DEVELOPMENT OF A MINIATURE PROBE FOR VELOCITY-PRESSURE CORRELATION MEASUREMENT

Yoshitsugu Naka, Shinnosuke Obi and Shigeaki Masuda
Department of Mechanical Engineering,
Keio University
Yokohama, 223-8522, Japan
y09774@educ.cc.keio.ac.jp

ABSTRACT

A novel technique for simultaneous measurements of fluctuating velocity and pressure is proposed which combines a miniature total pressure probe and an X-wires hot-wire anemometer. The basic performance of the present method is investigated in detail, such as the spatial resolution, effect of the angle of attack and frequency response. The effect of the interference of probes is examined as well. The correlation between the fluctuating pressure and velocity in transverse as well as streamwise directions has been measured in a two-dimensional turbulent mixing layer. The results show partly good agreement with those obtained by other techniques, although substantial improvements are found to be necessary.

INTRODUCTION

Simultaneous measurements of fluctuating velocity and pressure in turbulent flows have long been one of challenging problems in experimental fluid mechanics. Most of the difficulties come from the intrusive nature of the measurement technique for fluctuating pressure at arbitrary locations inside the flow. Various types of methods have been proposed in the past to measure fluctuating pressure; e.g., a shrouded condenser microphone by Fuchs (1972), a bleed-type probe by Spencer(1970), a probe based on fluctuating lift principle by Elliot (1972) and a "Quad-Disc" probe by Nishiyama (1991). However, the number of reports on velocity-pressure correlation measurement is only few.

A pioneering work was done by Kobashi (1957) who evaluated the contribution of pressure diffusion in the wake of a circular cylinder. Based on his work, Shirahama and Toyoda (1993) developed a tiny static pressure probe for accurate measurement of pressure fluctuation, and the possibility of simultaneous measurement of velocity and pressure was demonstrated. This technique was successfully extended to several applications. Iida et al. (1999) investigated aerodynamic sound source in the wake of a circular cylinder. Tsuji and Ishihara (2003) measured pressure spectra and the PDF in a turbulent jet. Tsuji et al. (2005) also evaluated velocity-pressure correlation in a turbulent boundary layer.

In our previous study (Naka et al., 2006), a miniature static pressure probe according to Shirahama and Toyoda (1993) and an X-wires hot-wire probe were applied to measure fluctuating velocity and pressure in a turbulent mixing layer. The individual terms in the transport equation for Reynolds stress, including pressure related terms, were directly measured in the turbulent mixing layer close to self-similar state and compared with DNS. It was found, however, that there was certain limit in the spatial resolution of the combined probe, and its application was limited to statistically two-dimensional flows.

In the present study, we choose another method where total and dynamic pressure measurements are combined; the idea is similar to that in previous studies by Giovanangeli (1988) and Nasseri and Nitsche (1991). The novelty in the present study lies in a better spatial resolution accomplished by using an extremely thin pipe for the pressure probe; a miniature probe has been manufactured by means of precision machining, and combined with an X-wires hot-wire probe. The applicability of this technique to the evaluation of velocity-pressure correlation is addressed.

MEASUREMENT TECHNIQUE

Instrumentation and data processing

The instruments used in the present study were common to those used in (Naka et al., 2006). In this study, a total pressure probe shown in Fig. 1 was employed in addition to a static pressure probe whose performance was investigated by Omori et al. (2003) and Naka et al. (2006). An extremely thin pipe with the inner- and outer-diameters being 0.4mm and 0.5mm, respectively, was used, and its tip was rounded in order to minimize the disturbance on fluid flow and to achieve a high spatial resolution. A condenser microphone (RION UC-29) was attached to the end of the probe.

