MODELING OF A WALL OF RANDOM ROUGHNESS FOR CFD

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ABSTRACT

A new idea is proposed in this work to convert an original rough surface of irregularly indented complicated geometry to a simplified model surface. The purpose is to provide a substitutive simple solid boundary surface which allows a practical computer of ordinary memory size and computational speed to conduct LES/DNS of a flow over a rough surface. As tall and comparatively large roughness peaks are expected to affect dominantly the flow over an irregularly indented surface, it is intended here to pick up those peaks adequately and substitute them by equivalent hemispheres. The first task is to coarsen the original surface to remove fast fluctuating component of surface indentation. In place of coarsening by a filtering function as is common in LES, wavelet multi-resolution analysis is introduced. Then, in order to pick up concerned peaks, we specify a threshold height on the coarsened surface and obtain a map of islands of peaks taller than the threshold plane. These peaks are converted to hemispheres of same volume. Among various kinds of canonical roughness elements, hemisphere is least sensitive with respect to interference with surrounding elements and individual hydrodynamic performance is most easily evaluated. Validation of the model is given by comparing wall drag of channel flows having original complex walls and simple model walls, on the basis of RANS simulation. The simulation results confirmed that the model surface of proposed idea represents original wall with reasonable accuracy.

INTRODUCTION

A vast number of reports have been published on flows over rough walls in the last several decades. The earliest notable works are those written by Nikuradse(1933) and Sclichting(1936) which appeared almost seven decades ago. Most of the reports of flows over rough walls in the literature however, deal with the case of discretely distributed roughness elements. In practice of engineering, randomly indented surfaces appear frequently. In particular, the case where roughness height is not so high as to modify the logarithmic layer substantially is of highest concern, in practice of engineering. For these walls, adequately simplified model of the surfaces is indispensable, since ordinary work-stations used in the practice of engineering do not afford acceptable simulation results within a reasonable time, say a couple of hours. The purpose of modeling is to avoid prohibitively large computer memory and computational speed which is required to resolve complex geometry of a wall. For this, we pick up high and big peaks by a suitable method. Thereafter, we substitute the peaks by roughness elements of a simple shape, namely, hemispheres in the present work. In a simulation of a flow, the roughness elements can be represented only by their fluid force, as demonstrated by Miyake et al. (2000). Our final goal is to compress the forces to shear stresses on a boundary surface, which allow us to reduce the solid boundary to a plane wall. The framework given above requires huge task, before obtaining an established tool for practical applications. The first task we should do is to confirm the validity of the idea to substitute the complex geometry by a group of elements of a simple shape. This paper describes the first half stage of the whole project, namely, methodology of modeling a rough wall of random indentation. Validation tests of the proposed modeling are also conducted by performing a large scale computation.

EXTRACTION OF DOMINANT PEAKS

Turbulent flow over a rough surface of regularly distributed hemispheres was measured by Chen(1975) in a pipe flow of circular cross section. The flow just downstream of a hemisphere was confirmed to have a shear layer originated around at the top of a hemisphere. Meanwhile, recent measurement of Subramanian et al. (2004) of a flow close to a randomly indented surface revealed similar shear layer as Chen observed, at any location above the wall. This suggests that even over a randomly indented rough wall, peaks which control dominantly the local flow can be identified. If those peaks are adequately picked up, they are expected to represent the flow over the original rough wall. So, the first task to do is to identify the concerned peaks.

Coarsening

Height fluctuation of an irregularly indented surface has various wavelength components. But spear-like fine peaks of short wavelength may be of small influence on the flow. In order to remove less important height fluctuation, coarsening of the surface is needed. For this purpose, filtering similar to that employed in LES formulation is used. Wavelet multi-scale analysis is applied here. As a representative example of a rough surface, a square surface of width L is considered. The width in two normal direction x and y may be different, but for simplicity, a square surface is chosen here.

Two-dimensional scaling function $\Phi(x,y)$ is defined as $\Phi(x,y)=N_4(x)N_4(y)$ where $N_4(x)$ is B-spline function of 4th rank (Chui(1992), Sakakibara (1995)). The height is assumed to be given discretely at every grid points separated by δ in both directions. The grid number in each direction is assumed to be 2^J where J is an integer and the larger, the better. The most dense data, i.e., a set of height data given at every grid point, is denoted as $f^{(0)}(k,l)$ where k,l are integral numbers and specify location of grid points, and superscript (j) means the level of resolution. Leveling down from (j) to (j-1) means to cut the components of upper half wavelength in level (j) in each direction, as illustrated in Fig.1. So, wave number area shrinks to a lower quarter corner of the previous level. The removed components are wavelet components of the new level. Height distribution of coarsened surface $f^{(j)}(x,y)$, j=0,-1,-2・・・ is given by

