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HYBRID RANS-LES APPROACHES FOR THE PREDICTION OF TURBULENT HEAT FLUX

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ABSTRACT

In the engineering applications, various types of thermal boundary conditions can be imposed on the velocity field. It is thus necessary to obtain the practically important quantities, such as the turbulent heat flux and the heat transfer coefficient. Because of some success, it is still believed that the most reliable prediction methods are those based on the LES approaches. However, at high Reynolds numbers, LES is considered too CPU-demanding for complex industrial applications. Therefore computations based on a RANS model in the near wall regions, and on LES in some other regions, where explicit computation of the large scale structure is required, are needed as a hybrid RANS-LES(HRL) computation. This paper reports on application of PITM(Partially Integrated Transport Model), the seamless HRL, to the simulation of heat transfer of non-equilibrium flow. In order to reproduce the sub-grid scale eddy viscosity, PITM are based on v^2-f model for the sub-grid scale eddy and temperature field is analyzed by eddy diffusivity model. The proposed models are validated against the DNS data of the fully-developed turbulent channel flow with uniform heat flux.

INTRODUCTION

In the numerical analysis of flow, it would be an important issue to predict the effect of wall region properly. However, this region could not generate sufficient grids to simulate a desired volume of vortex flow and various wall functions have been developed to overcome this problem. Of these, the Reynolds Averaged Navier-Stokes (RANS) method allows obtaining a satisfactory result by applying an elliptic equation while considering the influence of wall. As for the types of RANS model, there are Elliptic Relation Model (ERM) and Elliptic Blending Model (EBM). Recently, these elliptic approach methods have been applied in the calculation of passive scalar amount. Shin et al. have applied EBM to the transport equation for the prediction of turbulent heat flux and obtained a good result in the analysis of non-rotating and rotating channel and square duct flow. This study intends to analyze the heat transfer by using the HRL that utilizes ERM.

Since HRL an analysis method of economic and efficient turbulent flow can be an alternative of LES, this method has been studied in various directions. Of these, the Partially Integrated Transport Model (PITM) refers to the method that has modeled the high frequency energy by integrating the turbulence energy spectrum through modeling turbulent flow smaller in size than grid and simulating other turbulent flows. On the other hand, Hanjalić et al. have conducted a study of flow regarding the influence of buoyancy or Lorentz force by using the concept of Time-dependent Reynolds Average Navier-Stokes (T-RANS). T-RANS as the method used by dividing the turbulent stress into deterministic part and modeled part could be regarded as a similar method of PITM. Although T-RANS as the one that has applied the time averaged concept on the entire spectrum region shows a large effect when the flow is greatly influenced from an external force, it is not appropriate for the elaborate reproduction of abnormal flow due to its size limitation of eddy that can be reproduced. Kenjereš et al. have applied slightly modified PITM to the flow analysis of high Rayleigh number and obtained very satisfactory result in comparison to the results of LES and T-RANS.

PITM not only explains well about the turbulent energy theory but also makes easy of application in RANS by using the eddy dissipation transport equation modified slightly from the RANS equation. Currently, the $k-\varepsilon$ model (Schiestel et al., 2005) and RSM (Reynolds Stress Model) (Chaouat et al., 2005; Fadai-Ghotbi et al., 2007) have been developed while basing on the RANS model and this study has developed a model by using the $\overline{v^2}-f$ model. The $\overline{v^2}-f$ model as an eddy viscosity model of using elliptic relaxation can be applied widely.

This study treats the fully-developed low Reynolds number channel flow and heat transfer that holds the uniform heat flux on the surface of a wall by using HRL. The turbulent channel flow with heat transfer has the Reynolds number (Re_τ) of 395 and Prandtl number (Pr) of 0.71 on the basis of friction velocity and channel half width. We have compared the result with the DNS result of Moser et al.(1999) and Kawamura et al.(1999).

MATHEMATICAL MODELS

Velocity Field

PITM has the advantage that can reproduce a large eddy in the non-equilibrium flow holding a grid larger than LES in a way that models subgrid scale eddy by using RANS and reproduces the turbulent flow larger in size than the grid.

