OPTIMIZATION AS A VEHICLE FOR LES MODELING IN COMPLEX TURBULENT FLOWS

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ABSTRACT

In the context of LES modeling, we consider a high-Reynolds-number complex turbulent flow to be one in which the assumptions of a Kolmogorov inertial range and small-scale isotropy are not valid. This may be due, for example, to geometric "complexity" such as a solid wall, or due to complexity of the physics involved, such as turbulent combustion. In either case, since these assumptions underly virtually all subgrid models, such flows require further modeling. To address this problem, we consider optimization as a tool to formulate LES models. This so-called optimal LES formulation has the advantage that it is formulated independent of any assumptions regarding the structure of the small scales. It also makes explicit the statistical information that underlies the resulting model. The primary inputs into the model formulation are several small-separation multi-point correlations. The assumptions of small-scale isotropy and the Kolmogorov inertial range scaling are sufficient to model these correlations, and produce an optimal LES model. In complex turbulent flows, where these assumptions are not valid, the optimization approach essentially reduces the LES modeling problem to one of representing the anisotropic, inhomogeneous multi-point correlations with small separations. Another advantage to this approach is that the multi-point correlations provide a way to directly recover turbulence statistics from the LES statistics.

In this paper, we discuss both the general optimization approach and its application to isotropic turbulence and turbulent channel flow.

INTRODUCTION

One of the most promising techniques for the prediction of turbulent flows is that of Large Eddy Simulation (LES), in which an under-resolved representation of the turbulence is simulated numerically by modeling the effects of the unresolved small-scales on the simulation. While such simulations have been applied in a number of flows with reasonable success, there are several outstanding problems limiting LES's applicability as a general purpose engineering tool. Chief among these is the application of LES to so-called "complex" turbulent flows. In such flows, the assumptions of small-scale isotropy and a Kolmogorov inertial range on which many LES models are predicated, are not valid, because strong inhomogeneity and/or additional physical phenomena occurring at the small scales mean that dissipation of kinetic energy is not the dominant feature of the small-scale turbulence. The best known example of this complication to LES modeling is near-wall turbulence, but there are numerous other examples, including turbulent combustion, where the chemical reaction occurs at the small scales, and turbulent particle laden flows, in which the interaction with inertial particles occurs at small scales.

The optimal LES formulation (Langford and Moser, 1999; Langford, 2000) provides a framework in which to address these issues and to develop and analyze LES models and simulations. Optimal LES modeling has been found to produce accurate LES simulations when based on reliable statistical information (Langford and Moser, 2004; Zandonade *et al.*, 2004; Volker *et al.*, 2002), so the primary challenge in the development of LES models in the optimal LES framework is obtaining models for this statistical information. What is required are models for multi-point velocity correlations. When small-scale isotropy is a valid assumption, the Kolmogorov theory and isotropy are sufficient to construct expressions for the correlations that allow optimal LES models to be formulated (Zandonade, 2007; Chang and Moser, 2007), and the LES models perform well (Zandonade, 2007). In essence, the optimal LES formulation yields models valid for small-scale isotropy that are comparable to commonly used models, because they are based on the same assumptions regarding the small-scales. For complex turbulent flows