$$f^{(j)}(x,y)=\sum_{k,l}c_{k,l}^{(j)}\Phi_{k,l}^{(j)}(x,y)$$
$$=\sum_k\sum_l c_{k,l}^{(j)}\phi(2^j x-k)\phi(2^j y-l) \qquad (1)$$

where $\Phi(x)=N_4(x)$. The coefficient of the first level surface $c_{mn}^{(0)}$ can be determined by given data $f^{(0)}(k,l)$, by

$$c_{mn}^{(0)}=\sum_k\sum_l f^{(0)}(k,l)\beta_{m+2-k}\beta_{n+2-l} \qquad (2)$$
$$\beta=\sqrt{3}(\sqrt{3}-2), \qquad m=-5\sim 5$$

(Chui(1992), Sakakibara, (1995)). Coefficients of succeeding lower level can be calculated recurrently by

$$c_{mn}^{(j-1)}=\sum_k\sum_l \frac{1}{4}c_{k,l}^{(j)}g_{2m-k}g_{2n-l} \qquad (3)$$

where g_k is decomposition sequence which is specified for each individual scaling function, and for $N_4(x)$, it is given

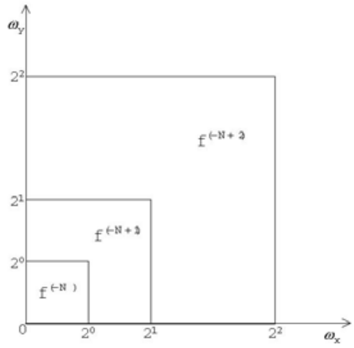

Figure1. Wavenumber area covered by level (j). Level down by one rank makes the area a lower quarter of the previous level.

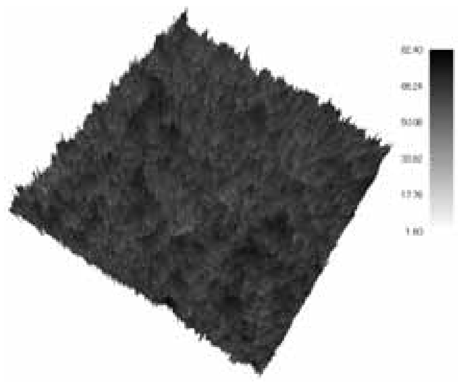

Figure2. Measured rough surface (Sand Paper #1200).

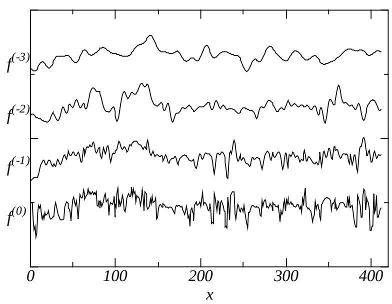

Figure3. Height fluctuation of the measured rough surface of each level, along a straight line passing through the center of the square.

in a table(Sakakibara (1994)). Equation (2) gives $c_{mn}^{(0)}$ and following simple algebra according to Eq.(3) gives whole resolution. Therefore, multi-resolution analysis goes on without laborious work.

Rough Surface

The sample rough surface used in the analysis in the following will be described in this section. Figure 2 is the surface $f^{(0)}(k,l)$ which is taken from a sand paper #1200 and is obtained by measuring its height by a laser height meter.

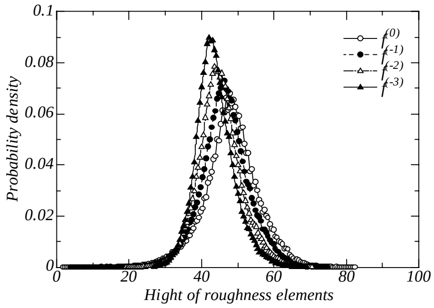

Figure4. Probability density distribution of the height fluctuation, of each level.

$f^{(0)}(k,l)$ is represented by height at 512×512 points of every $2\mu m$ apart in each direction in a square. The finest fluctuation $f^{(0)}(k,l)$ has no wavy component in this particular rough surface. Figure 3 is an example of coarsening and demonstrates one-dimensional height fluctuation along a straight line through the center of the square, of $f^{(0)} \sim f^{(-3)}$ from the bottom to the top. The fluctuation becomes milder gradually as the coarsening advances. Figure 4 shows probability density distribution of absolute value of height fluctuation of each level. It is observed that the fluctuation is of nearly normal distribution in every level *j* and that fluctuation becomes milder as the level comes down. Figure 5 is the coarsened surface of level *j*=-3, *i.e.*, $f^{(-3)}(x,y)$. Compared with $f^{(0)}$, we find that the surface looks pretty coarse.

