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A TWO-FLUID MODEL OF TURBULENT LIQUID-SOLID FLOW IN A HORIZONTAL CHANNEL

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ABSTRACT

This paper reports a prediction of fully-developed turbulent liquid-solid flow in a horizontal channel using a two-fluid model. The liquid phase is water while the solids phase consists of sand particles. The experimental measurements of Daniel (1965) of the mean mixture velocity and mean concentration are used to evaluate the numerical results. The two-fluid model of Bolio *et al.* (1995), originally developed for dilute gas-solid flows, was used to simulate the horizontal channel flow. The liquid-phase stresses were calculated using a low Reynolds number k - ε turbulence model, modified to include the effects of the particle phase. The solids-phase stresses were computed from a constitutive model based on the kinetic theory of granular flow; it includes a transport equation for the granular temperature, which represents the solids velocity fluctuations. Predictions are reported for fully-developed liquid-solid flows with mean bulk solids concentrations as high as 20 percent. Comparing the numerical predictions with the experimental data, it was observed that the mixture velocity profiles were in reasonable agreement, whereas the simulations failed to reproduce specific features of the measured concentration profiles, such as the location of the peak value. The simulations indicate that as the concentration in the lower region of the duct increases, the turbulence and related transport is almost completely suppressed. Further improvements in modeling, such as including the interstitial fluid effects while computing the solids-phase stress, are needed to improve the predictive capability of the two-fluid models for these relatively dense liquid-solid flows.

INTRODUCTION

Two-phase turbulent flows are relevant to many engineering and industrial applications including hydraulic transport of granular materials such as pulverised coal, and sediment transport in open channel flows. A granular flow model, which is analogous to the kinetic theory of molecular collisions, is often employed to account for the solids-phase stress based on inter-particle and particle-wall collisions. The Eulerian/Eulerian two-fluid formulation treats both phases as inter-penetrating continua with inter-phase interactions. Turbulence in the liquid-phase is typically calculated using a single-phase turbulence model modified to account for the effects of the particle phase on the fluid turbulence. Most of the existing two-fluid studies have investigated turbulent gas-solid flows in vertical ducts, e.g. Bolio *et al*. (1995) and Cao and Ahmadi (1995). Furthermore, the majority of these simulations were limited to dilute flows. For example, Bolio *et al.* (1995) compared their predictions for the phasic mean and fluctuating velocities with the experimental data of Lee and Durst (1982) and Tsuji *et al*. (1984) for different dilute flow conditions. A characteristic of these flows is that the mean solids velocity and concentration do not vary dramatically across pipe section, and the mean flow is symmetric with respect to the duct centreline.

In comparison, relatively fewer studies have considered liquid-solid flows e.g. Krampa-Morlu *et al*. (2004) and Hadinoto and Curtis (2004), or horizontal orientations, e.g. Cao and Ahmadi (2000). Note that unlike the case of the gas-solid flows described above, for dense liquid-solid flows, the solids volume concentration can become sufficiently high that one would reasonably expect the particles to significantly modify the instantaneous flow structures characterising the near-wall turbulence. This calls into question the validity of the single-phase turbulence models typically used in such studies, or at least suggests the need for more extensive analysis of their performance in such flows.

The original motivation for the present research program was the need to develop improved models for the dense coarse-particle slurries used to transport oil sands. The thesis research of Yerrumshetty (2007) investigated gas-solid and liquid-solid turbulent flows, including both vertical pipe and horizontal channel geometries. For the case of the horizontal channel, two-fluid model predictions for the mixture velocity and the solids concentration profiles were compared with the experimental data of Daniel (1965). This paper will specifically report a subset of these results, which are based on the application of the two-fluid model formulation developed by Bolio *et al*. (1995). Although it would be surprising if this model, which was developed for gas-solid flows, could capture all of the distinctive features of liquid-solid flows, it is useful to first identify the limitations of existing two-fluid model formulations before proposing further modifications.