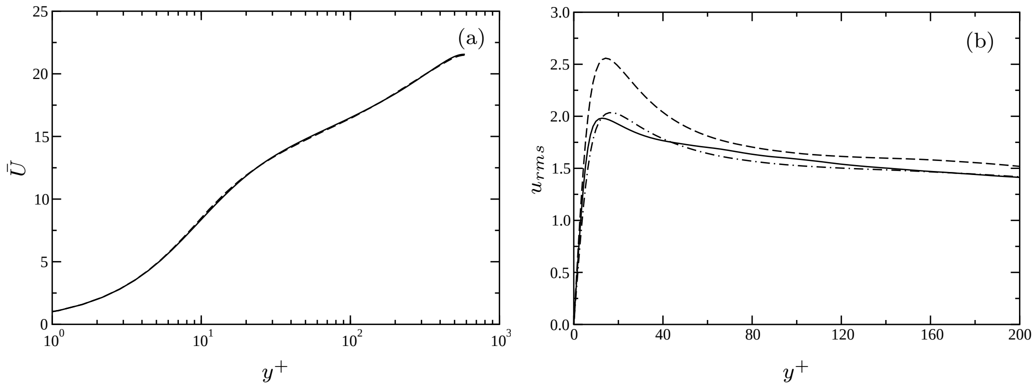

Figure 1: Mean and rms velocity profiles in wall coordinates from LES (———), DNS of Moser *et al.* (1999) (— — —) filtered DNS (– · – · –). Shown are (a) mean velocity, (b) rms streamwise velocity $u_{\rm rms}$ in channel flow at $Re_\tau = 590$, with Fourier truncation filter widths of $\Delta x^+ = 116$ and $\Delta z^+ = 58$

however, the optimal LES formulation provides a framework to include affects of anisotropy and complex small-scale physics, provided information regarding the small-scale correlations is available.

In this paper, we briefly describe the optimal LES formulation and its ability to model complex turbulent flows. The problem of representing small-scale multi-point correlations is then discussed, with results from complex (wall-bounded) turbulence. The use of such correlation models to recover turbulence statistics from LES statistics is described, and finally, some concluding remarks are provided.

OPTIMAL LARGE EDDY SIMULATION

The starting point for the development of LES is the definition of a spatial filter $\tilde{\cdot}$, which can be applied to the Navier-stokes equations to obtain an equation for the filtered velocity $\tilde{u}_i$:

$$\frac{\partial \tilde{u}_i}{\partial t} = -\frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j} - \frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{\rm Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} + M_i, \qquad (1)$$

Where M_i is the sub-grid model (force) term, which includes the divergence of the sub-grid stress as well as terms that arise when the filter does not commute with differentiation. The problem in LES, of course, is to model M_i. However, in this context, the filter is a mapping from the infinite-dimensional space of Navier-Stokes solutions to a finite dimensional space of computable LES solutions. In essence, in this definition we are including discretization in the filter operator. This makes the filter (mapping) non-invertible, and so there is insufficient information in the filtered field $\tilde{\mathbf{u}}$ to uniquely determine the evolution of $\tilde{\mathbf{u}}$. It is thus appropriate to consider this evolution to be stochastic, and define the LES model statistically. Let $\mathbf{w}$ be the output of an LES, as distinct from the filtered turbulence. An important result due to Pope (Pope, 2000; Langford and Moser, 1999) is that an LES $\mathbf{w}$ will match the one-time statistics of filtered turbulence $\tilde{\mathbf{u}}$ if and only if the model $\mathbf{m}(\mathbf{w})$ of $\mathbf{M}$ is given by

$$\mathbf{m}(\mathbf{w}) = \langle \mathbf{M}(u) | \tilde{\mathbf{u}} = \mathbf{w} \rangle \qquad (2)$$

This model also minimizes the difference between $\mathbf{M}$ and $\mathbf{m}$ (in the mean-square sense), and so it has all the properties that one could ask of a sub-grid model. We therefore call it the ideal sub-grid model.

Unfortunately, the conditional average in (2) cannot practically be determined, since the number of conditions is the number of degrees of freedom in the LES. However, it can be estimated using stochastic estimation (Adrian *et al.*, 1989) which is a well-established technique for estimating conditional averages. The result is a class of optimization-based LES models as first proposed by Adrian (1990). Such models have been named "optimal LES" models because the stochastic estimation yields an optimum (minimum mean-square error) approximation of the conditional average.

