SHEAR-IMPROVED SMAGORINSKY MODEL FOR LARGE-EDDY SIMULATION OF WALL-BOUNDED TURBULENT FLOWS

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ABSTRACT

A refinement of the standard Smagorinsky model is proposed in order to take into account the inhomogeneities of the mean flow. First results concerning turbulent plane-channel flows and backward-facing step flows indicate that the model possesses a good predictive capability, essentially equivalent to the dynamic Smagorinsky model, but with a computational cost and a manageability comparable to the original Smagorinsky model.

INTRODUCTION OF THE MODEL

On the importance of mean velocity gradients

Turbulence that occurs in nature is usually not, even approximately, homogeneous nor isotropic. There are frequent important variations of the mean (ensemble-averaged) velocity $\langle \mathbf{u}(\mathbf{x}, t) \rangle$ with the position $\mathbf{x}$ in the flow. In that case, the gradients of the average properties of the fluctuating velocity, $\mathbf{u}'(\mathbf{x}, t) \equiv \mathbf{u}(\mathbf{x}, t) - \langle \mathbf{u}(\mathbf{x}, t) \rangle$, have a dominant effect on the evolution of the mean flow through the action of the *Reynolds stresses* (Pope, 2000). Despite the importance of these effects, it is usually thought that the small-scale properties of general turbulent flows should be considered as homogeneous and isotropic. This hypothesis is rooted in the idea that eddies of sufficiently small size undergo strong non-linear interactions, which result in a cascade of kinetic energy (towards smaller scales) where all statistical information about the large-scale inhomogeneities is lost. This is the classical framework of the Kolmogorov's theory (Frisch, 1995). Within this framework, the size-scale of eddies should be small compared to the length-scale associated with the mean velocity gradients, *i.e.*,

$$L_{\mathcal{S}} \sim \frac{u'}{\mathcal{S}} \sim \frac{\sqrt{\langle (u_i')^2 \rangle}}{\sqrt{\left(\dfrac{\partial \langle u_i \rangle}{\partial x_j} \right)^2}} \tag{1}$$

where u' and $\mathcal{S}$ may be viewed as the characteristic values of the fluctuating velocity and of the mean-velocity-gradient tensor, respectively; $\mathcal{S}$ is usually termed the shear and may be re-written as

$$\mathcal{S} = |\langle \mathrm{S} \rangle| \equiv \sqrt{2 \langle \mathrm{S}_{ij} \rangle \langle \mathrm{S}_{ij} \rangle} \tag{2}$$

where S denotes the rate-of-strain tensor (the symmetric part of the velocity-gradient tensor). Finally, eddies of size larger than $L_{\mathcal{S}}$ have no time to adjust dynamically via non-linear interactions but are strongly distorted by the shear.

Implications concerning large-eddy simulations

In a large-eddy simulation (LES), only the large-scale dynamics of the flow are resolved, while the subgrid-scale (SGS) motions are parametrized (Sagaut *et al.*, 2006). The effect of SGS dynamics appears in the governing equations through an additional stress that needs to be modeled in terms of the grid-scale velocity field in order to close the equations (Lesieur, 1997).

According to the previous reasoning, SGS fluctuations may be considered as homogeneous and isotropic in flow-regions where the grid-scale Δ (fixed by the resolution of the simulation) remains smaller than the shear length-scale $L_{\mathcal{S}}$. In that situation, the Smagorinsky's model (1963) is a relevant proposal: An additional *eddy-viscosity* (related to the SGS motions) is introduced as

$$\nu_T^{\mathrm{Smag.}}(\mathbf{x}, t) = (C_{\mathrm{S}} \Delta)^2 \, |\mathrm{S}_{\Delta}(\mathbf{x}, t)| \tag{3}$$

where S_Δ is the resolved rate-of-strain (at the resolution Δ). However, the condition $\Delta < L_\mathcal{S}$ cannot hold everywhere in real flows, for instance, near a solid boundary where $L_\mathcal{S}$ necessarily vanishes ($u' \to 0$ and $\mathcal{S} \to \mathcal{S}_\text{wall} \neq 0$ in the viscous sub-layer) because of the no-slip boundary condition. In that regions, shear effects should therefore be taken into consideration.

