COMPARATIVE STUDY OF LAGRANGIAN EVOLUTION OF DISTURBANCES: VORTICITY VERSUS MATERIAL LINES.

Barak Galanti
Dmitri Gendler-Fishman
Alex Liberzon
Arkady Tsinober
Tel Aviv University
69978 Ramat Aviv, Israel
barakg@post.tau.ac.il
gendler/alexlib/tsinober@eng.tau.ac.il

ABSTRACT

The evolution of disturbances in turbulent flows is studied here using direct numerical simulations (DNS) and experimentally using three dimensional particle tracking velocimetry (3D-PTV) through a Lagrangian tracking of pairs of particles. The disturbances are presented as an evolving differences between the vorticity vectors of the initially closed particles and also infinitesimal material lines, attached to the particles. There is a striking dissimilarity of the evolution of active turbulent quantities, such as vorticity as compared to the passive objects such as material lines. The gained insight in the differences is important for progress in understanding of the mechanisms involved in genuine turbulence and evolution of passive objects.

INTRODUCTION

The usual premise of studying the evolution of disturbances is that in many cases one is unable to reproduce precisely the initial (and boundary) conditions. This approach takes its beginning from the predictability problem in meteorology (Holloway and West, 1984; Lorenz, 1985; Novikov, 1959), but is of more general importance (Born, 1958; Boffetta et al., 2002 and references therein) since *turbulence is a state of continuous instability* (Tritton, 1998). In other words, it is of interest to look at the processes of evolution of a disturbance of some flow realization **u** in a statistically steady state and corresponding quantities. Some aspects of this problem were addressed by Tsinober and Galanti (2003) in Euler setting. This included comparative study of disturbances of vorticity and passive vector (magnetic field). Another aspect of special interest is a comparative study of disturbances of vorticity and material lines. Such a study is by its very nature has to be performed in a Lagrangian setting. This is the main concern of the present report.

METHOD

We used both a DNS of NSE in a box with periodic boundary conditions and experimental approach enabling to access velocity derivatives. The solution of

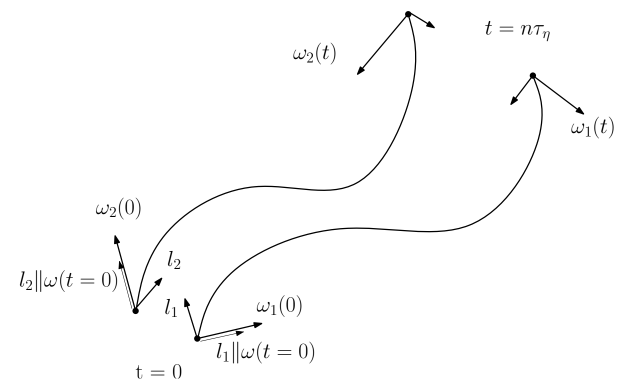

Figure 1: Schematic view of the problem: two initially closed points (at $t = 0$), their Lagrangian trajectories, and the vectors under investigation

the NSE is obtained using a pseudo–spectral method on a uniform grid. The time marching scheme is a second order Adams–Bashforth where the linear term (Laplacian) is computed exactly. The non–linear terms are computed in the real space while derivatives are found in the Fourier space. Using the information of the velocity field the Lagrangian dynamics of a fluid particle is obtained from the $d\mathbf{X}/dt = \mathbf{u}$, where $\mathbf{X}$ is the particle location, and $\mathbf{u}$ is the particle velocity. The time marching scheme is a 4th order Runge–Kutta and the intermediate values of the velocity field are found by interpolation using the field found by the Eulerian computations (Galanti et al., 2006). The experimental approach is based on the 3D-PTV system enabling access to the spatial velocity derivatives (i.e vorticity and strain), see Lüthi et al. (2005), Liberzon et al. (2006) and references therein.

Definitions

The emphasis in the present report is given to the comparison and study of the similarities and differences in the evolution of disturbances of vorticity and material lines in the Lagrangian setting. This done by defining