It has been found that it is generally sufficient to approximate the subgrid model term $\mathbf{M}$ as linear in the LES variables (Zandonade, 2007; Volker *et al.*, 2002). In this case, the optimal LES model is expressed:

$$\mathbf{m}' = \sum_k \mathbb{L}_k \cdot \mathbf{w}'_k \qquad (3)$$

$$\langle \tilde{\mathbf{u}}'_j \mathbf{M} \rangle = \sum_k \mathbb{L}_k : \langle \tilde{\mathbf{u}}'_j \tilde{\mathbf{u}}'_k \rangle \qquad (4)$$

where the array of second-ranked tensors $\mathbb{L}_k$ are the linear estimation coefficients, and the indices j and k specify which vector degree of freedom is used for the estimation (e.g. point value or Fourier coefficient). Note that the $\mathbb{L}$ are determined from the correlations of the filtered velocities with themselves and with the model term $\mathbf{M}$. It is these correlations that encode the characteristics of the unrepresented small scales, and that reflect whether (or not) they are isotropic or exhibit an inertial range. In the optimal LES formalism, it is these correlations that we, as modelers, are responsible for providing.

Tests of Optimal LES in Channel Flow

To test the ability of such optimal LES models to produce accurate LES, the correlations appearing in the equation for $\mathbb{L}$ (4) were determined from available DNS data for both isotropic turbulence and turbulent channel flow (Langford and Moser, 2004; Zandonade *et al.*, 2004; Volker *et al.*, 2002; Moser *et al.*, 2007). Here we recall example results for the channel flow at $Re_\tau = 590$.

In the study by Volker *et al.* (2002), filtering was based on Fourier truncation in the wall-parallel directions, while no filtering in the wall-normal direction was employed. As pointed out by Härtel and Kleiser (1998), in the absence of wall-normal filtering, the contribution of the subgrid term to the resolved-scale energy equation is positive near the wall, which is due to the subgrid contribution to the transport of energy from the production peak toward the wall. To ensure that this transport property of the LES model was well represented, Volker *et al.* (2002) formulated the model to estimate the subgrid stress (rather than its divergence) in terms of the Fourier-transformed local (in y) velocities

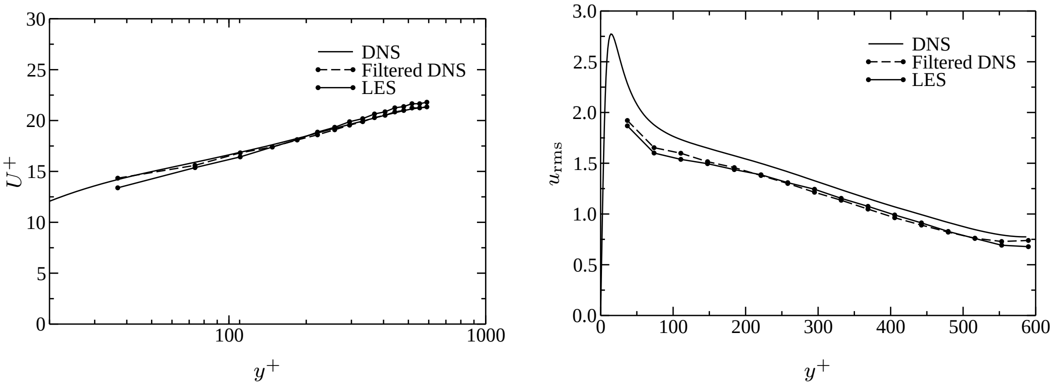

Figure 2: Mean velocity (left) and rms streamwise velocity fluctuations (right) in a turbulent channel at $Re_\tau = 590$. The LES was performed using the filtered boundary optimal LES formulation of Moser *et al.* (2007).

and their y-derivatives. The result was a remarkably good LES, which accurately represented the mean velocity, filtered Reynolds stress tensor and spectra (see figure 1 for example).

The optimal LES formulation was also shown to yield good simulations when wall normal filtering is applied (Moser *et al.*, 2007). In this case, in addition to Fourier truncation filters in the wall-parallel directions, a Fourier truncation in the wall-normal direction was used, in a domain that extended beyond the walls. In this way, the wall was filtered in addition to the turbulence. This approach can be considered to be an LES version of an embedded boundary formulation. See Moser *et al.* (2007) for further discussion of the wall treatment. In this formulation, optimal LES based on DNS statistical data from Moser *et al.* (1999) is used for the volumetric subgrid term, and the results are again remarkably good (see figure 2).