The instantaneous static pressure is related to total and dynamic pressure by

$$\hat{p}_t = \hat{p} + \frac{1}{2}\rho\hat{u}^2 \tag{1}$$

where $\hat{p}_t$ and $\hat{p}$ are instantaneous total and static pressure, respectively. ρ is fluid density and $\hat{u}$ stands for the instantaneous streamwise velocity. Since condenser microphone only senses the fluctuation, it is necessary to extract the relationship for fluctuating component from Eq. (1). By applying the Reynolds decomposition to pressure and velocity in Eq. (1), and extracting the fluctuating part, one obtains an expression for the fluctuating total pressure p_t:

$$p_t = p + \frac{1}{2}\rho(2Uu + u^2 - \overline{u^2}) \tag{2}$$

where lower and upper cases stand for fluctuating and mean quantities, respectively, and the over-bar denotes the time-averaging. Thus, the fluctuating static pressure p can be obtained by subtracting the dynamic pressure from the fluctuating total pressure p_t that is measured by a condenser microphone in the present study.

Calibration of yaw-angle effect

The flow direction varies in turbulent flow; hence, the sensitivity of the probe to the direction of the oncoming flow

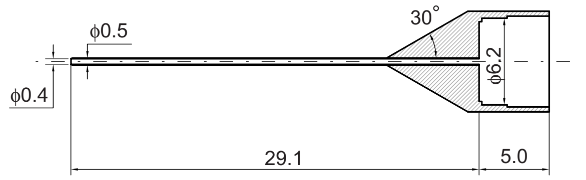

Figure 1: Schematic of the total pressure probe (dimensions in mm).

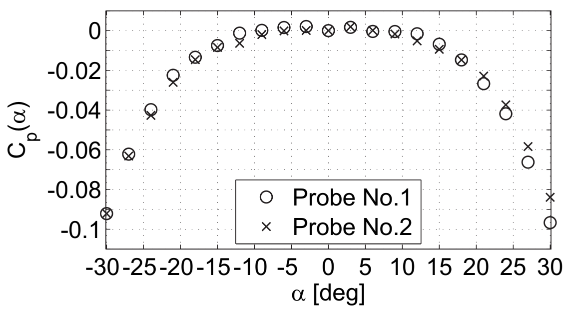

Figure 2: Effect of the angle of attack.

was investigated prior to the actual measurement using the condenser microphone. The total pressure probe was placed in a uniform flow, and the direction of the probe axis was varied relative to the flow direction. The pressure variation was measured by a low range pressure transducer (Validyne DP45-18) that was connected to the probe in place of the condenser microphone.

The total pressure coefficient C_p was calculated for various angles of attack α:

$$C_p(\alpha) = \frac{P_t(\alpha) - P_t(0)}{P_t(0)} \tag{3}$$

with P_t being the mean total pressure measured at various angles. The deviation of C_p from zero increased with the increasing α as illustrated in Fig. 2, though the pressure remained constant within 1% for -15 °$\leq \alpha \leq 15$ °.

This range of α corresponds to that of the instantaneous flow angle attack calculated by $\tan^{-1}(\hat{v}/\hat{u})$ in the middle of the turbulent mixing layer. It is thus expected that the measurements of total pressure are not much affected by the fluctuation of the flow direction in the present case.

Interference of probes

Next, the interference of the pressure and velocity probes was experimentally investigated. A series of simultaneous measurements of fluctuating total pressure and velocity was conducted with the pressure probe put in the center of the X-wires sensors (Dantec 55P54), as shown in Fig. 3. The effect of the probe interference was examined from the measurements where the total pressure probe was gradually moved in the direction of the probe axis.

The streamwise distance between the virtual crossing point of two wires of X-wires sensors and the tip of total pressure probe was defined as Δx, cf. Fig. 3. As shown in Fig. 4, the streamwise mean velocity U, Reynolds stresses $\overline{u^2}$, $\overline{v^2}$, $\overline{uv}$, fluctuating pressure $\overline{p_t^2}$, correlation of velocity and pressure, $\overline{up_t}$ and $\overline{vp_t}$, all varied against Δx. All variables were normalized by the reference value measured by each probe independently. The correlation $\overline{up_t}$ and $\overline{vp_t}$ were normalized by $\left(\sqrt{\overline{u^2}}\sqrt{\overline{p_t^2}}\right)$ and $\left(\sqrt{\overline{v^2}}\sqrt{\overline{p_t^2}}\right)$, respectively.

