RAPID AXISYMMETRIC CONTRACTION OF GRID-GENERATED TURBULENCE AND THE REYNOLDS-STRESS MODELS AT RAPID DISTORTION LIMIT

Özgür Ertunç[a], Çagatay Köksoy[a], Subhashis Ray[b], Franz Durst[a]

[a] LSTM-Erlangen, Institute of Fluid Mechanics,
Friedrich-Alexander-Universität Erlangen-Nürnberg, Cauerstrasse 4, D-91058 Erlangen, Germany
ertunc@lstm.uni-erlangen.de, ckoeksoy@lstm.uni-erlangen.de, durst@lstm.uni-erlangen.de

[b] Department of mechanical Engineering, Jadavpur University, Kolkata 700 032, India
juhp_sray@yahoo.co.in

ABSTRACT

Rapid distortion of mean-velocity in turbulent flows can be encountered in many technological applications, and hence, it is important to know the prediction performance of the available and widely used Reynolds-stress models under conditions where rapid distortion theory (RDT) is valid. Axisymmetric contraction of grid-generated turbulence is one class of homogeneous turbulent flows, which can approach to the RDT limit under highly strained condition and it is selected for the validation of various Reynolds-stress models in this study. For this purpose, available data on axisymmetric contraction are analyzed in terms of their accuracy and rapidity. The experimental data of Ertunç (2007) is found to be appropriate for this kind of validation at RDT limit. The Reynolds-stress models of Launder *et al.*(1975) , Speziale *et al.* (1991) , Sjögren and Johansson (2000) , Jovanović *et al.* (2003) are compared with the experimental data of Ertunç (2007) and the modified rapid distortion theory of Sreenivasan and Narasimha (1978). Since the rapid pressure strain term in the dynamic equation of Reynolds-stresses dominates the development of turbulence at the RDT limit, special emphasize is given to the differences in the models of this term and their consequences.

AXISYMMETRIC CONTRACTION AT RAPID DISTORTION LIMIT

Rapid distortion theory of Ribner and Tucker (1953) and Batchelor and Proudman (1954) provides an analytical solution of turbulence with which one can calculate the effect of rapid irrotational mean-velocity distortions on turbulence. According to this theory, only after assuming that the turbulence interacts strongly with the mean flow but only weakly with itself under rapid mean velocity distortions, can the effects of viscous dissipation and non-linear processes be neglected and the development of the velocity fluctuations and/or the vorticity fluctuations under superimposed distortions be calculated. Distortion of mean-velocity in turbulent flows can be encountered in many technological applications, and hence, it is important to know the prediction performance of the available and widely used Reynolds-stress models under conditions where RDT is valid.

Axisymmetric contraction of grid-generated turbulence is one class of homogeneous turbulent flows. This kind of turbulence can approach to the RDT limit under highly strained condition and it can be generated by passing the flow through a grid to generate turbulence and contract it downstream of the grid with a symmetric nozzle as shown in Figure 1. Due to spatial homogeneity, the equations govern-

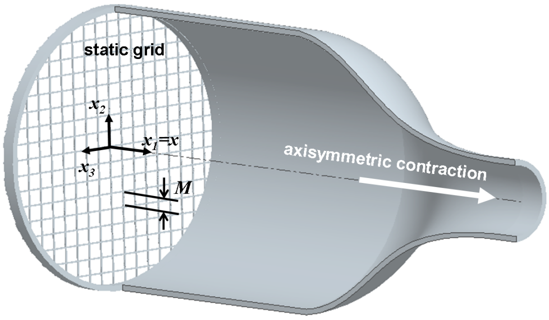

Figure 1: Illustration of axisymmetric contraction of grid-generated turbulence. Note that $x = x_1$ in the present study and M is the mesh size.