The results described above demonstrate that the optimal LES formalism does yield accurate LES models even in complex turbulent flows in which assumptions of isotropy and a Kolmogorov inertial range are invalid. However these assumptions were replaced with statistical data obtained from DNS, so by itself, the optimal formulation does not provide a practical predictive model for wall-bounded LES. The challenge of LES modeling in such complex turbulent flows has been reduced to that of modeling the input correlations.

CONSTRUCTING MODELS FOR OPTIMAL LES CORRELATIONS

The correlations appearing in (4) are essentially two-point correlations of the filtered velocities with themselves and the model term M. In Cartesian tensor notation, we need to know the following correlations:

$$\langle \tilde{u}_j(\mathbf{x}')\tilde{u}_k(\mathbf{x}'')\rangle \qquad \text{and} \qquad \langle M_i(\mathbf{x})\tilde{u}_k(\mathbf{x}'')\rangle. \tag{5}$$

where $\mathbf{x}'$ and $\mathbf{x}''$ are the spatial locations associated with the state variable indices j and k respectively in (4). We do not, however, generally have theory for these two-point correlations of filtered and model quantities. But we do have theory for multi-point velocity correlations, the LES correlations (5) can be obtained by applying the filtering operator to the following two and three-point correlations (Chang and Moser, 2007):

$$\begin{aligned} \mathbb{R}(\mathbf{r},\mathbf{s}) &= \langle \mathbf{u}(\mathbf{r})\mathbf{u}(\mathbf{s})\rangle & (6)\\ \mathbb{B}(\mathbf{r},\mathbf{s}) &= \langle \mathbf{u}(\mathbf{r})\mathbf{u}(\mathbf{s})\mathbf{u}(\mathbf{s})\rangle & (7)\\ \mathbb{T}(\mathbf{r},\mathbf{s},\mathbf{t}) &= \langle \mathbf{u}(\mathbf{r})\mathbf{u}(\mathbf{s})\mathbf{u}(\mathbf{t})\rangle & (8) \end{aligned}$$

Thus, models for these correlation tensors are needed. When the small scales are isotropic, Kolmogorov inertial range theory applies, and this will be sufficient to determine all but $\mathbb{T}$, and an extended inertial-range model to determine $\mathbb{T}$ has also been developed. Furthermore, correlations of filtered velocities with themselves can be computed directly from an LES. It is therefore possible to determine some of the necessary correlations dynamically in a running LES. In this case, a model is only needed for the two-point third order correlations $\mathbb{B}$ (Zandonade, 2007).

Correlations in Isotropic Turbulence

The required correlations can be obtained in the case of simple LES (i.e. isotropic small scales) as is shown below.

Isotropy and the continuity constraints are sufficient to determine the second- and third-order two-point correlation tensors from the second and third-order structure functions respectively. Using the Kolmogorov expressions for these quantities, the correlation tensors are:

$$\begin{aligned} \mathbb{R}_{ij}(\mathbf{r}) &= u^2\delta_{ij} + \frac{C_2}{6}(\epsilon r)^{2/3}\left(\frac{r_i r_j}{r^2} - 4\delta_{ij}\right) & (9)\\ \mathbb{S}_{ijk}(\mathbf{r}) &= \frac{\epsilon}{15}\left(\delta_{ij}r_k - \frac{3}{2}(\delta_{ik}r_j + \delta_{jk}r_i)\right) & (10) \end{aligned}$$

where the homogeneous two-point correlations are defined:

$$\begin{aligned} \mathbb{R}_{ij}(\mathbf{r}) &= \langle u_i(\mathbf{x})u_j(\mathbf{x}+\mathbf{r})\rangle & (11)\\ \mathbb{S}_{ijk}(\mathbf{r}) &= \langle u_i(\mathbf{x})u_j(\mathbf{x})u_k(\mathbf{x}+\mathbf{r})\rangle, & (12) \end{aligned}$$

and u^2 is 2/3 the turbulent kinetic energy, which is also the velocity variance. The result for the second-order correlation is well known. The expression for the third-order two-point correlation shown here was reported by Chang and Moser (2007), is implicit to the derivation of the Kolmogorov 4/5 law, and is alluded to by Frisch (1995).