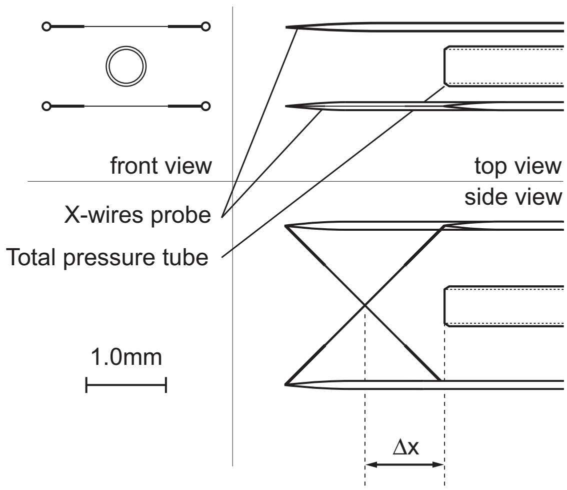

Figure 3: Arrangement of the total pressure probe and the X-wires hot-wire probe.

The streamwise mean velocity U decreases from unity to 0.97 as Δx decreases from $4D$ to $1.2D$, before increasing up to 1.02 at $\Delta x = 0$, where D is the outer diameter of the pressure probe. For reference, the axial variation of the velocity towards the stagnation is calculated by potential theory; two profiles are shown, one for the streamwise velocity along the stagnating streamline, and the other based on the velocity magnitude along a line that crosses the position of the hot-wire sensor. It is indicated that the velocity to be sensed by the hot-wire probe decreases as the pressure probe approaches to it, and an effect is visible for $\Delta x \leq 4D$. The difference between the measured value and the profile of potential theory may be explained by the heat loss from the hot-wire due to radiation.

Among the second-moments of fluctuation, $\overline{v^2}$ seems to be most sensitive to the probe interference, showing an increase at $\Delta x = 0$ by 10% relative to the value without probe interference. The profiles of $\overline{u^2}$ and $\overline{uv}$ show constant decrease as Δx decreases from $4D$ to $1.2D$ followed by an increase in $\Delta x \leq 0.8D$. On the other hand, $\overline{up_t}$ and $\overline{vp_t}$ vary more monotonically for entire range of Δx. It is obvious that $\overline{up_t}$ increases linearly with the decreasing Δx but the slope changes at $\Delta x \simeq 4D$. A similar change of the slope is also seen in $\overline{vp_t}$.

According to these observations, the effect of the probe interference is estimated to be on the order of 5% for turbulence statistics, when the probe distance is chosen for $2D \leq \Delta x \leq 4D$. In the subsequent measurements, the probe distance was set to $4D$.

Phase correction practice

For the measurement of correlation between fluctuating variables, the phase lag among their signal should be removed or at least its magnitude must be known. To this end, a few possible factors causing the delay of pressure signal are investigated. Here, the frequency response of the new pressure probe as well as the effect of the distance between the measurement locations of velocity and pressure is explored. It should be noted that the delay due to the electric circuit of the condenser microphone has been already addressed in our previous study (Naka et al., 2006), hence it is not repeated here.