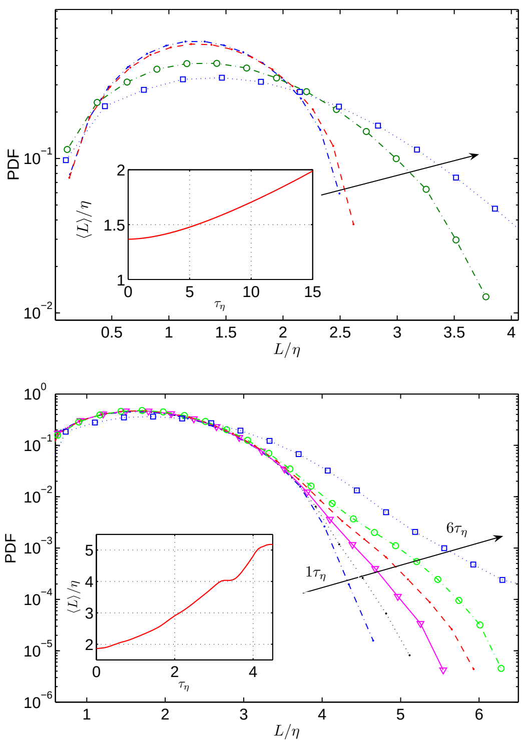

Figure 2: PDF of the distance between the two particles $L = \|\mathbf{L}\|$. Inset: average $\langle L\rangle$, normalized with the Kolmogorov length scale η. In all the Figures the top panel is DNS results and the bottom panel is 3D-PTV.

an initial disturbance of vorticity

$$\Delta\omega^0 = \omega_2^0 - \omega_1^0$$

for two initially close (compared with Kolmogorov length scale, η) fluid particles (say 1 and 2) having 'initial' vorticity values $\omega_1(t=0) = \omega_1^0$ and $\omega_2(t=0) = \omega_2^0$ and following the evolution in time of $\Delta\omega(t)$ along Lagrangian trajectories of the two particles. We compare the evolution of vorticity disturbance to those of the infinitesimal material lines. The infinitesimal material lines represent a passive vector field that evolves according to the following equation:

$$\frac{D\mathbf{l}}{Dt} = \mathbf{l}\cdot\nabla\mathbf{u}.$$

The evolution of passive vector field is compared here to the evolution of a genuine turbulent quantity of vorticity that evolves according to:

$$\frac{D\omega}{Dt} = \omega\cdot\nabla\mathbf{u} + \nu\nabla^2\omega.$$

Our definitions are shown schematically in Fig. 1.

We make two (intelligent) choices of material lines that provide better insight on the comparative behavior of disturbances. Thus, we select initially randomly oriented material elements and material elements that at $t=0$ coincide with the vorticity vector. In other words we study $\Delta\mathbf{l}$ such that $\Delta\mathbf{l}^0 = \mathbf{l}_2^0 - \mathbf{l}_1^0$, and *i)* random $\Delta\mathbf{l}^0$ and *ii)* $\mathbf{l}_{1,2}^0 = \omega_{1,2}^0$ (it means that $\Delta\mathbf{l}^0 = \Delta\omega^0$ with the proper units).

Subsequent following the evolution of $\Delta\omega$ and $\Delta\mathbf{l}$ along particle trajectories 1 and 2 (as well a number of related quantities relevant for the evolution of disturbances entering the corresponding equations for their evolution) provides the information which forms the basis for the main goal of this work as mentioned above.

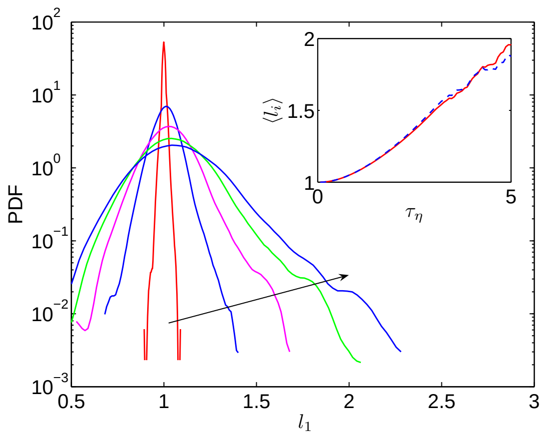

Figure 3: Typical time evolution of the infinitesimal material lines attached to a single particle shown as PDFs at different time instants. Inset shows the average growth of the material lines.

RESULTS

Distance between the particles

We would like to stress out that we study the evolution of disturbances comparing the respective quantities along the trajectories of **two**, initially close particles (otherwise called 'pairs'). At first, we emphasize that even at relatively low Reynolds numbers ($Re_\lambda = 50$ both in experiment and simulation) the genuine dispersive nature of turbulent flows is present, as it is shown in Fig. 2 in which the distance between the two particles is shown as probability density function (PDF) at different time instances and as an average (over all pairs) versus time. It is noteworthy that after short time of $\sim 5\tau_\eta$ (τ_η is the Kolmogorov time scale) the particles that initially distant $1 \div 2\eta$ (note the PDF at $\sim 0\tau_\eta$) separate to distances of tens of Kolmogorov length scales (with non-negligible probability) and on average to $\sim 7\eta$.