A representation for the isotropic three-point third-order correlation $\mathbb{T}$ is not as straight-forward. Proudman and Reid (1954) determined the most general isotropic tensor form satisfying all the relevant symmetry and continuity constraints, and when transformed to physical space it yields

$$\begin{aligned} \mathbb{T}_{ijk}(\mathbf{r},\mathbf{s}) = {} & P^t_{im}P^s_{jn}P^r_{kp}[\delta_{np}\partial^s_m\psi(r,s,t) \\ & +\delta_{mp}\partial^r_n\psi(t,r,s) + \delta_{mn}\partial^s_p\psi(t,s,r)] \end{aligned} \tag{13}$$

where the separation vectors are interrelated $\mathbf{t} = \mathbf{r} - \mathbf{s}$, and spatial derivatives are denoted $\partial^r_i = \left.\frac{\partial}{\partial s_i}\right|_{\mathbf{r}}$, $\partial^s_i = \left.\frac{\partial}{\partial r_i}\right|_{\mathbf{s}}$, $\partial^t_i = -\left.\frac{\partial}{\partial r_i}\right|_{\mathbf{s}} - \left.\frac{\partial}{\partial s_i}\right|_{\mathbf{r}}$, the operators $P^\alpha_{ij} = \delta_{ij}\partial^\alpha_k\partial^\alpha_k - \partial^\alpha_i\partial^\alpha_j$ and symmetries require $\psi(r,s,t) = -\psi(s,r,t)$. This form for

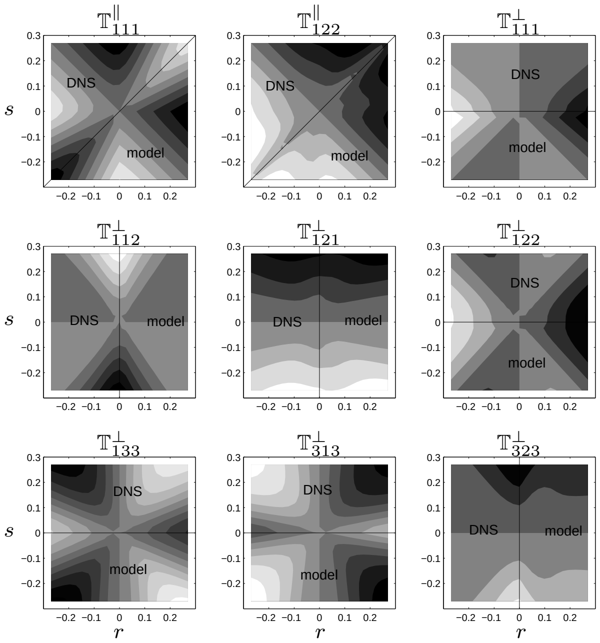

Figure 3: Contours of the DNS data of Langford and Moser (1999) and tensor model for $\mathbf{r}$ and $\mathbf{s}$ co-linear ($\mathbb{T}^{\parallel}$) and orthogonal ($\mathbb{T}^{\perp}$), in the r–s plane. Each component has a symmetry, which is used to allow the data and the model to be displayed side-by-side, as shown. The heavy black lines are lines of symmetry for each component.

$\mathbb{T}$ is a linear combination of seventh derivatives of ψ, where ψ is a scalar function of scalar separations r, s and t. Further, for inertial range separations, the Kolmogorov 4/5 law implies a linear dependence on separation for the two-point third order correlation. Assuming that ψ is a polynomial in r, s and t, consistency with the 4/5 law then requires that the polynomial have overall order eight. Thus, we may consider ψ of the form $(r^a s^b - r^b s^a)t^c$, where $a+b+c=8$. If the exponents are non-negative, there is a 20-dimensional space of possible expressions, in which there is only a 5-dimensional subspace of $\mathbb{T}$ that are non-zero, non-singular, and continuous. The four unknown constants (not counting the overall magnitude) were determined by fitting to the DNS data of Langford and Moser (1999). The resulting model for the three-point third-order correlation is a remarkably good representation of the DNS data (see figure 3).