First, the instantaneous velocity (U_i) is divided as in the Eq. (1).

$$U_i = \langle U_i \rangle + u_i' + u_i'' = \overline{U_i} + u_i'' \tag{1}$$

Here, $\overline{U_i}$ represents the spatially averaged velocity, $\langle U_i \rangle$ the time averaged velocity, u_i' the large scale fluctuation, and u_i'' the subgrid scale fluctuation. And for all the components, $^{-}$ and $\langle\ \rangle$ refer to the average values for space and time.

In order to model the sub-grid scale turbulence, the governing equation that has applied $\overline{v^2} - f$ model is shown as in the following. Since the original $\overline{v^2} - f$ model of Durbin can cause numerical instability in the wall boundary condition of f-value, we have used the Lien-Durbin Model (LDM) developed by Lien et al. (1996) in order to prevent this.

$$\frac{Dk_{sgs}}{Dt} = P - \varepsilon + \left(\nu + \frac{\nu_{sgs}}{\sigma_k}\right)\frac{\partial^2 k_{sgs}}{\partial x_j \partial x_j} \tag{2}$$

$$\frac{D\varepsilon}{Dt} = \frac{C_{\varepsilon 1}P - C_{sgs\varepsilon 2}\varepsilon}{T} + \left(\nu + \frac{\nu_{sgs}}{\sigma_\varepsilon}\right)\frac{\partial^2 \varepsilon}{\partial x_j \partial x_j} \tag{3}$$

$$\frac{D\overline{v_{sgs}^2}}{Dt} = k_{sgs}\overline{f} - 6\varepsilon\frac{\overline{v_{sgs}^2}}{k_{sgs}} + \left(\nu + \frac{\nu_{sgs}}{\sigma_{\overline{v^2}}}\right)\frac{\partial^2 \overline{v_{sgs}^2}}{\partial x_j \partial x_j} \tag{4}$$

$$\overline{f} - L_{sgs}^2 \frac{\partial^2 \overline{f}}{\partial x_j \partial x_j} = \frac{C_{sgs1} - 1}{T}\left(\frac{2}{3} - \frac{\overline{v_{sgs}^2}}{k_{sgs}}\right) + C_2\frac{p}{k_{sgs}} + 5\varepsilon\frac{\overline{v_{sgs}^2}}{k_{sgs}} \tag{5}$$

Here, k_{sgs} represents the subgrid scale turbulent kinetic energy and $\overline{v_{sgs}^2}$ represents the scalar amount corresponding to the subgrid scale wall-normal Reynolds stress component. ν_{sgs} as the subgrid scale eddy viscosity is identical to $C_\mu \overline{v_{sgs}^2} T$ and P is $2\nu_{sgs}\overline{S_{ij}}\,\overline{S_{ij}}$ by the production of spatially averaged velocity field. $\overline{S_{ij}}$ is the strain rate tensor.

The characteristic time scale T and length scale L_{sgs}^2 have applied a limiter as in the Eq. (6) and Eq. (7). This as a prevention measure for the stagnation point anomaly is given by $2k \geq \overline{u^2} \geq 0$ and is also effective in the channel flow. Application of such limiter in this analysis blocks the flow from being laminarized by preventing ν_{sgs} from getting large.

$$T = \min\left(\max\left(\frac{k_{sgs}}{\varepsilon}, C_T\sqrt{\frac{\nu}{\varepsilon}}\right), \frac{0.6k_{sgs}}{\sqrt{6}\,\overline{v_{sgs}^2}C_\mu|S|}\right) \tag{6}$$

$$L_{sgs} = C_L \max\left(\min\left(\frac{k_{sgs}^{3/2}}{\varepsilon}, \frac{k_{sgs}^{3/2}}{\sqrt{6}\,\overline{v_{sgs}^2}C_\mu|S|}\right), C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}\right) \tag{7}$$

Here, $|S| = \sqrt{\overline{S_{ij}}\,\overline{S_{ij}}}$.