A shear-improved Smagorinsky model

We propose a shear-improved Smagorinsky model (SISM) in which the magnitude of the shear is subtracted from the magnitude of the instantaneous resolved rate-of-strain:

$$\nu_T^\text{SISM}(\mathbf{x},t) = (C_\text{S}\Delta)^2 \left(|S_\Delta(\mathbf{x},t)| - \mathcal{S}(\mathbf{x},t)\right) \qquad (4)$$

$\mathcal{S}(\mathbf{x},t)$ denotes the shear at the position $\mathbf{x}$ and time t, C_S is the Smagorinsky constant for homogeneous and isotropic turbulence ($C_\text{S} \approx 0.17$ according to Lilly (1967)) and $\Delta = (\Delta x.\Delta y.\Delta z)^{1/3}$ is the local grid spacing. It is assumed that the flow is well enough resolved in the direction of the shear, so that

$$\mathcal{S}(\mathbf{x},t) \approx |\langle S_\Delta(\mathbf{x},t)\rangle| \qquad (5)$$

It should be noticed that the angle brackets $\langle\ \rangle$ *a priori* imply ensemble average, however, space average over homogeneous directions and/or time average will be considered in practice.

The main concerns of the SISM are, firstly, to take into account shear effects in the exchanges of momentum to the SGS motions (without any kind of adjustment) and, secondly, to make the eddy-viscosity automatically vanish in laminar-flow regions (without *ad hoc* damping function). The SISM stems from theoretical arguments that were initially put forward by Toschi *et al.* (2000) and developed by Lévêque *et al.* (2007).

In flow regions where the fluctuating part of the rate-of-strain is much larger than the shear, *i.e.* $|S'_\Delta| \gg \mathcal{S}$, the grid scale $\Delta \ll L_\mathcal{S}$ by assuming that $|S'_\Delta| \approx u'/\Delta$. In that case, turbulence can be considered as homogeneous and isotropic at scales comparable to Δ. The SISM then reduces to the original Smagorinsky model, which is known to perform reasonably well. In flow regions where $|S'_\Delta| \ll \mathcal{S}$, the grid scale $\Delta \gg L_\mathcal{S}$ and therefore shear effects are significant at scales comparable to Δ. In that case, the SISM yields a SGS energy flux of order $\Delta^2 \mathcal{S}\langle|S'_\Delta|^2\rangle$, which is fully consistent with the SGS energy budget that can be derived from the Navier-Stokes equations in the case of a (locally homogeneous) shear flow (Lévêque *et al.*, 2007).

It should be stressed that the SISM exhibits apparent similarities with the model originally proposed by Schumann (1975), which relies on a two-part eddy-viscosity accounting for the interplay between the non-linear energy transfer and the shear effects associated with anisotropy. However, our model clearly differs from Schumann's proposal, which additionally, requires an empirical prescription for the *inhomogeneous eddy-viscosity*. Also, the expression (4) is not a simplification of Schumann's formulation.

TESTS OF THE MODEL

plane-channel flow

Over the last twenty years, LES of wall-bounded flows have received considerable attention, with the turbulent plane-channel flow being the prototypical case. This flow
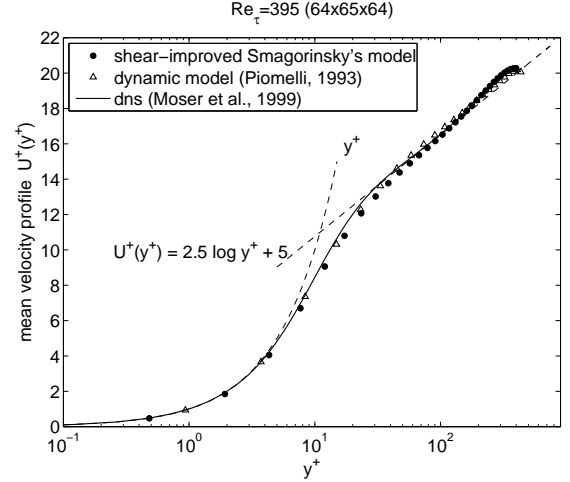

Figure 1: ($\bullet$) mean-velocity profile (in wall units) at $Re_\tau = 395$. The computational domain (in outer units) is $4\pi H \times 2H \times 2\pi H$ with $64 \times 65 \times 64$ grid points. In comparison with ($-$) the DNS data obtained by Moser et al. (1999) in the domain $2\pi H \times 2H \times \pi H$ with $256 \times 193 \times 192$ grid points, and ($\triangle$) a computation of the dynamic Smagorinsky model (Germano *et al.*, 1991) carried out by Piomelli (private communication) in the domain $5\pi H/2 \times 2H \times \pi H/2$ with $48 \times 49 \times 48$ grid points (using a pseudo-spectral solver).