Correlations in Turbulent Channel Flow

When the assumption of small-scale isotropy and homogeneity are not valid, the models for the required correlations must be more complex. The most common example of this situation is near-wall turbulence. In the log-layer of wall-bounded turbulence, however, self-similarity of the correlations (Oberlack, 1997) can be used to represent the inhomogeneity. To model anisotropy, we extend the description of single-point anisotropy in terms of structure tensors (Kassinos *et al.*, 2001) to the two-point correlation.

Structure tensors are second rank tensors obtained from single point moments of derivatives of fluctuating stream functions and are related to integrals of the two point correlation over separations $\mathbf{r}$, and therefore contain information about the anisotropy of R_{ij}. For homogeneous turbulence, the independent structure tensors are given by the componentality (or Reynolds Stress) $B_{ij} = \epsilon_{ipq}\epsilon_{jts}\left\langle \psi'_{q,p}\psi'_{s,t}\right\rangle$, dimensionality $Y_{ij} = \left\langle \psi'_{n,i}\psi'_{n,j}\right\rangle$ and strophylosis Q_{ijk}[1]. While R_{ij} is not uniquely determined by the structure tensors, our modeling approach is to use the theory of invariants (Spencer, 1971) to formulate the most general linear representation of R_{ij} in terms of the structure tensors, i.e. $\Delta R_{ij}(\mathbf{x},\mathbf{r}) = \frac{R_{ij}(\mathbf{x},\mathbf{r})-B_{ij}}{q^2} = F_{ij}(\mathbf{r},\mathbf{b},\mathbf{Q},\mathbf{y})$, where $q^2 = B_{kk}$, $b_{ij} = B_{ij}/q^2 - \delta_{ij}/3$, $y_{ij} = Y_{ij}/q^2 - \delta_{ij}/3$. Under these assumptions, and with the additional assumption that the dependence on r is a power law, we obtain (dependence on $\mathbf{x}$ will be implicit in the rest of the section):

$$
\begin{aligned}
\Delta R_{ij}(\mathbf{r}) \;=\; & r^{\alpha_I}\left[a_1\delta_{ij} + a_2\hat{r}_i\hat{r}_j\right] \qquad (14)\\
& + r^{\alpha_b}[a_3 b_{ij} + a_4\hat{\mathbf{r}}\cdot\mathbf{b}\cdot\hat{\mathbf{r}}\delta_{ij} + a_5\hat{\mathbf{r}}\cdot\mathbf{b}\cdot\hat{\mathbf{r}}\hat{r}_i\hat{r}_j \\
& \quad + a_6(\hat{r}_i(\hat{\mathbf{r}}\cdot\mathbf{b})_j + \hat{r}_j(\hat{\mathbf{r}}\cdot\mathbf{b})_i)] \\
& + r^{\alpha_y}[a_7 y_{ij} + a_8\hat{\mathbf{r}}\cdot\mathbf{y}\cdot\hat{\mathbf{r}}\delta_{ij} + a_9\hat{\mathbf{r}}\cdot\mathbf{y}\cdot\hat{\mathbf{r}}\hat{r}_i\hat{r}_j \\
& \quad + a_{10}(\hat{r}_i(\hat{\mathbf{r}}\cdot\mathbf{y})_j + \hat{r}_j(\hat{\mathbf{r}}\cdot\mathbf{y})_i)] \\
& + r^{\alpha_Q}[a_{11}(\epsilon_{imk}Q_{klj} + \epsilon_{jmk}Q_{kli})\hat{r}_l\hat{r}_m \\
& \quad + a_{12}(\hat{r}_j\epsilon_{ink} + \hat{r}_i\epsilon_{jnk})Q_{klm}\hat{r}_l\hat{r}_m\hat{r}_n]
\end{aligned}
$$