The phase delay caused by the elastic response of the

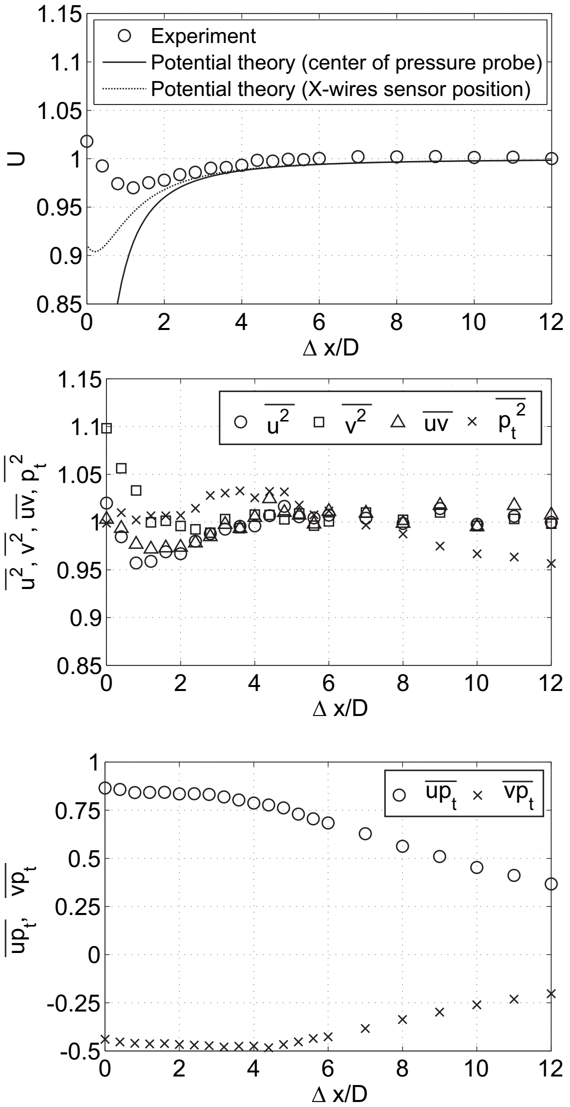

Figure 4: Interference of the probes. Mean streamwise velocity U (top); second moment of fluctuation (center), correlation between velocity and total pressure (bottom).

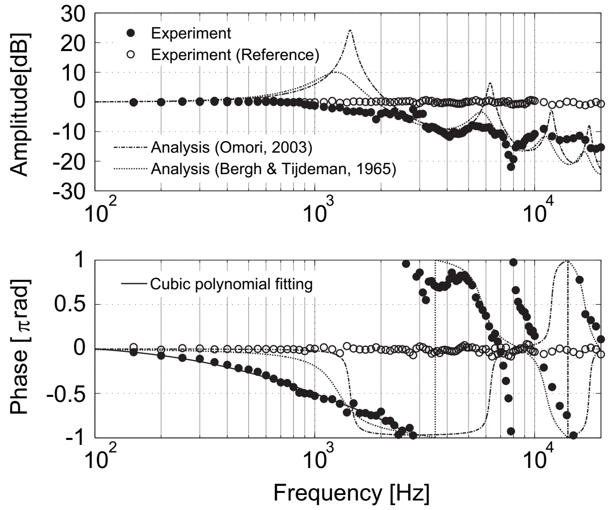

Figure 5: Frequency response of the pressure probe. Amplitude ratio (top); phase lag (bottom)

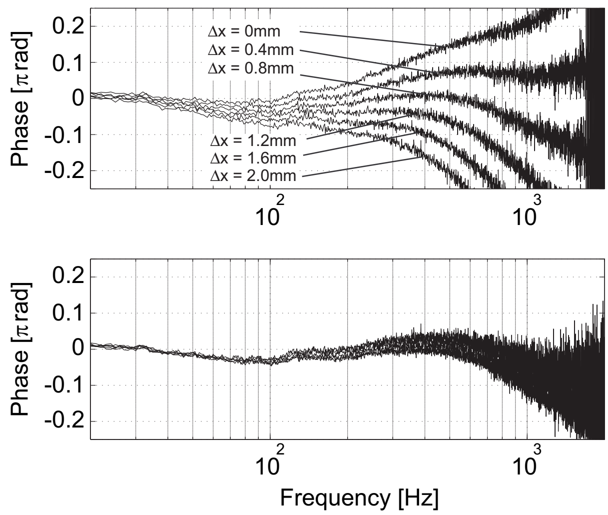

Figure 6: Phase lags due to probe distance. Un-corrected (top); corrected by Eq. (4) (bottom).

air inside of the pressure probe was analyzed by methods proposed by Omori et al. (2003) and also by Bergh and Tijdeman (1965), and the results are compared with the present experiment. The sound signal generated by a loudspeaker was measured by the condenser microphones with and without the pressure probe. The signal was varied from 100Hz to 20kHz and the measurements were undertaken in a quasi-anechoic box. The reference data taken without pressure probe exhibit the flat frequency response of the condenser microphone. The phase remains also unchanged for the entire frequency range, see Fig. 5. In contrast, mounting the pressure probe on the condenser microphone caused a gradual decrease in amplitude for frequency higher than 1kHz. There is an obvious phase difference for the entire frequency range examined here.

