = 1.80, C_\mu = 0.09, \sigma_k = 1.0, \sigma_\epsilon = 1.0 \quad ,$$

where $Re_t = \frac{k^2}{\nu\epsilon}$ is turbulent Reynolds number and x_n is wall-normal coordinate. The simulations have been performed for identical Reynolds numbers as in the above DNS(s). The computational domain is $X_2 \in [-1, 1]$, since in turbulence modeling only a one-dimensional problem persists. The mesh consists of 128 cells. It should be mentioned that fully turbulent field in non-rotating case has been used as initial condition for rotating cases. All model computation have been conducted with the in-house computer code FASTEST based on a finite volume numerical method for solving the RANS equations on block-structured meshes. Comparison the results obtained from Chien model with the DNS data shows a good agreement between them (Figure.$6-8$). It means that the model is able to predict the turbulent wall-normal rotating channel flow. It should be noted that only in the case of wall-normal rotation (in contrast to the streamwise and the spanwise rotation), the Coriolis terms, which are the results of rotation appear directly in the mean flow equations. This can be seen easily in the following RANS (Reynolds averaged Navier-Stokes) equations, writing for an incompressible steady channel flow (there is no mean velocity in wall normal direction) in a wall-normal rotating reference frame:

$$-\frac{\partial \bar{P}}{\partial x_1} + \frac{1}{Re_{\tau 0}}\frac{\partial^2 \bar{U}_1}{\partial x_2^2} - \frac{\partial \overline{u_1'u_2'}}{\partial x_2} - Ro\bar{U}_3 = 0 \quad , \quad (3)$$

$$\frac{1}{Re_{\tau 0}}\frac{\partial^2 \bar{U}_3}{\partial x_2^2} - \frac{\partial \overline{u_3'u_2'}}{\partial x_2} + Ro\bar{U}_1 = 0 \quad . \quad (4)$$

CONCLUSION

The general purpose of the present investigation is first to explore the turbulent channel flow with wall-normal rotation using DNS and second to study the ability of relative simple turbulence model to predict the results. It has been established that the flow is very sensitive to the wall-normal rotation and is highly affected by a very small rotation rate. It has been shown that a relative simple turbulence model is able to predict the flow. It is possible that, since the Coriolis terms appear directly in the mean flow equations in the streamwise and the spanwise directions, relative simple turbulence models are able to capture the main effects of the wall-normal rotation.

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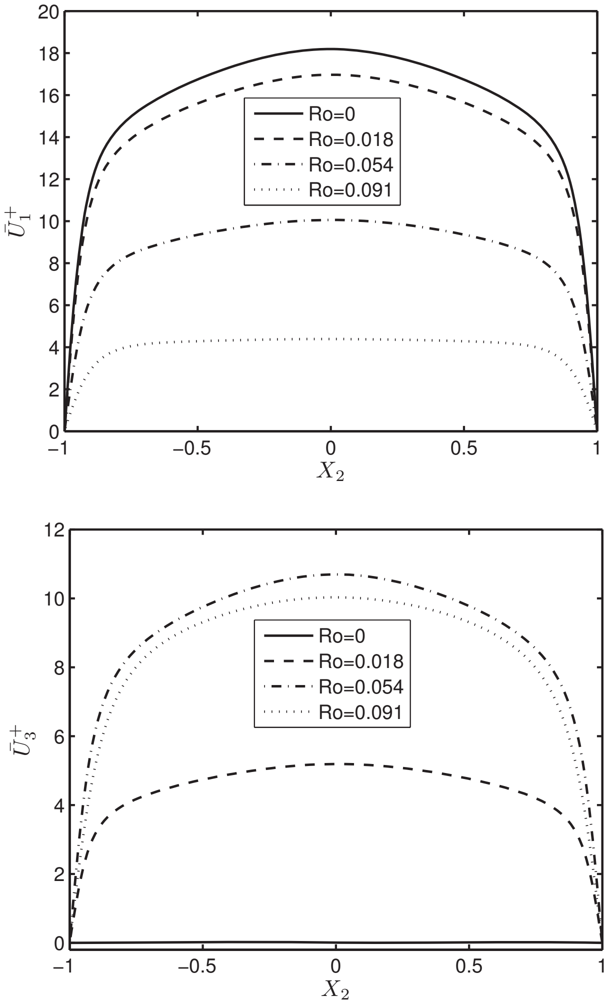

Figure 2: Mean velocity in the streamwise (upper) and in the spanwise (lower) directions at different rotation numbers for $Re_{\tau 0} = 180$ from DNS.