Here $\hat{\mathbf{r}} = \mathbf{r}/r$ and α_s, $s \in \{I, b, y, Q\}$ are power-law indices. The number of free constants $(a_1 - a_{12})$ are reduced to 4 by enforcing the continuity constraint (i.e. $\frac{\partial \Delta R_{ij}(\mathbf{r})}{\partial r_j} = 0$) and a self-consistency constraint, which requires that when $\mathbf{Q}$, $\mathbf{b}$ or $\mathbf{y}$ are zero, the values of the respective tensors calculated directly from the representation F_{ij} should be zero. We fit the representation to DNS data over a space spanned by the 4 free constants, 4 power law indices and 10 free components of $\mathbf{y}$ and $\mathbf{Q}$. The representation captures many features of the exact correlation, like the inclination of the principle axis of the isocontours and the shape of the isocontours (figure 4). While the model captures the anisotropy of the correlations quite well, there are some obvious differences between the model and the DNS data. Most notable is the effects of inhomogeneity, especially on the R_{12} component.

Given a model for the anisotropy and inhomogeneity of the two-point second order correlation, an expression for the two-point third order correlation can be developed from the two-point correlation equation (assuming the turbulence is stationary). This along with a dynamic process to estimate the filtered three-point correlations (Zandonade, 2007) is enough to formulate the optimal LES model.

ESTIMATING TURBULENCE STATISTICS

One of the more subtle difficulties associated with LES is that formally, the quantities that are being simulated (the filtered velocities) are of no particular interest. Instead, one wants to know the statistics of the actual turbulence the LES is supposed to represent. For this, one must devise a way to determine the turbulence quantities of interest from the statistics of the LES solution. Some LES models (e.g. the stretched-vortex model of Misra and Pullin, 1997; Voelkl *et al.*, 2000) do this naturally, and this is the case for the optimal formulation.

Using a model of the small separation multi-point correlations, as discussed above, many turbulence statistics can be approximated, given the multi-point LES statistics. Any multi-point correlation of the LES fields can be written as the filter applied multiple times to the corresponding turbulence correlations. For example, the two-point correlation of the LES velocities $\tilde{R}_{ij}(\mathbf{x},\mathbf{x}') = \langle w_i(\mathbf{x})w_j(\mathbf{x}')\rangle$ is written in

[1] $Q_{ijk} = \left(Q^*_{ijk} + Q^*_{jki} + Q^*_{kij} + Q^*_{kji} + Q^*_{jik} + Q^*_{ikj}\right)/q^2$, where $Q^*_{ijk} = -\langle u'_j\psi'_{i,k}\rangle$