MATHEMATICAL MODEL

The numerical model adopted involves solving six coupled partial differential equations to predict the velocities of the liquid and solid phases, the liquid-phase turbulence kinetic energy and its dissipation rate, the solids-phase granular temperature, and solids volume fraction. For steady fully developed flow in a horizontal channel, with x and y denoting the horizontal and vertical directions, respectively, the transport equations may be written as follows:

Fluid phase:

Momentum -

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\left(\mu+\mu_t\right)\frac{\partial u}{\partial y}\right)-\beta\left(u_f-u_s\right) \qquad (1)$$

Turbulence kinetic energy -

$$0=\frac{\partial}{\partial y}\left\{\left(1-c_s\right)\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right\}+\left(1-c_s\right)\mu_t\left(\frac{\partial u_f}{\partial y}\right)^2 - \rho_g\varepsilon\left(1-c_s\right)-\beta\left(2k-\overline{u'_{fi}u'_{si}}\right) \qquad (2)$$

Dissipation rate -

$$0=\frac{\partial}{\partial y}\left\{\left(1-c_s\right)\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right\} + C_{\varepsilon 1}\frac{\varepsilon}{k}\left(1-c_s\right)\mu_t\left(\frac{\partial u_f}{\partial y}\right)^2 - C_{\varepsilon 2}f_2\rho_g\left(1-c_s\right)\frac{\varepsilon^2}{k}-C_{\varepsilon 3}\beta\left(2k-\overline{u'_{fi}u'_{si}}\right)\frac{\varepsilon}{k} \qquad (3)$$

Particle-turbulence interaction –

$$-\overline{u'_{fi}u'_{si}}=\sqrt{2k}\sqrt{3T} \qquad (4)$$

Particle phase:

Momentum -

$$0=\frac{\partial}{\partial y}\left(\frac{\partial \sigma_{xy}}{\partial y}\right)+\beta\left(u_f-u_s\right) \qquad (5)$$

$$0=\frac{\partial}{\partial y}\left(\sigma_{yy}\right)-\rho_s c_s g \qquad (6)$$

Granular temperature -

$$0=-\frac{\partial}{\partial y}\left(\Gamma\frac{\partial T}{\partial y}\right)-\sigma_{xy}\frac{\partial u_s}{\partial y}-\gamma+\beta\left(\overline{u'_{fi}u'_{si}}-3T\right) \qquad (7)$$

In these equations, the flow parameters are as follows: u is the mean (phasic) velocity in the x-direction, p is the fluid phase pressure, μ is the viscosity, ρ is the density, g is the gravity force, β is the drag coefficient, c_s is the solids volume fraction (or concentration), k is the turbulence kinetic energy of the fluid phase and ε is its dissipation rate, σ_{xy} and σ_{yy} are components of the particle stress tensor, T is the granular temperature, Γ is a diffusion coeffient for the granular temperature and γ is its dissipation rate. The subscripts f and s are used to denote the fluid and solid phases, while t indicates a turbulent property. The quantity $\overline{u'_{fi}u'_{si}}$ represents the correlation between the fluid and solids phase velocity fluctuations, and is calculated here using the model of Sinclair and Mallo (1998). The k – ε model adopted is the low Reynolds number version of Myong and Kasagi (1990), modified in a somewhat *ad hoc* manner to include a turbulence modulation term. The eddy viscosity for the fluid phase is then calculated from the value of k and ε and includes a wall damping function. Otherwise, all of the model coefficients and underlying constitutive relations for the solids phase stress tensor derived from kinetic theory are given in the paper by Bolio (1995). Substitution of the constitutive model relation for σ_{yy} into equation (6) for the vertical (or wall-normal) momentum balance for the solids phase results in an equation which is then solved for the solids volume fraction. This approach for determining c_s from a momentum balance opposed to a mass conservation equation is special to the case of fully-developed flow.

A no-slip boundary condition was specified for the fluid-phase at both walls. The turbulence kinetic energy was set to zero at the wall, while the dissipation rate was calculated to balance the net diffusion of k to the wall. The boundary condition given in Bolio *et al*. (1995) was used for the solids-phase velocity, which effectively implements a finite slip condition at the wall. It is derived from the balance between the momentum flux due to particle-wall collisions and the particle stress at the wall. The wall boundary condition used for the granular temperature in Bolio *et al*. (1995) was also implemented in the horizontal flow at both walls. It is derived from a balance between the energy transfer to the wall due to particle-particle collisions and the