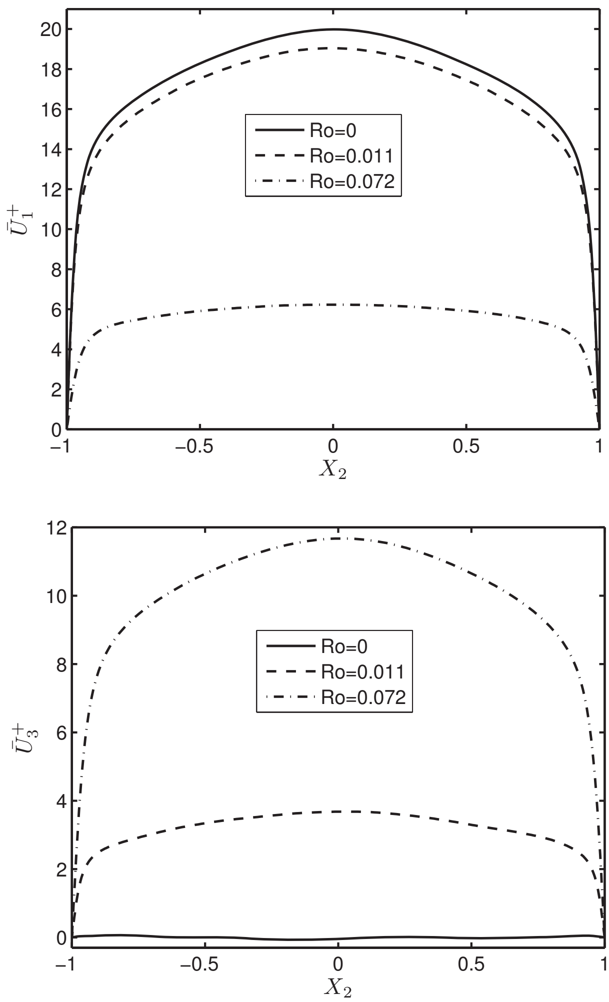

Figure 3: Mean velocity in the streamwise (upper) and in the spanwise (lower) directions at different rotation numbers for $Re_{\tau 0} = 360$ from DNS.

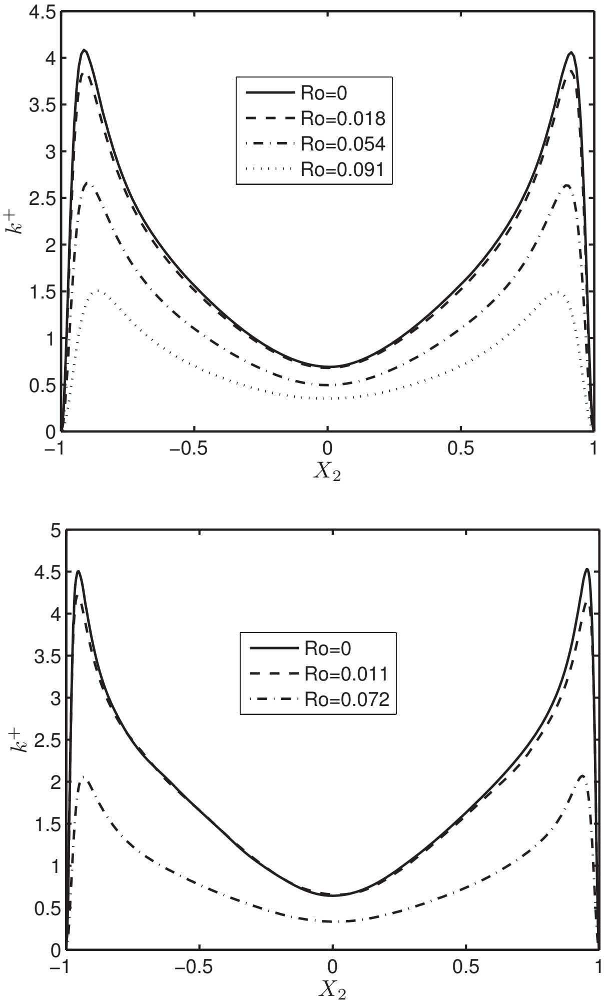

Figure 4: Turbulent kinetic energy at different rotation rates for $Re_{\tau 0} = 180$ (upper) and for $Re_{\tau 0} = 360$ (lower) from DNS.