ing this turbulence reduce to an initial value problem. For example, the ensemble averaged transport equations for the Reynolds-stresses ($\overline{u_i u_j}$) become

$$\frac{\partial \overline{u_i u_j}}{\partial t} = \underbrace{-\overline{u_j u_k}\frac{\partial \overline{U}_i}{\partial x_k} - \overline{u_i u_k}\frac{\partial \overline{U}_j}{\partial x_k}}_{P_{ij}} + \underbrace{\frac{1}{\rho}\left(\overline{p\frac{\partial u_j}{\partial x_i}} + \overline{p\frac{\partial u_i}{\partial x_j}}\right)}_{\Pi_{ij}=\Pi^f_{ij}+\Pi^s_{ij}} - 2\,\nu \underbrace{\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}}_{\epsilon_{ij}}, \quad (1)$$

where the unknown terms in this equation system are only the pressure-strain correlations (or pressure-velocity gradient correlations) (Π_{ij}) and the dissipation correlations (ϵ_{ij}). Π_{ij} can be further splinted to fast $\left(\Pi^f_{ij}\right)$ and slow $\left(\Pi^s_{ij}\right)$ pressure-strain terms. At the RDT limit, it is expected that the dissipation and slow pressure-strain correlations vanish, i.e.

$$\epsilon_{ij} \cong 0, \quad (2a)$$
$$\Pi^s_{ij} \cong 0. \quad (2b)$$

Hence, the only unknown correlation remains to be Π^f_{ij}. Moreover, since this kind of turbulence is axisymmetric in terms of its statistical properties, the Reynolds-stress tensor

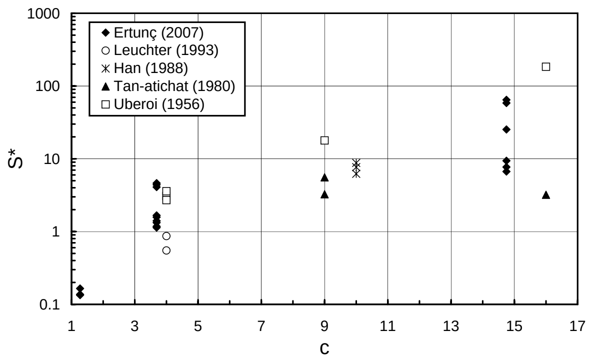

Figure 2: Mean strain parameters applied in the present study and in the literature.

composed of only diagonal elements:

$$\overline{u_i u_j} = \begin{pmatrix} \overline{u_1 u_1} & 0 & 0 \\ 0 & \overline{u_2 u_2} & 0 \\ 0 & 0 & \overline{u_3 u_3} \end{pmatrix}. \tag{3}$$

Moreover, the mean value of the transverse and lateral velocities are zero and their fluctuations deliver equal Reynolds-stresses, i.e. $\overline{u_2 u_2} = \overline{u_3 u_3}$.

The axisymmetric contraction of grid-generated turbulence is selected for the validation of various Reynolds-stress models at the RDT limit because of the reduction in complexity of dynamics and kinematics of turbulence mentioned above. The rapidity of the distortion can be quantified by the mean strain rate parameter S^*, which is defined as the ratio of the time scale of distortion to the time scale of the turbulence:

$$S^* = \frac{Sq^2}{\epsilon} \tag{4}$$

where $S = (0.5 S_{ij} S_{ij})^{1/2}$, $S_{ij} = \partial \overline{U}_i / \partial x_j$ and $\epsilon = \epsilon_{ii}$ is the trace of the dissipation tensor of turbulence and $q^2 = \overline{u_i u_i}$ is the trace of $\overline{u_i u_j}$. S reads $\sqrt{3}/2 \partial \overline{U}_1 / \partial x_1$ for axisymmetric strain. It is accepted that for $S^* >> 1$, RDT can describe the evolution of the turbulent velocity fluctuations.

Available data in the literature on axisymmetric contraction are analyzed in terms of their rapidity. The S^* values of the experiments in Ertunç (2007), as well as, of other important contraction experiments in the literature are shown in Figure 2. For the calculation of S^*, the average strain rate along the whole nozzle was considered, whereas the turbulent energy and the dissipation were chosen to be the ones at the inlet of the contractions. As may be observed in Figure 2, except the experiments of Leuchter (1993), for all experiments in the literature S^* is larger than 1 and applied strains are not uniform, i.e. S_{ij} is not constant along the contraction. In the experiments of Uberoi (1956) with $c = 16$, S^* reaches values close to 200. As far as the experimental validation of RDT with Uberoi's data is concerned, good agreement was observed only up to contraction ratio of 4, although his nozzle with $c = 16$ satisfied RDT requirements.