In order to correct the above-mentioned phase delay, the cubic polynomial fitting is introduced to approximate the experimental data for the range between 100Hz and 2.5kHz. On the other hand, the amplitude is not compensated because no significant attenuation is found for the frequency range that corresponds to the experiment in the turbulent flow which is below 1kHz in the present study.

Next, the phase delay related to the distance between the X-wires hot-wire sensor and the pressure probe is examined. We relate now such delay θ_d with the traveling distance, velocity, and frequency by

$$\theta_d = 2\pi \frac{\Delta x - \Delta x_0}{U} f \tag{4}$$

where Δx_0 denotes a reference distance.

In Fig. 6, phase delay is shown as a function of the frequency, with the probe distances Δx used as a parameter. We specify here Δx_0 to be 0.8mm ($1.6D$) at which the smallest phase lag is achieved. A reasonable correction is then made by the application of Eq. (4) as demonstrated in Fig. 6 (bottom). It is seen that the phase lag is reduced to less than 2% of one period for the frequency range of 20Hz $\leq f$ $\leq$ 600Hz.

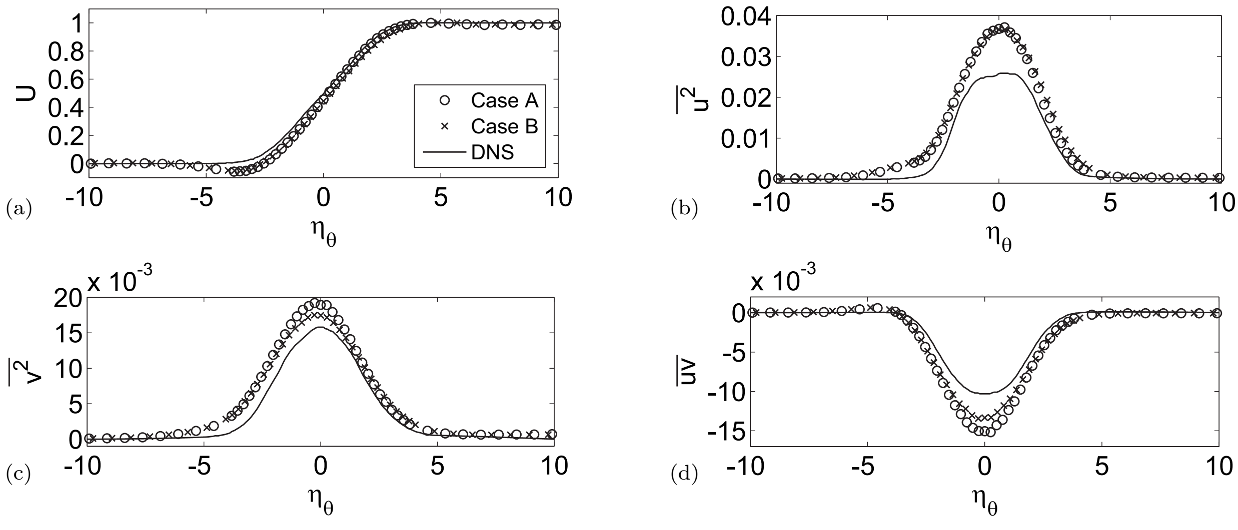

Figure 7: Profiles of fundamental statistics. (a) streamwise mean velocity U; second moment of velocity fluctuation: (b) $\overline{u^2}$, (c) $\overline{v^2}$, (d) $\overline{uv}$.

The assessment of the phase delay due to various factors is now accomplished. The results presented in the subsequent sections are all obtained after the phase correction practice introduced here are applied.

RESULTS AND DISCUSSIONS

Preliminary remarks

All measurements were conducted in a turbulent mixing layer at 100mm downstream from the edge of the splitter plate in the same wind tunnel used in our previous study (Naka et al., 2006). A slight modification was made in the inlet section of the wind tunnel to promote the development of oncoming turbulent boundary layers. The velocity ratio r ($\equiv U_l/U_h$) was 0.52, and other conditions of the inlet boundary layers were summarized in Table 1.