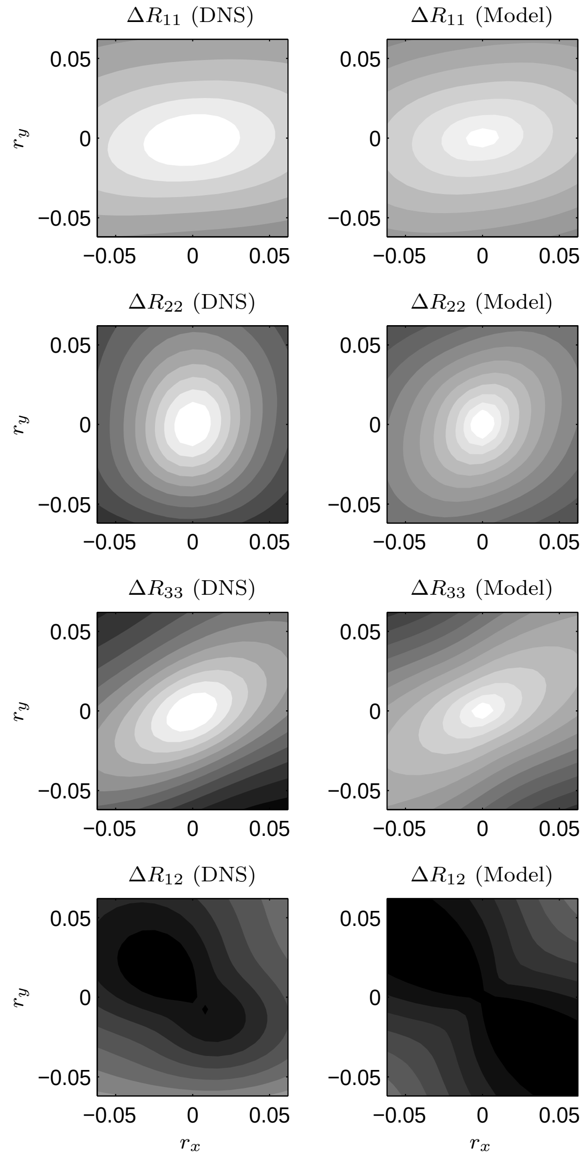

Figure 4: Isocontours of $\Delta R_{ij}(y, \mathbf{r}) = R_{ij}(y, \mathbf{r}) - R_{ij}(y, 0)$ at $y^+ = Re_\tau y/h = 114$, plotted for $r_z = 0$. The correlations in the left column were calculated from DNS of channel flow, and the correlation in the right column from the best fit of the model with DNS. The contour levels for given a correlation component (i.e. for the same row) have the same range.

terms of the turbulent two-point correlation as

$$\tilde{R}_{ij}(\mathbf{x}, \mathbf{x}') = \mathcal{F}^{\mathbf{x}}_{\mathbf{y}} \mathcal{F}^{\mathbf{x}'}_{\mathbf{y}'} R_{ij}(\mathbf{y}, \mathbf{y}') \tag{15}$$

where $\mathcal{F}^{\mathbf{x}}_{\mathbf{y}}$ is the filter operator that acts on the $\mathbf{y}$ space to yield the LES variable associated with the point $\mathbf{x}$. In particular, in the integral representation of the filter

$$\tilde{\phi}(\mathbf{x}) = \mathcal{F}^{\mathbf{x}}_{\mathbf{y}} \phi(\mathbf{y}) = \int F(\mathbf{x}, \mathbf{y}) \phi(\mathbf{y})\, d\mathbf{y} \tag{16}$$

where F is the filter kernel associated with the operator $\mathcal{F}$. The tensor models described above for the small separation correlation are expressed in terms of a hand full of parameters. Such a model for $R_{ij}(\mathbf{y}, \mathbf{y}')$ implies, through the relationship (15), a parameterization of the LES correlation $\tilde{R}_{ij}$. The LES correlation $\tilde{R}_{ij}$ is presumed to be known as a statistical output of an LES. Parameters can therefore be fit to the LES statistical data, which then yields a model for the unfiltered turbulence correlation.

This reconstruction of the turbulence statistics is natural in the context of the optimal LES formulation because it makes direct use of models for the small separation correlations. However, regardless of the LES model being used, the links between filtered and unfiltered statistics are these small-separation correlations. Indeed the reconstruction of turbulence statistics that occurs in the context of the stretched-vortex LES model (Misra and Pullin, 1997; Voelkl *et al.*, 2000) can be cast as a model for the small separation correlations.

CONCLUSIONS

Optimization is clearly a viable vehicle for the development of LES models for complex turbulent flows. Here, the optimization is the minimization of the error in representing the ideal LES given by (2). This so-called optimal LES formulation requires as input knowledge or models of several small-separation (on order the filter width) multi-point correlations. One might protest that this requirement is much more burdensome than the inputs required for other models. However, we saw that the assumptions of small scale isotropy and the presence of an inertial range is sufficient to define the required statistics, and these are the assumptions upon which virtually all LES models are based.

When these assumptions are not valid, then other inputs are clearly needed. It appears that, as burdensome as they may be, the correlations upon which optimal LES are based are what is needed. There are a number of ways that models for these correlations might be constructed, such as a mechanistic model as in the spiral vortex formulation of Misra and Pullin (1997) and Voelkl *et al.* (2000). Alternatively, we are exploring models based on simplified tensor forms. The vision is that these correlation models can be parameterized dynamically based on statistics of the running LES, to provide the optimal LES subgrid model for the LES. Having this model of the correlations also allows the statistics of the underlying turbulence to be reconstructed from the LES statistics.

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