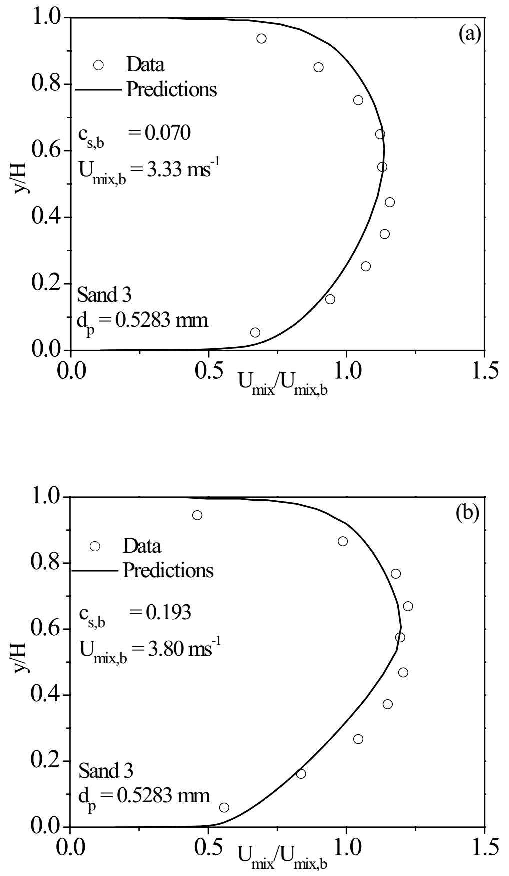


Figure 1: Effect of concentration on mixture velocity profiles; experimental values from Daniel (1965).

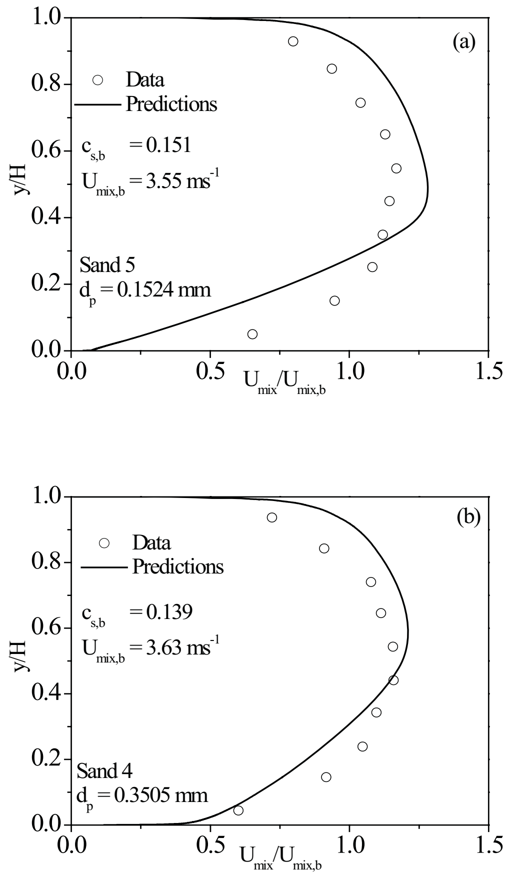


Figure 2: Effect of particle diameter on mixture velocity profiles; experimental values from Daniel (1965).

net production of energy by particle-wall interactions. The particle-particle and particle-wall collisions are characterised by coefficients of restitution of $e = 0.94$ and $e_w = 0.7$, respectively. The specularity coefficient of the particle collisions with the wall was given by $\varphi = 0.002$. Although the form of the transport equations given above is greatly simpified for fully-developed flow, the couplings remain very complex and still present substantial challenges for numerical solutions.

SOLUTION METHOD

The finite volume technique was used to discretise the transport equations for the one-dimensional solution domain considered. The channel cross-section was meshed with a non-uniform grid using 60 control volumes. Further grid refinement was observed to not significantly change the solution fields. A Tri-Diagonal Matrix Algorithm (TDMA) was used to solve the transport equations, and a pseudo-transient solution method was implemented, i.e. the solution was iterated with a false time step until a converged steady state solution was obtained. Typically the normalised residuals of the discrete equations were reduced to a value of 10^{-4} or less. In the overall solution procedure, the value of the solids volume fraction at the top wall of the channel and the pressure gradient were supplied as initial values, and the solution process was iterated until the correct values were obtained for the bulk solids concentration and mixture velocity.