It is well-known from the vortex stretching theory of Prandtl (1932) and RDT that positive strain of nearly isotropic turbulence promotes the anisotropies of the turbulent stresses such that the mean square of the longitudinal (axial) velocity fluctuation ($\overline{u_1 u_1}$) decreases and the transverse and lateral fluctuations ($\overline{u_2 u_2} = \overline{u_3 u_3}$) increase by increased contraction ratio ($c = A_{outlet}/A_{inlet}$) of nozzles. Moreover, direct numerical simulation studies of Rogallo (1981) and Lee (1985), on the effect of strain on homogeneous turbulence, have shown continuous increase of Reynolds-stress anisotropy, irrespective of the applied strain.

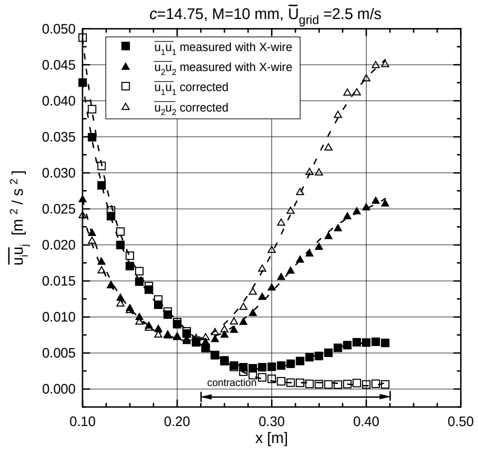

Figure 3: Anomalous and corrected Reynolds-stress measurements in a 14.75:1 contraction.

In contrary to analytical and numerical findings, in all experimental investigations known to authors with nozzles having contraction ratios higher than 9, initially $\overline{u_1 u_1}$ decreased and $\overline{u_2 u_2}$ increased, but, after a while, $\overline{u_1 u_1}$ started to increase along the contraction and, consequently, anisotropy of $\overline{u_i u_j}$ decreased. Experimental studies showing this kind of Reynolds-stress development are those reported by Uberoi (1956), , Hussain and Ramjee (1976), Ramjee and Hussain (1976), Tan-atichat *et al.* (1980) and Han (1988). Ertunç (2007) and Ertunç and Durst (2006) showed that the increase of $\overline{u_1 u_1}$ occurs because of experimental contaminations in the measured data like the flow rate fluctuations intrinsic in the flow facility, the electronic noise of the measurement system and, the most important one, the spatial resolution problem of the X-wire probe. Ertunç and Durst (2006) termed this phenomenon as *high contraction ratio anomaly* and corrected the contraction measurements for these artifacts and, consequently, showed that the longitudinal stress component monotonically decreases and the transverse and lateral stresses increases as shown in Figure 3.

The axisymmetric contraction experiments mentioned above, which satisfy the conditions of RDT ($S^* >> 1$) and show high contraction ratio anomaly, do not agree with RDT, since the experimental contaminations, discussed in detail by Ertunç and Durst (2006), damp the measured amplification of the anisotropy of Reynolds-stresses in contractions. Among the experimental measurements of Ertunç (2007), cases with contraction ratio of 14.75 can fairly be accepted to be rapid enough such that RDT applies to the turbulent flow. Hence, these measurements are used as reference data within the present work for checking the prediction performance of various turbulence models at RDT limit.