allows us for an investigation of the SISM in a simple geometry. In this configuration, we have performed two LES at $Re_\tau = 395$ and $Re_\tau = 590$, where Re_τ is the Reynolds number based on the friction velocity u_τ ($Re_\tau = u_\tau H/\nu$ with H being the half width of the channel). In the present article, we will only report the results at $Re_\tau = 395$. Details about the simulations are provided by Lévêque *et al.* (2007). Let us only mention that a pseudo-spectral Fourier-Chebyshev (de-aliased) method has been used to limit (numerical) discretization errors and therefore concentrate on modelling errors. In the following, the grid-scale velocity components are $U + u'$, v' and w' along the streamwise, wall-normal and spanwise directions, respectively.

First of all, it has been observed that the SISM exhibits a transition (or drag crisis) from the initially perturbed Poiseuille profile to the appropriate turbulent regime, as the
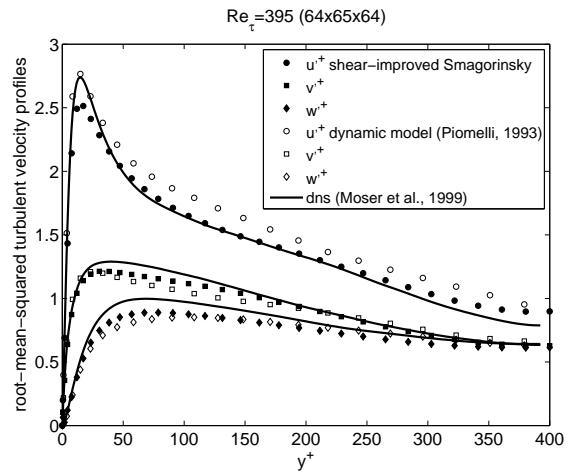

Figure 2: Turbulent intensity profiles (normalized by the squared friction velocity) in comparison with DNS data and LES data obtained with the dynamic Smagorinsky model.
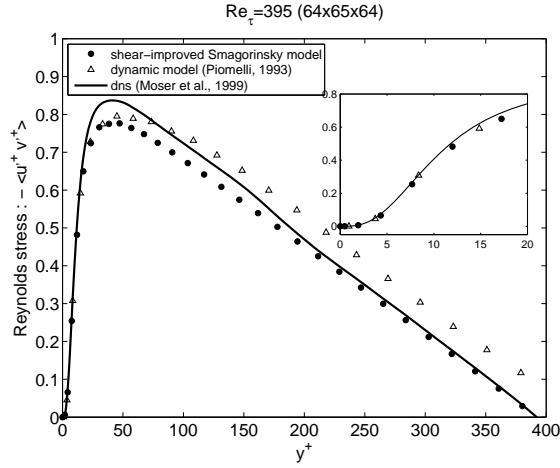
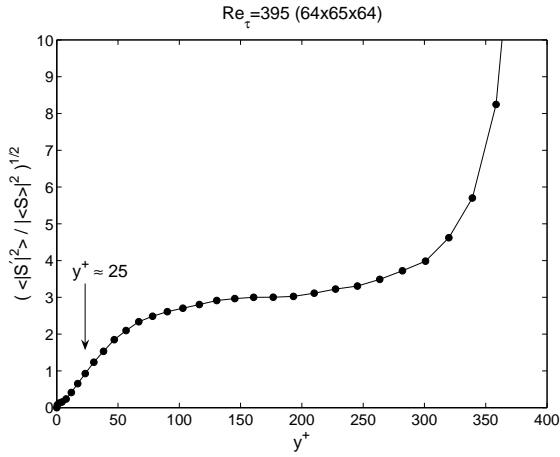Figure 3: Reynolds stress computed from the resolved velocity. The inset focuses on the near-wall behaviour.

Figure 4: The ratio $\sqrt{\langle |S'_\Delta|^2 \rangle}/\mathcal{S}$ is displayed as a function of the wall-normal distance (in wall units). For $y^+ < 25$, the shear dominates over the fluctuating part of the rate-of-strain. In that region, our eddy-viscosity differs from the original Smagorinsky model.

integration progresses with the prescribed ν_T^{SISM} viscosity. This feature is a first major improvement over the Smagorinsky model, for which such transition is not captured.