The expression proposed by Chaouat et al. (2007) was used as the spectral model for $C_{sgs\varepsilon 2}$. That is,

$$C_{sgs\varepsilon 2} = C_{\varepsilon 1} + \frac{k_{sgs}}{k_{tot}}\left(C_{\varepsilon 2} - C_{\varepsilon 1}\right) \tag{8a}$$

where,

$$\frac{k_{sgs}}{k_{tot}} = \frac{1}{\left(1 + \beta_N N_c^3\right)^{2/9}} \tag{9}$$

The Eq. (9) as the model obtained by integrating a spectrum curve is theoretically better in the limiting behavior with RANS on the surface of a wall than the model (Schiestel, 2005) computed by the Kolmogorov law. Where, β_N is an empirical constant and $N_c = \kappa_c L = \kappa_c K y$ is the dimensionless cutoff wave number. κ_c represents $\pi/(\Delta x \Delta y \Delta z)^{1/3}$, K the Von Karman constant, and y the vertical distance from the wall.

C_{sgs1} a coefficient for the slow part of redistribution term in the subgrid scale turbulent stress transport equation was prepared to complement the properties of return-to-isotropy from the center of the channel that the property of LES gets large by multiplying C_1 to the Eq. (10) as in the existing PITM model.

$$C_{sgs1} = \frac{(1 + \alpha_N N_c^2)C_1}{(1 + N_c^2)} \tag{10}$$

Here, α_N also is an empirical constant.

The boundary conditions for respective values at the wall are $k_{sgs} = \overline{v_{sgs}^2} = 0$, $\lim_{y \to 0} \varepsilon = 2\nu k_{sgs} / x_n^2$ and $\overline{f} = 0$. x_n represents the vertical distance from the wall.

Temperature Field

The governing equation for the computation of spatially averaged temperature field is shown as in the Eq. (11).

$$\frac{D\overline{\Theta}}{Dt} = \frac{\partial}{\partial x_j}\left(\frac{\nu}{\Pr}\frac{\partial \overline{\Theta}}{\partial x_j} - \tau_{\theta i}\right) \tag{11}$$

In case of the Eddy diffusivity model, the turbulent heat flux $\tau_{\theta i}$ is shown as in the Eq. (12). Here, Pr the Prandtl number is 0.71 and $\Pr_t$ the turbulent Prandtl number is 0.9 in RANS and 0.4 in LES. We use the new $\Pr_t$, Eq. (13) with Eq. (9) to varify the value continuously between RANS and LES region.

$$\tau_{\theta i} = -\frac{\nu_{sgs}}{\Pr_t}\frac{\partial \overline{\Theta}}{\partial x_j} \tag{12}$$

$$\Pr_t = 0.4 + \frac{k_{sgs}}{k_{tot}}(0.9 - 0.4) \tag{13}$$

$\tau_{\theta i}$ by the algebaic heat flux model is shown as in the Eq. (14). Here, τ_{ij} is the turbulent stress tensor with the value of $2/3 k_{sgs} - 2\nu_{sgs}\overline{S_{ij}}$.

$$\tau_{\theta i} = -C_\theta\left(\tau_{ij}\frac{\partial \overline{\Theta}}{\partial x_j} + \tau_{\theta j}\frac{\partial \overline{U_i}}{\partial x_j}\right)T \tag{14}$$

The constants used in the expressions above are shown on the Table 1.

Table 1: Constants for modeling of subgrid scale kinetic energy and turbulent heat fluxes

C_μ	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_1	C_2
0.22	$1.4\left(1 + 0.05\sqrt{k_{sgs} / \overline{v_{sgs}^2}}\right)$	1.9	1.4	0.3
C_T	C_L	C_η		
6.0	0.23	70		

Table 2: Details for channel flow simulation

Domain	L_x	L_y	L_z
-	4	2	4
Grid	N_x	N_y	N_z
1	16	64	32
2	32	84	64

NUMERICAL METHOD

The model developed in this study has done a numerical analysis for the channel flow of Re_τ=395 and the size and number of grids are shown on the Table 2. Chaouat et al. (2005). In order to solve each governing equation, finite volume method is used. As for the grid that is based on a staggered method, a uniform grid was used in the streamwise and spanwise direction and the stretching grid was used in the wall-normal direction. In case of Grid-1, the first grid is positioned at $y^+ \approx 0.5$ from the wall. We have used the fractional step method for the analysis of pressure field. Time discretization has used the 2nd order Adams-Bashforth method for the convection term of momentum conservation equation and the 2nd Crank-Nicolson method for the diffusion term. And, the 2nd Crank-Nicolson method was used for the transport equation of each turbulent property. As for the spatial discretization, the 2nd order central difference scheme was used.