SUMMARY RESULTS

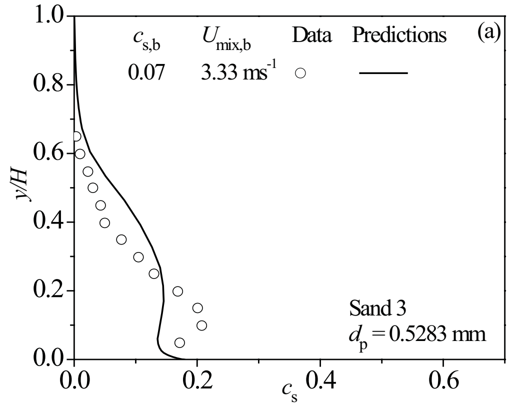
Fully-developed turbulent liquid-solid flow in a horizontal channel was analysed in terms of the predictions for the mixture velocity and means solids concentration profiles. The predictions were compared with the experimental measurements of Daniel (1965), who studied the transport of sand particles in a horizontal channel flow of water. The mixture velocities were measured by the use of a special "flow divider" device placed at different heights above the channel bed. The flow divider separated the flow into two streams, and the value of the mixture velocity was determined by measuring the volume flow rate of the lower stream at different heights above the channel bed. The local solids concentration was measured using a gamma-ray densitometer. In this paper, the base case relates to transport of water and sand particles of diameter d_p = 0.5382 mm and density ρ_s = 2632 kg m^{-3} (referred to as Sand 3). Our numerical study also considered the flow of particles of smaller diameter, i.e. d_p = 0.3505 mm and d_p = 0.1524 mm (referred to as Sand 4 and Sand 5, respectively.)

For comparison with the experimental values, the mixture velocity U_{mix} is calculated as follows based on the predicted properties for each phase:

$$U_{\text{mix}} = u_{\text{f}}\left(1 - c_{\text{s}}\right) + u_{\text{s}} c_{\text{s}} \tag{8}$$

Figure 1a compares the predicted and measured mixture velocity profiles for a bulk solids concentration of 0.070 and bulk mixture velocity of 3.33 ms^{-1} for the Sand 3 particles. Figure 1b presents the corresponding profiles for a higher bulk solids concentration of 0.193 and a similar bulk mixture velocity of 3.80 ms^{-1}. For the experimental data, the mixture velocity profile only becomes strongly asymmetric at the higher bulk solids concentration. In contrast, the numerical model predicted noticeable asymmetry for both concentrations, with a peak value located slightly above the centreline of the channel. Near the top wall, the predicted mixture velocity was consistently larger than the experimental value. The measured values closest to the wall may be less accurate since use of the flow divider device in this region is likely problematic. Figures 2a and 2b, show the predictions for the mixture velocity profiles for the Sand 5 and Sand 4 particles, respectively, at similar bulk velocities and higher bulk concentrations. As the particle diameter decreases, the steep gradient predicted in the lower region of the channel becomes even more pronounced.

The experimental and predicted mean solids concentration profiles for Sand 3 corresponding to the mixture velocity profiles given in Figures 1a and 1b are presented in Figure 3a and 3b, respectively. As would be expected, the experimental data indicates that the value of the mean concentration steadily increases with depth, with a more rapid increase in the lower region of the channel. Note that there is a peak value near (but a finite distance above and away from) the bottom wall. The predictions show a somewhat different behaviour. In general, the predicted concentration also increases with depth, but more quickly than for the experimental data. It reaches a maximum value closer to the centre of the channel and exhibits a more uniform profile over the bottom region of the channel. Finally, it exhibits a sharp peak at the bottom wall. Figure 4 presents the mean concentration profiles for the two smaller particle diameters at bulk mean concentrations close to those in Figure 3a. As the particle diameter is reduced, the measured concentration profile tends to become more uniform. For the smallest particle diameter (Figure 4a, Sand 5) finite concentration values extend all the way up to the top wall of the channel. In contrast, the model predicted negligible values for the particle concentration in this region for both of the two smaller particle diameters. Another observation based on the measured profiles is that as the particle size decreases, the peak value of the mean concentration moves closer to the bottom wall. For the smallest particle (Figure 4a, Sand 5), the peak value is located at the wall. In contrast, the concentration profiles predicted by the two-fluid model continue to exhibit peak values away from the wall as well as a sharp peak at the wall itself.