The anisotropy of Reynolds-stresses is increasingly being used to construct Reynolds-stress turbulence (RST) models; see for example Rotta (1951), Lumley (1978), Lee *et al.* (1986) , Sjögren and Johansson (2000) and Jovanović *et al.* (2003) . The anisotropy of the Reynolds-stress tensor is defined as:

$$a_{ij} = \overline{u_i u_j} - 1/3\delta_{ij} \tag{5}$$

The first component of the anisotropy tensor (a_{11}) of 6 ex-

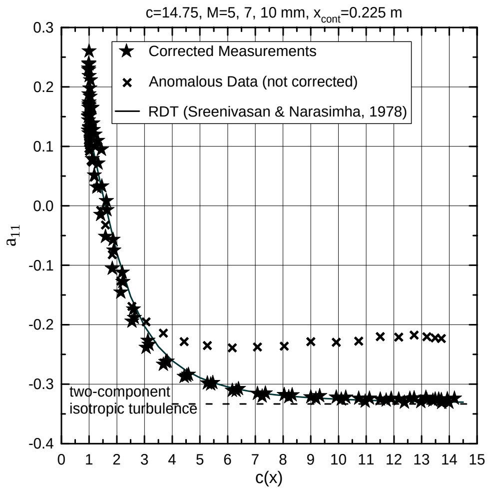

Figure 4: Anomalous and corrected Reynolds-stress measurements in a 14.75:1 contraction.

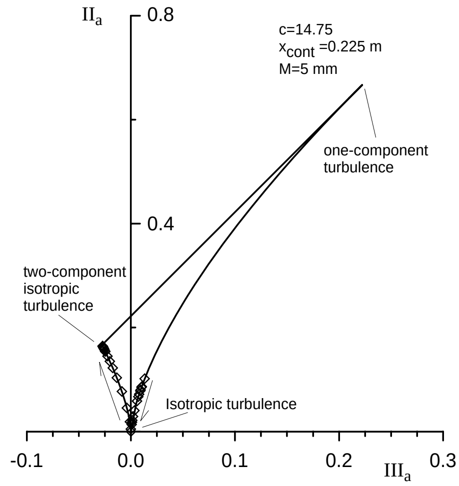

Figure 5: Anisotropy of the corrected Reynolds-stresses approaches to the two-component isotropic limit shown on AI-map.

periments of Ertunç (2007) (involving 2 different contraction location after the grid and 3 different meshes) are plotted with respect to the local contraction ratio $c(x)$ along the 14.75:1 contraction in Figure 4. It can be said that the anisotropy of turbulence are dominantly controlled by the local contraction ratio. Since the measured Reynolds-stresses have a slight anisotropy downstream of the nozzle, they are compared with the modified RDT theory of Sreenivasan and Narasimha (1978) which can handle anisotropic initial values. This comparison is shown in Figure 4 and depicts a fairly good agreement. Moreover, when the second and third invariants of a_{ij}

$$II_a = a_{ij}a_{ji}, \quad (6a)$$

$$III_a = a_{ij}a_{jk}a_{ki}, \quad (6b)$$

are plotted for one of the measured data on the anisotropy invariant map (AI-map) of Lumley (1978) (Figure 5), it can be seen that turbulence approaches to the two-component anisotropic state as it is contracted along the nozzle. According to the authors' best knowledge, these measurements are the first among its class which approached two-component isotropic state and for the first time Ertunç (2007) verified RDT for nozzles with contraction ratios higher than 9. In Figure 4, the uncorrected (anomalous) data is also shown to demonstrate why the data in the literature could not match with the predictions obtained by RDT.

The integral length scales are obtained by integrating the two-point correlation functions as follows:

$$L_{u_{1x}} = \int_0^\infty R_{11}(x)\mathrm{d}x, \quad (7a)$$

$$L_{u_{2x}} = \int_0^\infty R_{22}(x)\mathrm{d}x \quad (7b)$$

where the point-correlation functions are defined as:

$$R_{11}(x) = \frac{\overline{u_1(0)u_1(x)}}{\overline{u_1(0)u_1(0)}}, \quad (8a)$$

$$R_{22}(x) = \frac{\overline{u_2(0)u_2(x)}}{\overline{u_2(0)u_2(0)}}. \quad (8b)$$