The mean velocity profile is displayed as a function of the wall-normal distance (in wall units) in figure 1. The average is used in time and over horizontal planes (homogeneous directions). The agreement with the DNS data obtained by Moser *et al.* (1999) is very good. The comparison with the dynamic Smagorinsky model ((Germano *et al.*, 1991) is also satisfactory. The normalized turbulence intensity profiles are shown in figure 2. Also here, the profiles compare well with the DNS data and LES results with the dynamic Smagorinsky model; the positions of the peaks are captured well and the errors on the peak values remain acceptable. The grid-scale Reynolds stress profile is shown in figure 3. The agreement with the DNS and the LES based on the dynamic Smagorinsky model is fair and also the behaviour close to the wall is satisfactory.

The ratio $\sqrt{\langle |S'_\Delta|^2 \rangle}/\mathcal{S}$ is shown as a function of the wall-normal distance (in wall units) in figure 4. For $y^+ < 25$, the shear dominates over the fluctuating part of the rate-
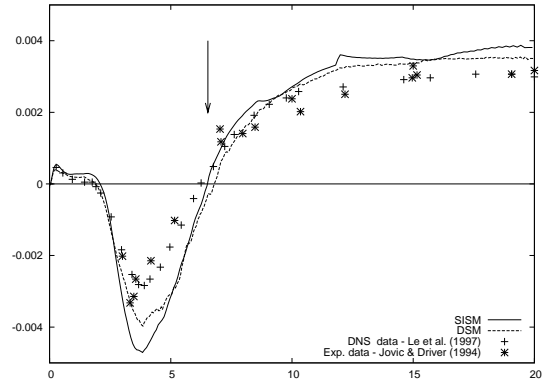

Figure 5: For the backward-facing step flow at $Re = 4800$ (based on the step height), the prediction of the skin-friction coefficient, here displayed as a function of the distance to the step, is in good agreement with experimental data, DNS data and with a LES based on the dynamic Smagorinsky model (DSM) at comparable Reynolds number. Notice that the strength of the secondary recirculation bubble near the corner (a sensitive diagnostic) is also correctly predicted.

of-strain, indicating the relevance of the shear component of our eddy-viscosity in that region. The transition distance $y^+ \approx 25$ is fully consistent with the empirical distance $A^+ = 25$ commonly used in the van Driest damping function (Pope, 2000). In the log-layer, $\sqrt{\langle |S'_\Delta|^2 \rangle}/\mathcal{S}$ increases slowly with y^+ (the classical description of the log-layer predicts a linear increase resulting from $\mathcal{S} \sim 1/y$ and $\sqrt{\langle |S'_\Delta|^2 \rangle} \sim u_\tau/\Delta$) and eventually diverges around the centerline of the channel. This behaviour indicates that our eddy-viscosity suitably bridges the situation where the shear prevails (close to the boundary) and the situation where fluctuating part of the rate-of-strain dominates (in the bulk of the channel).

backward-facing step flow

A more stringent test of the SISM has been been performed in the backward-facing step geometry by Toschi et al. (2006). In addition to wall effects, the flow over a backward-facing step is strongly affected by the detached shear layer and the recirculating motions behind the step. An adequate resolution of the instability of the shear layer is required to predict correctly the location of the reattachment point.

The SISM has been tested in a backward-facing step flow configuration with resolution $256 \times 96 \times 64$ in the x (streamwise), y (wall-normal) and z (spanwise) directions, respectively. The Reynolds number based on the height of the step and the bulk velocity was 4800. The simulation was preformed using jetCode, an incompressible-flow solver developed at Stanford university. Details about the simulations are provided by Toschi *et al.* (2006). It should be mentioned that the standard value $C_{\mathrm{S}} = 0.2$ has been used in simulations. The shear was evaluated by averaging in the spanwise direction and by using a running average over time with a weight recycling factor comparable to the eddy-turnover time of the flow.

The computation of the skin-friction coefficient is displayed in figure 5. The skin-friction coefficient changes sign from negative to positive at the reattachment point (indicated by the arrow). We observe that the prediction is in very good agreement with experimental and DNS results (at comparable Reynolds number). We also see that the SISM

performs well with respect to the DSM, but with the great advantage of being much less demanding in computational resources.

CONCLUSION

Our first tests indicate that the SISM possesses a good predictive capacity (essentially equivalent to the dynamic Smagorinsky model) with a computational cost and a manageability comparable to the original Smagorinsky model. Nonetheless, some issues need to addressed in the future. In particular, a proper *instantaneous shear* should be defined in the case of non-stationary flows. An average over a time window of width comparable to the local large-scale eddy-turnover time of the flow may be envisaged. How to exactly implement this procedure is the matter of present investigations.

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