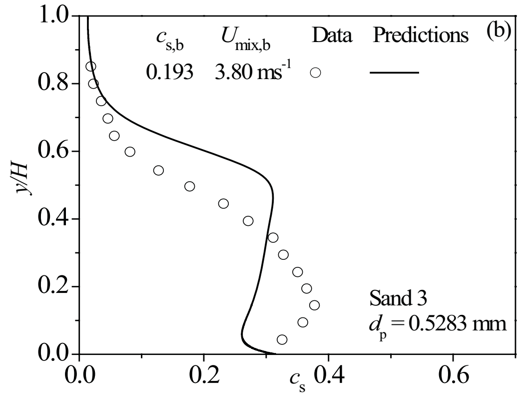


Figure 3: Mean solids concentration profiles; experimental values from Daniel (1965).

The predictions for the profile of the granular temperature for both bulk solids concentrations considered in Figure 1 for Sand 3 are presented in Figure 5. Unfortunately, no experimental measurements were available for comparison. Recall that the granular temperature is defined in terms of the solids phase fluctuating velocity, i.e.

$$T = \overline{u_s' u_s'} / 3 \tag{9}$$

As such, it is a measure of the fluctuating kinetic energy associated with the particles, and therefore to some degree analogous to the fluid phase turbulence kinetic energy (to be discussed below.) For both concentration values, the profile for T peaks near the walls, with a lower peak value in the bottom region. Increasing the bulk mean solids concentration is observed to suppress and enhance the granular temperature in the lower and upper parts of the channel, respectively. These profiles show much more variation than was observed in the predictions for the case of both gas-solid and liquid-solid flow in a vertical pipe (Yerrumshetty, 2007).

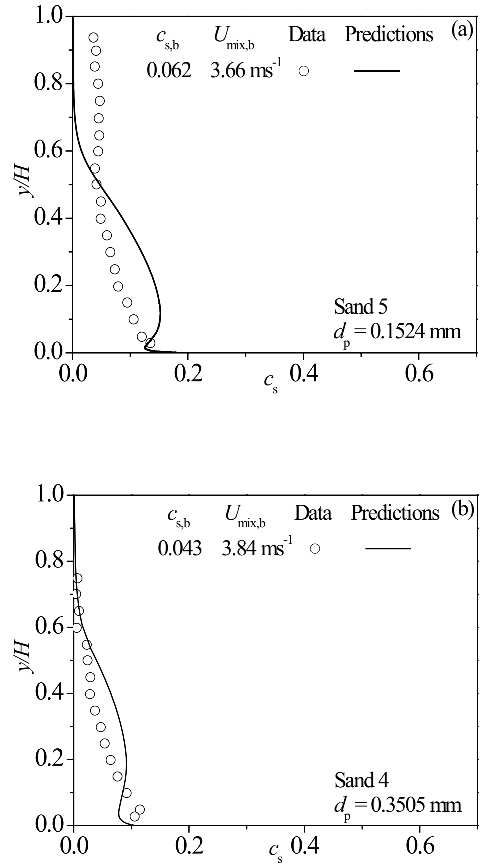


Figure 4: Effect of particle diameter on the mean solids concentration profiles; experimental data from Daniel (1965).

The predictions for the turbulence kinetic energy for the Sand 3 particles at two different bulk concentrations are presented in Figure 6. The profiles for both bulk concentration values retain the shape of the single-phase turbulence kinetic energy profile near the upper wall, although the dimensionless peak value is reduced to approximately one-half the value of the single phase flow ($k/u_\tau^2 \cong 4$). In the lower part of the channel, where the particle concentration is much higher, the peak in the k profile is blunter and relatively higher values of k extend into the flow. The effect of increasing the particle concentration is to reduce the level of k in the bottom half of the channel.

Finally, some representative profiles for the turbulent (or eddy) viscosity of the fluid phase (normalised by the kinematic molecular viscosity) are presented in Figure 7. In this case, the predictions pertain to the case of the smallest particle diameter, d_p = 0.1524 mm (Sand 5). For reference, the profile for the single phase flow is also shown. In the upper half of the channel, the effect of particles at the lowest value of the bulk concentration is to first decrease the peak profile below the single phase value. Thereafter, the peak value recovers as the bulk concentration increases.