SIMULATION OF FULLY TURBULENT CHANNEL FLOW

The PITM of using $\overline{v^2} - f$ model was tested in the channel of Re_τ=395 that holds Grid1 and Grid2 as shown on the Table 2. Figure 1 has shown the averaged flow direction velocity of channel flow that was analyzed by using DNS, LES, RANS and HRL (present). LES has used Smagorinsky model the simplest model. The Van Driest Damping was considered on the wall and the Smagorinsky constant is 0.1. As shown on the figure, we could see that the LES of using coarse grids in the log layer has shown a considerable difference from the DNS result. The Smagorinsky model requires a constant to be used in the computation of mixing length when predicting the eddy viscosity and researchers tend to use somewhat different values. Also since the scope of applying damping function can vary by the case, its application is not easy. Although there is the dynamic model that complements these disadvantages, this is also influenced by the shape of grid.

Modelling of Subgrid Scale Turbulence

As you can see in the expressions above, we need two empirical constants α_N and β_N in order to model the subgrid scale turbulent kinetic energy. Here, this study has used $\alpha_N = 1.5$ and $\beta_N = 35.0$ ($\beta_\eta = \beta_N / a_K^{9/2} \approx 0.162$, $a_K = 3.3$) which are quite similar to the values used by Chaouat et al. (2007). Here when β_N holds a very small value, it will have the result similar to the one that has analyzed the laminar flow by using RANS. We could know empirically that the theoretical value $\beta_N = 5.5$ ($\beta_\eta \approx 0.026$) presented by Chaouat et al. (2007) corresponds to the boundary value of β_N that the LES of using this spectral model can be expected. Figure 2 shows $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ and $C_{sgs\varepsilon 2}$. In the $\overline{v^2} - f$ model, $C_{\varepsilon 1}$ is obtained by the ratio of $k/\overline{v^2}$ as shown on the Table 1. This is to obtain an appropriate value of k in the free shear flow as well as in the boundary layer. However as shown on the Figure 2, it holds a very large value of $C_{\varepsilon 1}$ on the wall and accordingly, the value of $C_{\varepsilon 2} - C_{\varepsilon 1}$ gets very small. Therefore, a very small value of subgrid turbulent kinetic energy in this region is modeled.

Figure 3 shows the turbulent kinetic energy and resolved part of turbulent stresses that were simulated by using Grid1. The total turbulent kinetic energy is computed by $k_{tot} = k_{res} + k_{sgs}$ and $k_{res} = (\overline{u_i'^2})/2$. The subgrid scale turbulent kinetic energy around the wall is shown on the Figure 3.

Comparison of Simulation on Grid1, Grid2

Figure 4 shows the turbulent kinetic energy by using Grid2. Although the result shows a slight difference from the result of DNS, this show a relatively accurate result as compared to the value of using Grid1 (Figure 3). Figure 5 - Figure 8 show the results that have compared the values of normal stresses by large scale fluctuation, ε^+, and $\overline{f}^+$ by HRL respectively with the resulting values of RANS or DNS. On the Figure 5, the resolved part of stress of streamwise direction has held a considerably large value on Grid1 and the large total and resolved turbulent kinetic energy in Figure 3 was induced by the result. ε^+ in the Figure 6 shows that the resulting value of HRL is considerably smaller than that of DNS. The ε value obtained through HRL is the subgrid scale eddy dissipation rate ε^+_{sgs}. This is because the turbulent kinetic energy in the spectral domain is balanced with the value of turbulent kinetic energy that is delivered to the turbulent flow of higher frequency in the low frequency field.

Figure 7 shows the result that the turbulent flow close to the wall is modeled by the subgrid eddy viscosity ν^+_{sgs}. The value is very small since the overall small values of k_{sgs} and $\overline{v^2_{sgs}}$ are modeled by selecting a very large β_N. $\overline{f}^+$ showing the value corresponding to the redistribution term for $\overline{v^2_{sgs}}$ on the Figure 8 shows the characteristics of LES as it gets to the channel center. Since the isotropy property gets strong, we could see that the result of HRL becomes considerably larger than the result of RANS. This is attributable that C_{sgs1} in the center on the Eq. (10) is made about 1.5 times larger than C_1.