Evolution of infinitesimal material lines

Here we want to emphasize that in a stationary turbulent flow (in Eulerian sense, of course) the Lagrangian quantities, e.g. vorticity, rate-of-strain, are also stationary. This is not true for the two-point statistics, even for the short time interval under investigation

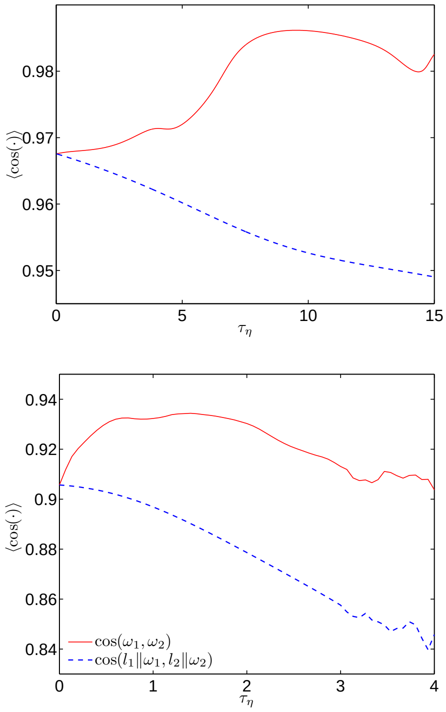

Figure 4: Time evolution of mean $\cos(\omega_1, \omega_2)$ (solid line) and $\cos(l_1, l_2)\|\omega$ (dashed).

(e.g. Fig. 2). Another example is the evolution of single-particle quantity shown in the following Fig. 3, in which the length of the vector $\mathbf{l}_1$ is presented as PDF and an average in the inset (the two overlapping lines emphasize that the definitions of $\mathbf{l}_1$ and $\mathbf{l}_2$ are interchangeable).

Relative evolution of quantities

The real reason of relative evolution being important will probably be seen in the following, but even a simple question of how long the two vectors (belonging to the initially two close fluid particles) are similar (or different from) each other deserves our curiosity. For example, we have learnt (e.g. Guala et al. 2005, among others) how different the material lines and vorticity in respect to their alignments (self- and related to other local vector fields such as strain eigenvectors). Of similar interest we are looking at the cosine of the angle between two vorticity vectors ω_1 and ω_2 used in our study of disturbances. This result is shown in Fig. 4. A noteworthy feature is the qualitative difference in the time evolution of the alignment between ω_1 and ω_2 and corresponding material lines which initially coincide with $\omega_1(t=0)$ and $\omega_2(t=0)$.

Evolution of disturbances

This, among other things, includes the comparative study of evolution of $(\Delta\omega)^2$ and $(\Delta\mathbf{l})^2$ (and $(\Delta s_{ij})^2 \equiv \Delta s_{ij}\Delta s_{ij}$) and a variety of quantities contributing essentially to their production and dissipation, alignments and other characteristics related to geometrical statistics. An example of first qualitatively similar results both from DNS and PTV experiments on evolution of disturbances of vorticity, strain and material lines with two choices of material elements mentioned above is shown in Fig. 6. Here too a noteworthy feature is the qualitative difference in the time evolution of the disturbances of $\Delta\omega$ (and strain rate, Δs) as compared to the disturbance of not only for for random choice of material lines Δl but especially for the material lines $\Delta l\|\omega(t=0)$. The qualitative difference shown in Fig. 4 and Fig. 6 (among others that are not addressed here) in time evolution of $\Delta\omega$ and Δl - both randomly chosen and more important even $\Delta l\|\omega$ - comprises the main message of this initial stage of a larger project.

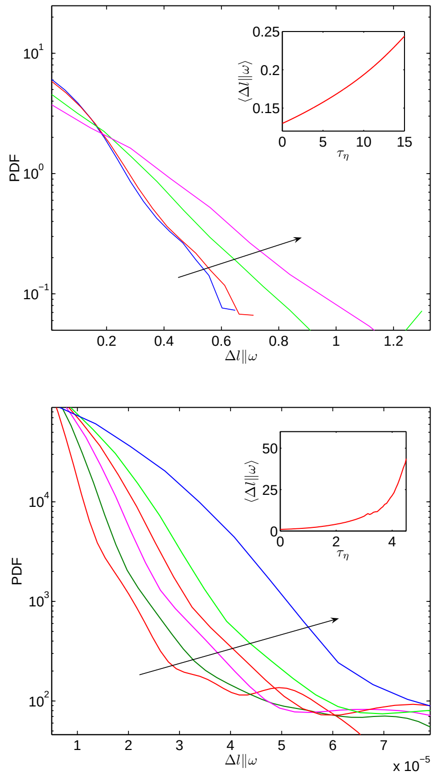

Figure 5: Time evolution of the disturbance $\Delta l\|\omega$ of the infinitesimal material lines, initially parallel to the vorticity vector.