It is well known from RDT and vortex stretching theory (Prandtl, 1932) that the longitudinal length scales increase along the contractions. The amount of elongation of these length scales along the contractions is quantified by the following elongation parameters

$$c_{L_{u_{1x}}} = \frac{L_{u_{1x}}(x)}{L_{u_{1x}}(0)}, \quad (9a)$$

$$c_{L_{u_{2x}}} = \frac{L_{u_{2x}}(x)}{L_{u_{2x}}(0)}. \quad (9b)$$

The correlation functions (8 a, b) were obtained from the auto-correlation measurements and the integral length scales in the longitudinal direction (7 a, b) were evaluated. The elongation of the integral length scales along 14.75:1 contraction are presented in Figure 6. The increase in the integral length scale of u_2 fluctuations $\left(L_{u_{2x}}\right)$ is the best indication of stretching in the flow direction and the elongation of this quantity is well predicted with the RDT (see Townsend, 1976 p.75). The integral length scale of u_1 fluctuations $\left(L_{u_{1x}}\right)$ also increased and, as can be seen in Figure 6, the amount of elongation can be approximated by $\sqrt{c}$ as the vortex stretching theory suggests. The deviation of $c_{L_{u_{1x}}}$ from $\sqrt{c}$ at high contraction ratios is related to the increasing effect of flow rate pulsations in the measured correlation coefficients and the spatial resolution problems of the X-wire probe.

The anisotropy of the one-dimensional spectra can be examined when the ratio of u_1 spectra $[E_{u_1}(k_1)]$ to u_2 spectra $[E_{u_2}(k_1)]$ is plotted versus normalized wavenumber as shown in Figure 7. In the same plot the theoretical ratio for isotropic spectra in the inertial subrange, which is 4/3, is also provided (see Pope 2000, p. 229, for details). This figure clearly shows that the anisotropy is more emphasized in the low wavenumber range and turbulence tends to be isotopic in the high wavenumber range. The high ratios at the exit of the 14.75:1 contraction indicate the vortex filaments are well aligned parallel to the symmetry axis and almost all of the energy of turbulence is transfered to the these filaments, which are responsible for u_2 fluctuations. The deviation the ratio from 4/3 line downstream of the contraction implies that the anisotropy penetrates deep into the smaller scales as turbulence is strained along the contraction.

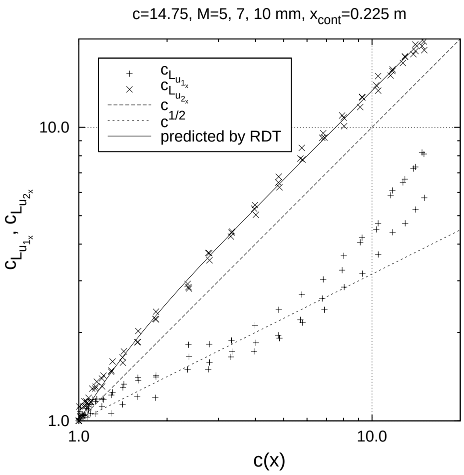

Figure 6: Integral length scale development along the 14.75:1 contraction (Ertunç, 2007).

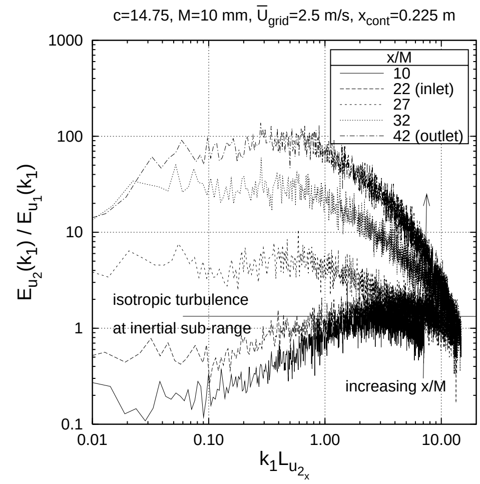

Figure 7: Development of the ratio of the transverse to longitudinal spectra along the 14.75:1 contraction (Ertunç, 2007).