Temperature field

Temperature field is analyzed by the eddy diffusivity model and the moderate choice of turbulent Prandtl number is a challenging problem. We use 0.4, 0.7, 0.9 and the new Pr_t by Eq. (13). Figure 9 shows the mean temperature profiles with various Pr_t on Grid1. Each result has the slight difference with DNS data.

CONCLUSIONS

This study has developed the PITM of using $\overline{v^2} - f$ model and has performed a numerical analysis in the channel flow. In order to obtain an appropriate value of turbulent kinetic energy on the wall, the $\overline{v^2} - f$ model gets the coefficient of production of eddy dissipation by using the ratio of turbulent kinetic energy and normal stress scalar quantity. At this time, the normal stress scalar quantity becomes quite small due to the anisotropic characteristics of turbulent stress and the coefficient of production of eddy dissipation gets very large. Therefore, very small subgrid scale turbulent kinetic energy can be modeled. However, it has an advantage of realizing LES with a smaller number of grids than various LES modeling techniques that have too many variables for the flow to be analyzed.

The heat transfer has been analyzed by eddy diffusivity model with various turbulent Prandtl number. But each constant for Pr_t has the limiation in order to analyze the exact temperature field. The study on this part is to be performed continuously and is remained as an assignment.

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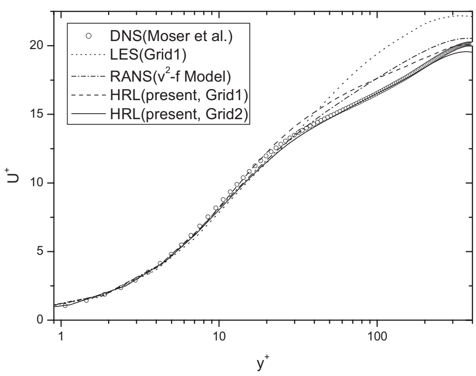

Figure 1: Mean velocity profile

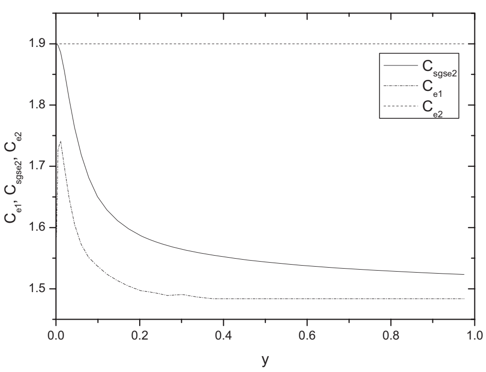

Figure 2: Coefficients for ε-equation

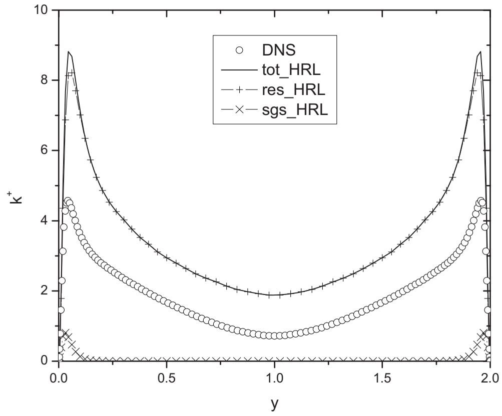

Figure 3: Total, resolved and subgrid scale turbulent kinetic energy of turbulent channel flow simulation on Grid1

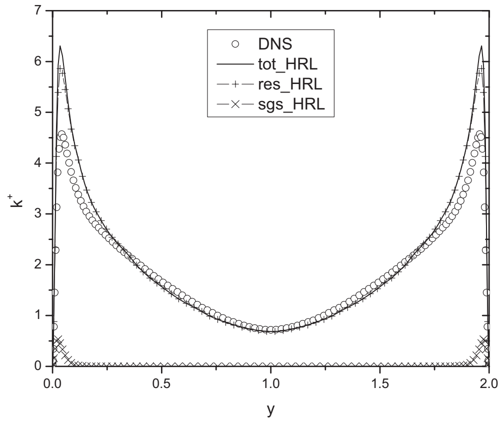

Figure 4: Total, resolved and subgrid scale turbulent kinetic energy of turbulent channel flow simulation on Grid2