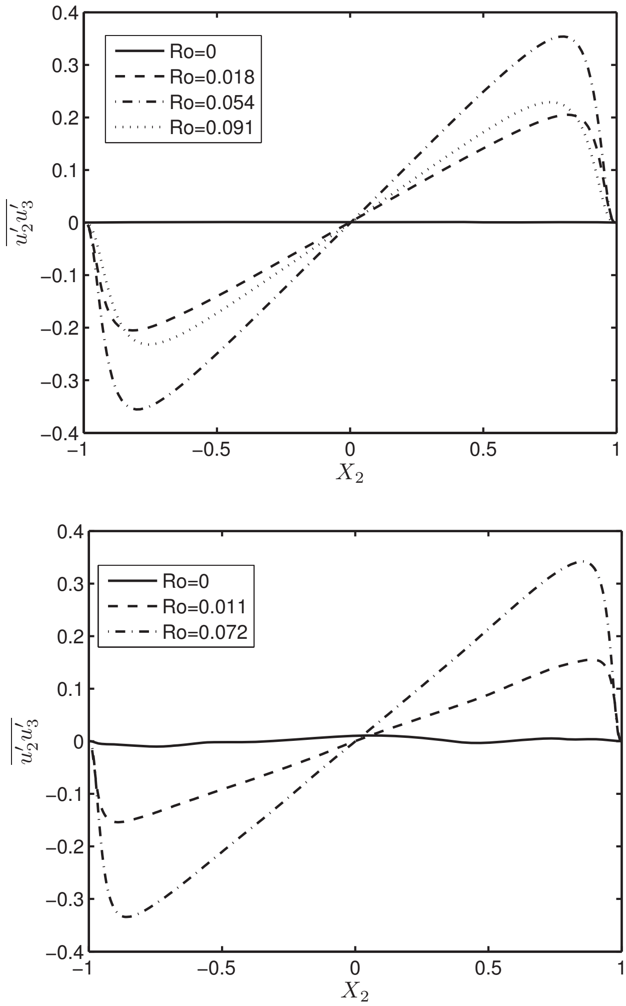

Figure 5: Shear stress $\overline{u_2'u_3'}$ at different rotation rates for $Re_{\tau 0} = 180$ (upper) and for $Re_{\tau 0} = 360$ (lower) from DNS.

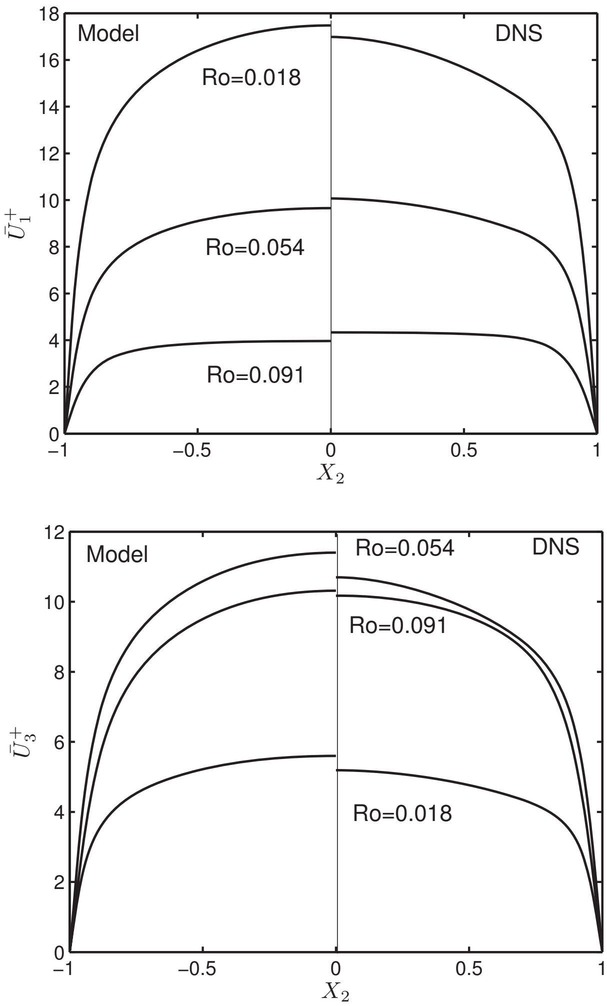

Figure 6: Comparison of the results obtained from Chien model for the streamwise (upper) and the spanwise (lower) mean velocity with DNS at different rotation rates for $Re_{\tau 0} = 180$ from DNS .