RST MODELS AT RDT LIMIT

The Reynolds-stress models of Launder *et al.* (1975) (LRR-model) , Speziale, *et al.* (1991) (SSG-model), Sjögren and Johansson (2000) (SJ) and Jovanović *et al.* (2003) (JOB) are compared with the experimental data presented above and the RDT of Sreenivasan and Narasimha (1978). The two LRR models are the so called isotropic production (LRR_IP) and quasi isotropic (LRR_QI) models (Pope 2000, p.423). The dissipation (ϵ_{ij}) and slow pressure-strain $\left(\Pi^s_{ij}\right)$ processes are still active at the inlet of the contraction, thus, the predictions made by activation and inactivation of these terms, are provided. The comparison of the Reynolds-stress anisotropy for inactive (zero) ϵ_{ij} and Π^s_{ij}, as in the case of RDT, is shown in Figure 8 a. As shown before, RDT and measurements matches very good. The model of Sjögren and Johansson (2000) also delivers very good anisotropy prediction. Other selected models show significant deviation from the experiments. Interestingly, anisotropies do not change remarkably when the slow pressure-strain and dissipation terms are active (Figure 8 b). This implies that under the selected flow conditions dissipation and slow pressure-strain

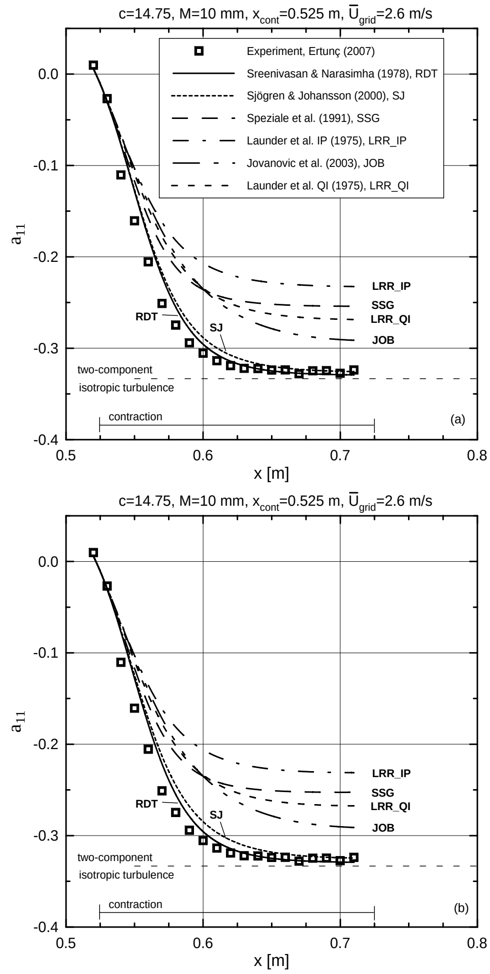

Figure 8: Comparison of a_{11} measurements with those predicted by various RST models and RDT (a) $\epsilon = 0$ and $\Pi^s_{ij} = 0$, (b) all terms are active. Legend of this figure is used in the following figures.

terms do not significantly influence the anisotropy development. The comparison of measured and predicted turbulent kinetic energy $\left(k = q^2/2\right)$ reveals the damping effect of dissipation and slow pressure-strain terms (Figure 9a and b). In Figure 9a, the measurements of k show slightly higher values than RDT downstream in the nozzle, which is not expected under the assumed conditions. It is suspected that either the non-uniform distortion in the nozzle or the uncertainties of the correction methods are causing this discrepancy.

In contrast to other correlations, the fast pressure-strain term $\left(\Pi^f_{ij}\right)$ becomes significant at RDT limit. It is defined as:

$$\Pi^f_{ij} = 2\overline{U}_{\ell,k}\left(M_{kji\ell} + M_{ikj\ell}\right) \qquad (10)$$

where the fourth order tensor for homogeneous turbulence is

$$M_{ijk\ell} = \int_{-\infty}^{\infty} \phi_{ij} \frac{\kappa_\ell \kappa_k}{\kappa_m \kappa_m} d\kappa, \qquad (11)$$

in which ϕ_{ij} is velocity spectrum tensor and κ is the wavenumber vector. Since $M_{ijk\ell}$ is linear in spectrum, the Π^f_{ij} should also be linear in Reynolds-stresses. However from the study of Shih *et al.* (1990), it is clear that realizable regions in the anisotropy map of the linear models are severely limited and higher order models are more suitable for the cases of high anisotropy rates. Figure 10 shows normalized