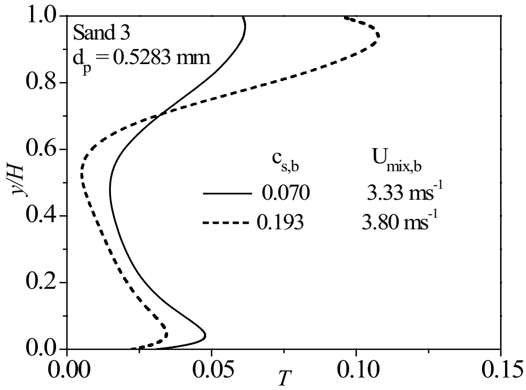


Figure 5: Prediction for granular temperature.

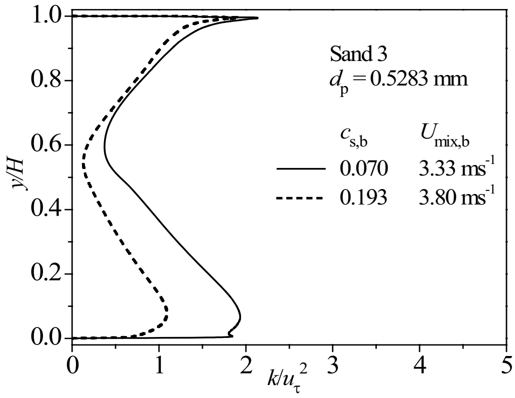


Figure 6: Predictions for turbulence kinetic energy.

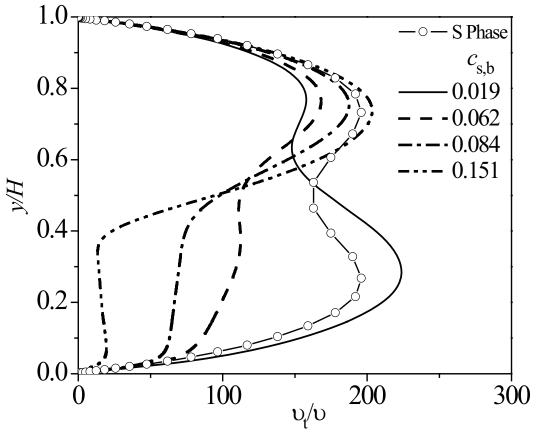


Figure 7: Effect of concentration on fluid phase eddy viscosity.

In the lower half of the channel, adding particles to the flow has quite the opposite effect. At the lowest value of the bulk concentration considered, the peak value of the eddy viscosity is initially enhanced above that of the single phase.

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Thereafter, as the bulk concentration is increased, the peak value of ν_t/ν is dramatically reduced, such that for the highest concentration the turbulent transport is almost entirely suppressed.

CONCLUSIONS

This paper reports the results of applying a two-fluid model developed for dilute turbulent gas-solid flow to the case of dense liquid-solid flow of relatively coarse particles in a horizontal channel. In this case, gravity creates a highly heterogeneous concentration field. Based on comparisons to the experimental data of Daniel (1965), some conclusions are as follows:

1. The effect of particle size on the mean mixture velocity profile is successfully reproduced by the model, although the degree of asymmetry is greater than that exhibited by the data.

2. The dramatic increase in mean solids concentration with depth shown by the data is captured. However, the specific shape and location of the peak value is not accurately reproduced by the model, partly due to an erroneous peak value at the bottom wall of the channel.

3. The granular temperature profile exhibits much more variation than in a vertical pipe, with peak values located near both walls.

4. The turbulence kinetic energy is in general reduced in the lower region of the channel at higher concentrations. The eddy viscosity for the fluid phase is likewise greatly diminished in the lower region at higher solids concentrations.

It appears that many features of the two-fluid model developed for gas-particle flows are also relevant to liquid-solid flows. One aspect that warrants further development is the model for the solids volume fraction, which must be capable of handling flow regions where the particle concentrations are sufficiently high to strongly inhibit the fluid motion and significantly suppress the fluid turbulence.

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