The orthogonal coordinate system was defined, originating from the trailing edge of the splitter plate, with the x-, y- and z-axes taken in streamwise, transverse and spanwise directions.

The mean velocity and Reynolds stress were normalized by the free stream velocity difference $\Delta U(\equiv U_h - U_l)$. The fluctuating pressure was normalized by the dynamic pressure based on the velocity difference, $\rho(\Delta U)^2$. The nondimensional transverse coordinate was defined as $\eta_\theta = y/\theta$, where θ stands for the momentum thickness of the mixing layer. In the present study, average value of θ was 2.29mm.

The spatial resolution was evaluated by the Kolmogorov length scale η_K which was estimated from the fluctuating streamwise velocity measurement under the assumption of local isotropy and Taylor's hypothesis. The inner diameter of the pressure probe corresponded to about $6\eta_K$, and the distance between the sensors of X-wires probe was about $15\eta_K$. The streamwise distance between the X-wires probe and the pressure probe Δx corresponded to about $30\eta_K$.

Mean velocity and Reynolds stress

The distribution of streamwise mean velocity U and Reynolds stress components $\overline{u^2}$, $\overline{v^2}$, $\overline{uv}$ are presented in Fig. 7. The results of DNS by Rogers and Moser (1994) are also plotted for comparison. The velocity deficit about 5% of ΔU was observed in the mean velocity profile, and slight asymmetry were found in the profiles of the Reynolds stress $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$. These features are common to our previous study (Naka et al., 2006).

The results are also compared between the measurements under the existence of the static pressure probe case-A and those with the total pressure probe case-B. The same sensor is used for the velocity measurement. The peak values of $\overline{v^2}$ in the case-A was larger than that of case-B by 9%. The difference in $\overline{uv}$ is as large as 12%. These differences are attributable to the different proximity effect of the pressure probes which is inevitable.

Total and static pressure fluctuation

The measurements of pressure fluctuation by two different pressure probes are now compared. The root mean square (RMS) values of the fluctuating total pressure are presented in Fig. 8. The comparison is made between the values measured directly by the total pressure probe, p_{tm}, and p_{te} that estimated by the sum of the static pressure measured by the static pressure probe and dynamic pressure measured by the hot-wire anemometry. The overall shape of the distribution is in good agreement with each other, though p_{tm} is significantly smaller than p_{te}.

One possible reason for the disagreement is the change in the response of condenser microphone: The full dynamic pressure loading on the diaphragm may have influenced the response in the case of the total pressure probe. This problem has been pointed out by Donaldson et al. (1971) who used the specially designed pressure probe for supplying the mean pressure to the back port of the condenser microphone in order to cancel the steady load of dynamic pressure. In the present case, however, such treatment is extremely difficult because of the small diameter of the pressure probe.

Provided that the value of p_{tm} suffers the unfavorable effect imposed on the sensor, an *ad hoc* correction factor is introduced here:

$$\Gamma = \frac{\sqrt{\overline{(p_m + p_{dm})^2}}}{\sqrt{\overline{p_{tm}^2}}} \quad (5)$$

Table 1: Inlet Conditions

	U [m/s]	Tu_f [%]	θ_b [mm]	Re	H
HSS	6.78	0.86	0.88	398	1.41
LSS	3.52	1.03	1.41	330	1.69

where p_m is the fluctuating static pressure that is directly measured by the static pressure probe, and p_{dm} is the dynamic pressure converted from the measured velocity data by hot-wire. p_{tm} stands for the total pressure that is directly measured by the total pressure probe. The values corrected by this Γ will be noted by the subscript c hereafter to identify "corrected" pressures.