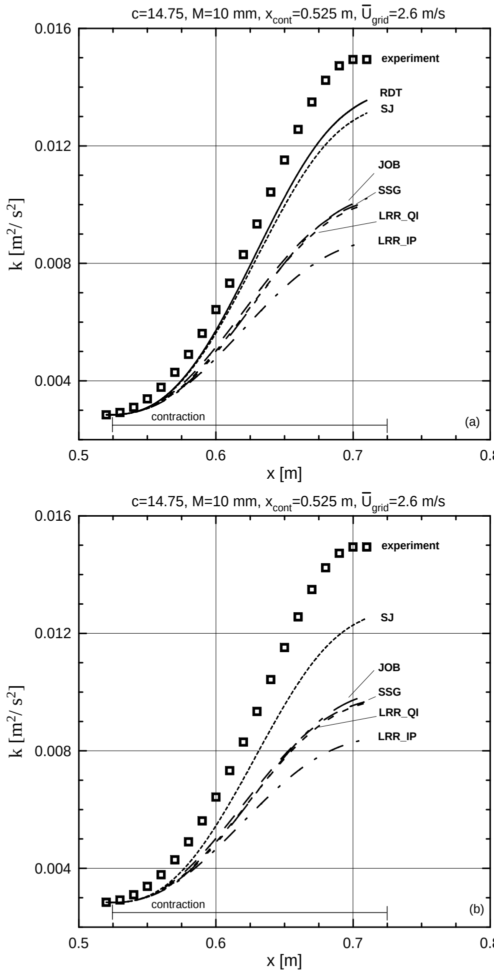

Figure 9: Comparison of turbulent kinetic energy $\left(k = q^2/2\right)$ measurements with those predicted by various turbulence models and RDT (a) $\epsilon = 0$ and $\Pi^s_{ij} = 0$, (b) all terms active.

fast pressure-strain term with respect to second invariant of a_{ij} for selected models and RDT. This comparison clearly shows that the major discrepancy between models, caused due to the differences in modeling of the fast pressure-strain term. All the linear models poorly predict fast pressure-strain term as the anisotropy increases, whereas the 4^{th} order model of Sjögren and Johansson (2000) shows very good agreement. It should be noted that SSG-model is also nonlinear, but it is also not capable of predicting Π^f_{ij}. Furthermore, Figure 10 depicts that the fast pressure-strain term of LRR models and SSG would not vanish at the two-component isotropic limit. The artifacts in the models of Π^f_{ij} has already been extensively discussed by Johansson and Hallbäck (1994) .

The positive normalized value of Π^f_{ij} in Figure 10 means that Π^f_{ij} takes the sign of the corresponding strain rate, i.e. it acts like a source for $\overline{u_1u_1}$ and sink for $\overline{u_2u_2}$ along a contraction. The Π^f_{ij} models, which delivers values at a much higher level than those predicted by the the RDT, consequently, prevents $\overline{u_2u_2}$ from increasing more and $\overline{u_1u_1}$ from decreasing further. In other words, failure of this kind in Π^f_{ij} damps the development of the anisotropy (Figure 8). Even though Π^f_{ij} does not appear in the transport equation of the turbulent kinetic energy $\left(k = q^2/2\right)$, the production

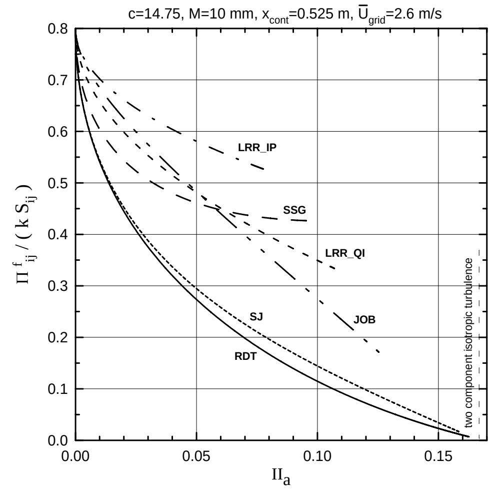

Figure 10: Comparison of the normalized Π^f_{ij} predicted by various turbulence models and RDT ($\epsilon = 0$ and $\Pi^s_{ij} = 0$).