ACKNOWLEDGEMENTS

We gratefully acknowledge a contribution of the research group of the Hydromechanics Laboratory in the Institute of Environmental Engineering at ETH Zurich where experimental data was obtained. The experiments were supported by ETH Zurich research funds, under Grant No. TH-18/02-4.

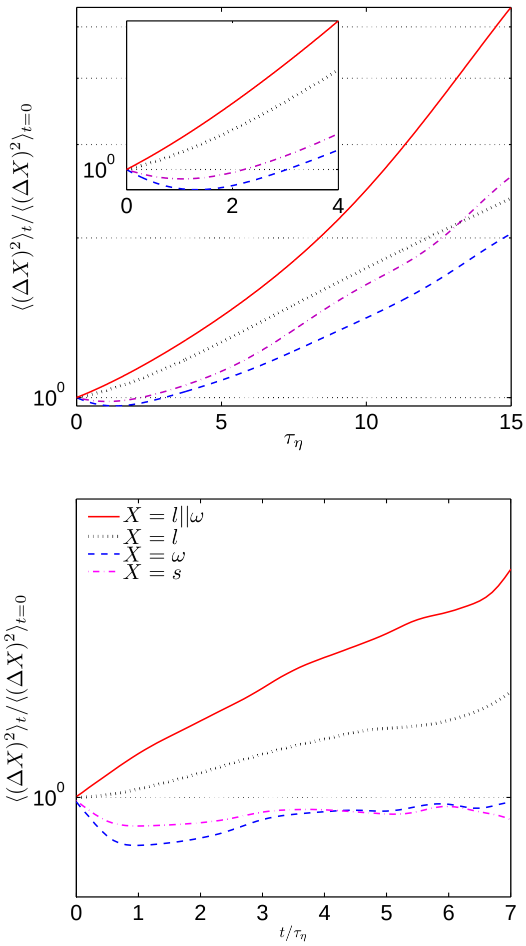

Figure 6: Time evolution of the mean square disturbance $\langle(\Delta X)^2\rangle$ of various quantities (X stands for ω, s_{ij}, randomly oriented infinitesimal material lines l and initially parallel to vorticity vector, $l \parallel \omega$), normalized with the 'initial disturbance' at $t = 0$. Left panel - DNS, right panel - 3D-PTV. Inset in the left panel is a zoom on the interval of $5\tau_\eta$.

REFERENCES

Born, M. 1958, "Vorhersagbarkeit in der klassischen Mechanik", *Zeitschrift für Physik,* **153**,372-388.

Boffetta, G., Cencini, M., Falconi, M. and Vulpiani, A., 2002, "Predictability: a way to characterize complexity", *Phys. Rep.,***356,** 367-474.

Galanti, B., Gendler– Fishman, D. and Tsinober, A., 2006, "Comparative Study of Vorticity and Material Lines Evolution in Numerical Turbulence – the Role of Forcing", *Turbulence, Heat and Mass Transfer* K. Hanjalić, Y. Nagano and S. Jakirlic (eds.), Dubrovnik, Croatia.

Holloway, G. and West, B.J., 1984, *Predictability of fluid motions*, AIP, NY.

Liberzon, A., Guala, M., Kinzelbach, W. and Tsinober, A., 2006, "On turbulent kinetic energy production and dissipation in dilute polymer solutions", *Phys. Fluids*, 18, pp. 125101

Lorenz, E. N., 1985, "The growth of errors in prediction", in M. Ghil, R. Benzi and G. Parisi, (eds.), *Turbulence and predictability in Geophysical fluid dynamics and climate dynamics,* North-Holland, pp.243-265.

Lüthi, B., Tsinober, A. and Kinzelbach, W., 2005, "Lagrangian measurement of vorticity dynamics in turbulent flow", *J Fluid Mech.,* **528**, pp. 87-118.

Novikov, E. A., 1960, "Contribution to the Problem of the Predictability of Synoptical Processes", *Proc. Acad. Sci. USSR, Geoph. Ser.,* **11**, pp. 1209-1211.

Tsinober, A. and Galanti, B., 2003, "Exploratory numerical experiments on the differences between genuine and 'passive' turbulence", *Phyics of Fluids*, **15**, pp. 3514-3531.