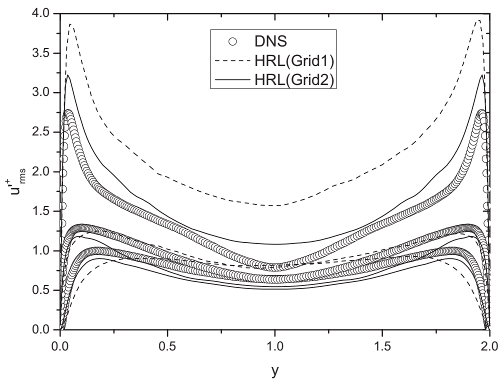

Figure 5: Resolved part rms of normal stresses of turbulent channel flow simulation

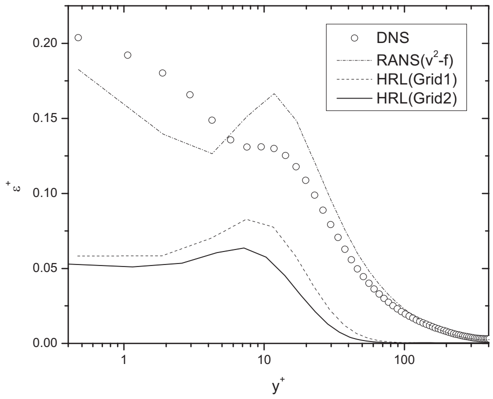

Figure 6: ε+ profiles of turbulent channel flow simulation

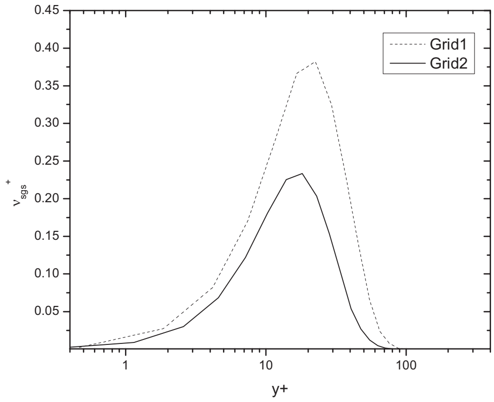

Figure 7: ν_{sgs}^{+} profiles of channel flow simulation by HRL on Grid1, Grid2

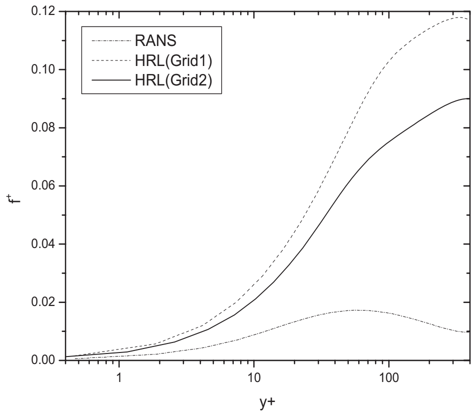

Figure 8: $\overline{f}^{+}$ profiles of turbulent channel flow simulation

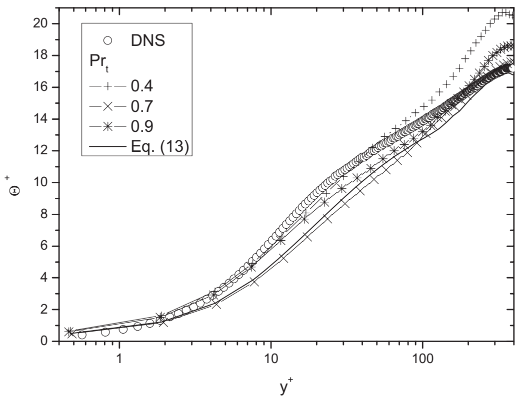

Figure 9: Mean temperature profile