of the turbulent kinetic energy (P_k) is effected because of Π^f_{ij}'s influence on the anisotropy development (Figure 11). This effect can be best understood when P_k is expanded for axisymmetric contraction case:

$$P_k = \left(\overline{u_2u_2} - \overline{u_1u_1}\right)\frac{\partial \overline{U}_1}{\partial x_1}. \tag{12}$$

Hence the model of Sjögren and Johansson can reach higher anisotropy in comparison to the other models and it predicts more turbulent kinetic energy production than the others.

The anomalous trend of $\overline{u_1u_1}$ in the measurements can also be observed in the prediction of the two LRR models and the SSG model downstream in the contraction as shown in Figure 12. However, it is made clear above, the anomalous increase is due to the inaccurate models of Π^f_{ij}.

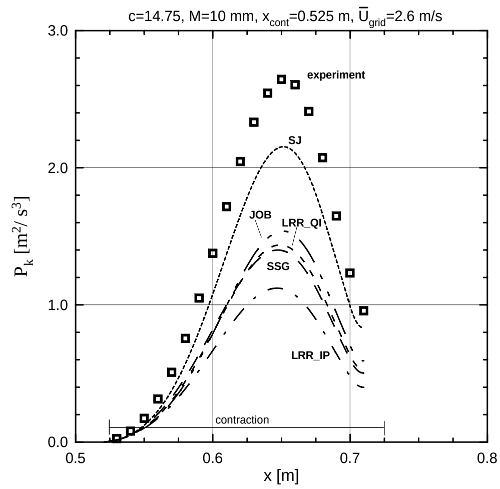

Figure 11: Comparison of turbulent kinetic energy production (P_k) measurements with those predicted by various turbulence models (all terms are active).

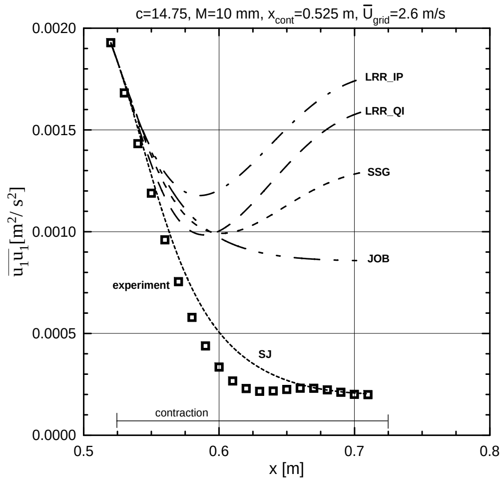

Figure 12: Comparison of $\overline{u_1 u_1}$ measurements with those predicted by various turbulence models (all terms active).

DISCUSSION AND OUTLOOK

Even though the solution provided by RDT is of limited predictive value, it does well describe limiting situations occurring under certain conditions. It is once more shown in this study that at these limiting situations the rapid distortion analysis together with relevant experiments give guidance for the development of Reynolds-stress closures, especially the models of the fast pressure-strain correlation. It is shown that RST models whose fast pressure-strain term can not reproduce that of RDT, influences the anisotropy development and, consequently, production of the turbulent kinetic energy. Furthermore, these can even deliver anomalous trends of Reynolds-stresses in the contraction.

Undistorted and distorted axisymmetric turbulence is an invaluable class of turbulent flows which should be further exploited for turbulence modeling. The fast pressure-strain models can be checked for other types of strains with the help of RDT. The slow pressure-strain correlation and dissipation correlations can be modeled in slowly and moderately distorted axisymmetric turbulence. For this purpose, experimental and direct numerical realizations of axisymmetric turbulence should cover slow and moderate strain rates in a wide range of turbulence Reynolds number and anisotropies. The results of this kind of investigations can easily be incorporated to the Reynolds-stress models using the anisotropy invariants which are well defined for axisymmetric turbulence.

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