The RMS values of fluctuating static pressure p_m, p_e and p_c are presented in Fig. 9. Both p_e and p_c are remarkably larger than p_m. The discrepancy between "measured" and "corrected" values indicates that the correction practice applied to p_{tm} does not work well. Possible reasons are, first, since the amplitude of p_m is considerably smaller than that of p_{tm} and p_{dm}, tiny noises may greatly affect the resultant static pressure. Second, the "true" total pressure signal may not be simply proportional to p_{tm}. In that case, Γ cannot be treated as a constant. Third, Γ contains some uncertainty because p_m and p_{dm} were measured at different positions.

Although a great deal of noise is still included in p_c, there is a certain effect found in representing the triple moments. In Fig. 10, the distributions of skewness factor of the fluctuating static pressure p_m, p_e and p_c are indicated. The correction procedure has provided a qualitatively different profile of p_c as compared to p_e, and the values of p_c are mostly negative across the shear layer as the distribution of p_m.

Among a limited number of studies that report on the statistics of the fluctuating pressure, Kim (1989) showed that the skewness takes negative values across the fully developed turbulent channel flow, and Tsuji and Ishihara (2003) reported that the probability density function of the static pressure inside the turbulent jet is negatively skewed. The present result of p_c and p_m are qualitatively consistent with these previous studies.

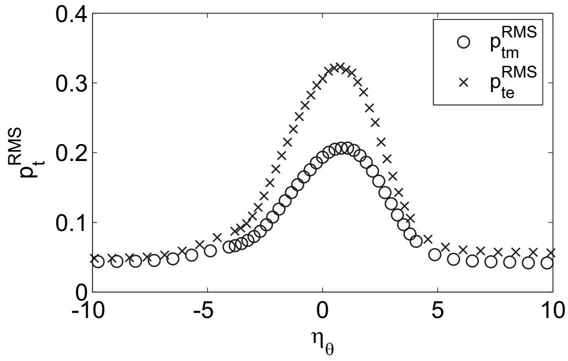

Figure 8: RMS values of the fluctuating total pressure.

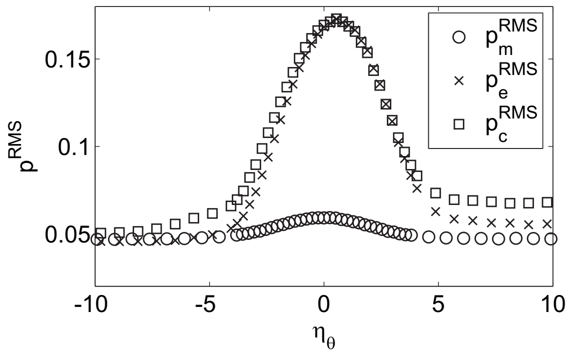

Figure 9: RMS values of the fluctuating static pressure.

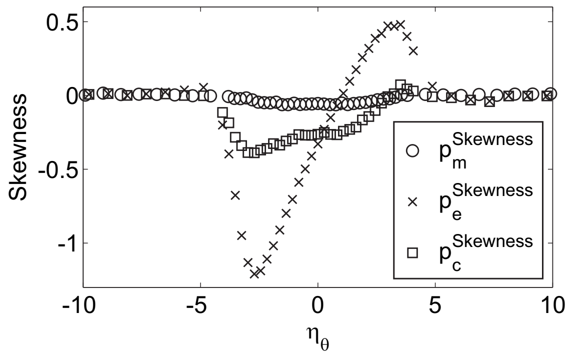

Figure 10: Skewness of the fluctuating static pressure.

Velocity-pressure correlation

The profiles of the velocity-pressure correlation are presented in Fig. 11. The comparison is made for the pressure values obtained in different manners. The distributions of $\overline{up_e}$ originating from the measurements by the total pressure probe have a large peak at the center of the mixing layer which is not similar to $\overline{up_m}$ that is measured by the static pressure probe. The value of $\overline{up_c}$ is closer to $\overline{up_m}$, indicating the correction to be somehow effective. The effect of the correction is more obvious when applied to $\overline{vp_e}$. It is seen in the profile that the corrected values $\overline{vp_c}$ have a similar distribution to that of $\overline{vp_m}$.