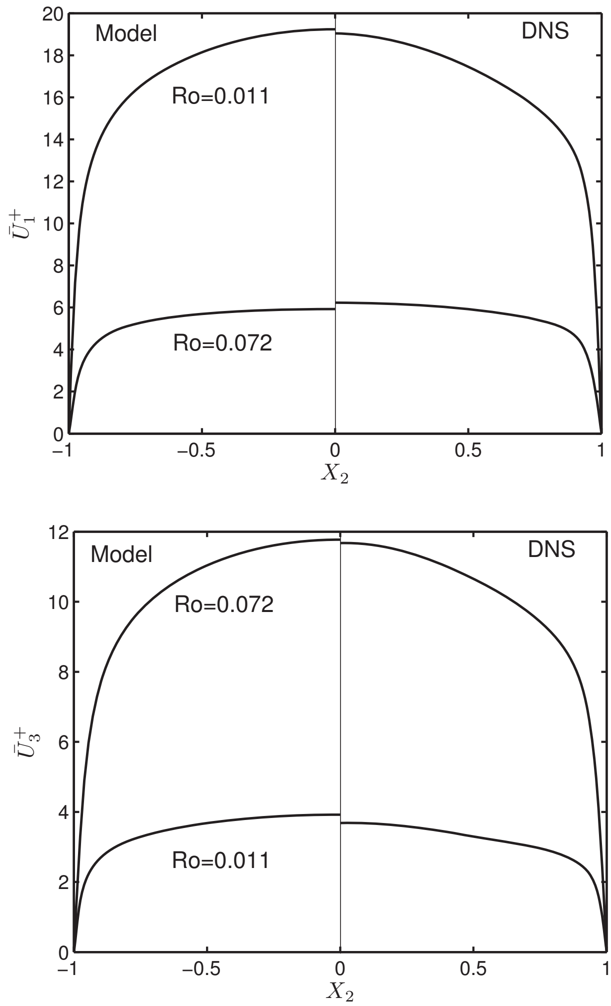

Figure 7: Comparison of the results obtained from Chien model for the streamwise (upper) and the spanwise (lower) mean velocity with DNS at different rotation rates for $Re_{\tau 0} = 360$ from DNS.

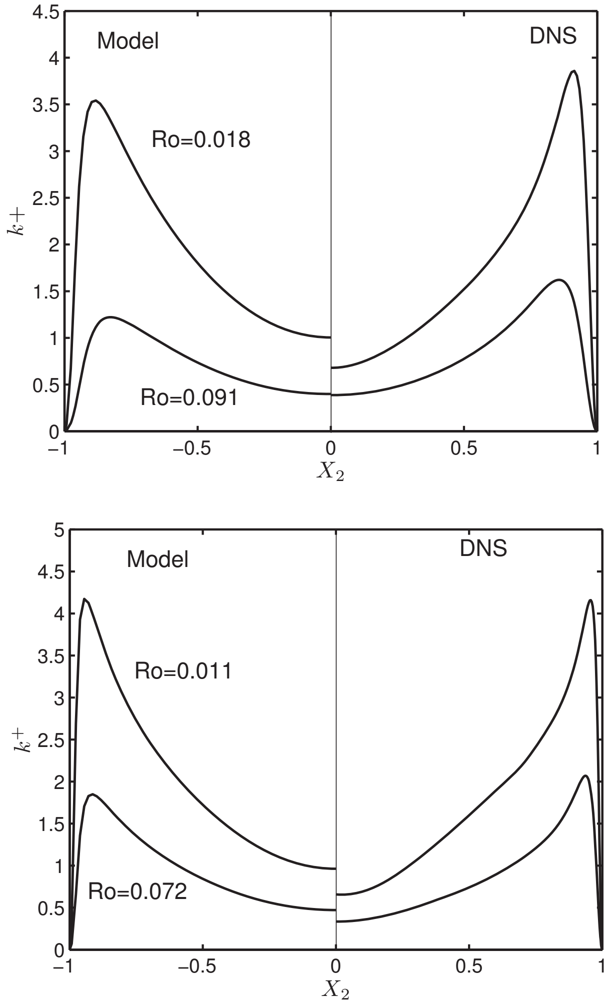

Figure 8: Comparison of the results obtained from Chien model for turbulent kinetic energy with DNS at different rotation rates for $Re_{\tau 0} = 180$ (upper) and for $Re_{\tau 0} = 360$ (lower) from DNS.