The consequence of the correction is now investigated for the $\overline{vp_c}$ component. According to Eq. (2), it is split into three parts:

$$\overline{vp_c} = \Gamma\,\overline{vp_{tm}} - \rho U \overline{uv} - \frac{\rho}{2}\overline{u^2 v} \tag{6}$$

The contributions of the terms on the right hand side are presented in Fig. 12. The first and second terms have opposite sign, but their magnitude are nearly the same. This indicates that the consequence of the correction by Γ is to balance these two terms, so that the velocity-pressure correlation becomes proportional to the triple correlation of the velocity. This happens to be reasonable because in simple shear flows the pressure diffusion is expressed by the turbulent diffusion (e.g., Lumley, 1978). However, this is merely a coincidence and the validity of the present technique should be assessed by further investigations.

CONCLUDING REMARKS

A novel technique for simultaneous measurement of fluctuating pressure and velocity is proposed. Its applicability is tested in a plane turbulent mixing layer through comparison with an already existing technique.

The fluctuating total pressure is measured by an extremely thin total pressure probe together with the velocity by an X-wires hot-wire probe. An indirect measurement of

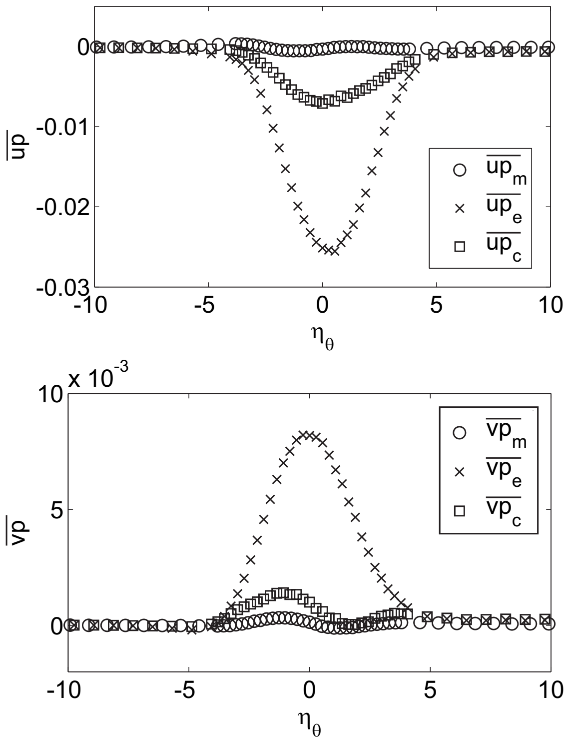

Figure 11: Velocity-pressure correlation: $\overline{up_{sm}}$, $\overline{up_{se}}$ and $\overline{up_{sc}}$ (top); $\overline{vp_{sm}}$, $\overline{vp_{se}}$ and $\overline{vp_{sc}}$ (bottom)

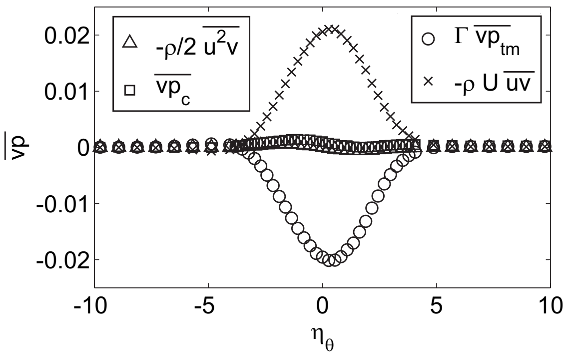

Figure 12: Balance of the terms in Eq. (6).

the fluctuating static pressure is made possible by combining these data. The fundamental performance of this method is examined, such as the spatial resolution, the effect of angle of attack, the probe interference as well as the frequency response.

The amplitude of fluctuating total pressure measured by the present technique is found significantly smaller compared with the values calculated as a sum of the static pressure and dynamic pressure measured by an already existing technique. A method to correct the measured total pressure is proposed based on this difference in RMS values of total pressure.

There are certain effects of the correction found in the statistics of the fluctuating static pressure, such as the skewness factor and the velocity-pressure correlation. However, further study is necessary to improve the accuracy of the present technique.

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