Model Surface Consisting of Hemispheres

Of several coarsened surface, we adopt the surface $f^{(-3)}(x,y)$ which contains lowest 1/8 wavenumber components of fluctuation in both *x* and *y* direction, of the original rough wall, as the base rough wall to make up model surfaces.

Dominant peaks are picked up from this coarsened surface. The scheme is illustrated in Fig.6 which is in one-dimensional version, for simplicity. Firstly, mean height $\bar{h}$ which is included in the measured height data is removed and new original plane *z*=0 is defined which is shifted upward by $\bar{h}$. Next, the plane of 0-height is further shifted upward by 0.3 h_{rms} where h_{rms} is rms(root-mean-square) value of height fluctuation of the adopted base wall $f^{(j)}(k,l)$. In the present case, the scheme is applied to *j*=-3. Then, the entities projecting above the plane is retained. Each continuous entity is regarded as individual island. Ofthese islands, those lower than 0.3 $h'_{\max}$ where $h'_{\max}$ is the maximum height measured from the finally shifted 0-height plane, are removed. The threshold criterion for the trimming employed here is not unique but alternatives may be available. Equivalent sand-grain height h_s introduced by Schlichting may be available as the threshold value, but it is not a good parameter because it's estimation requires laborious work. Islands of extracted peaks are shown in Fig.7. Since an island is of complicated shape, each is individually converted to a hemisphere whose volume is same as the corresponding individual island. The volume of the island is that above the above-mentioned 0-height plane. Second trimming of hemispheres is conducted to remove small hemispheres whose radius is smaller than that of 40% of the largest one. Finally obtained model surface consisting of hemispheres on a flat plate is shown in Fig.8 where the center of each hemisphere is located at the highest point of corresponding island. The surface is the prototype model and is named Model surface 1, here.

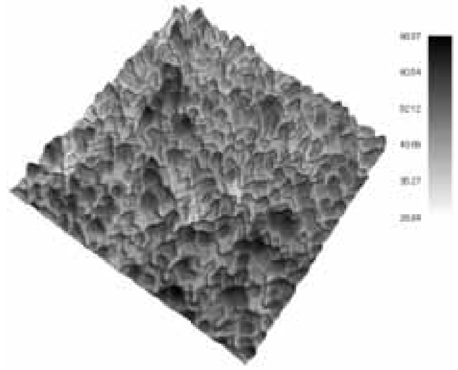

Figure5. The coarsened surface of level *j*=-3, $f^{(-3)}(k,l)$, which is used as the base rough wall in the succeeding procedure.

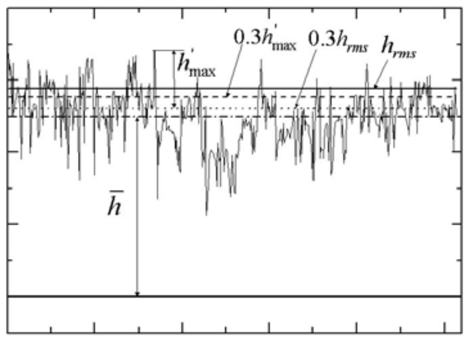

Figure6. Scheme of picking up dominantly influential peaks, illustrated in one-dimensional version, for brevity.

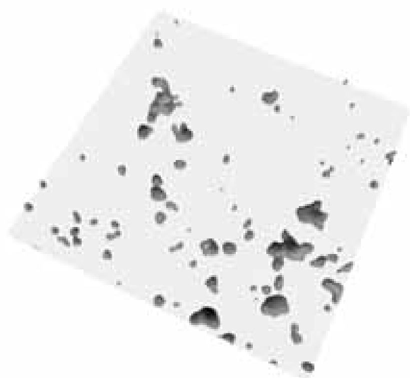

Figure7. Tall roughness islands retained for modeling, from $f^{(-3)}(k,l)$.

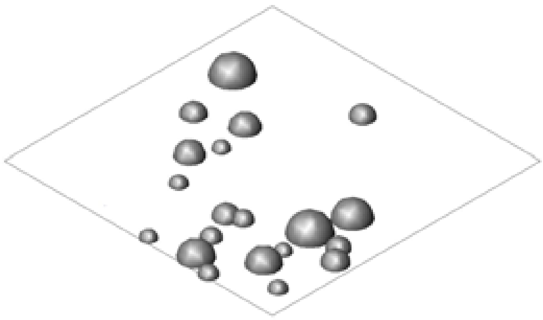

Figure8. Model surface consisting of hemispheres, converted from Fig.7. Model surface 1(prototype model).

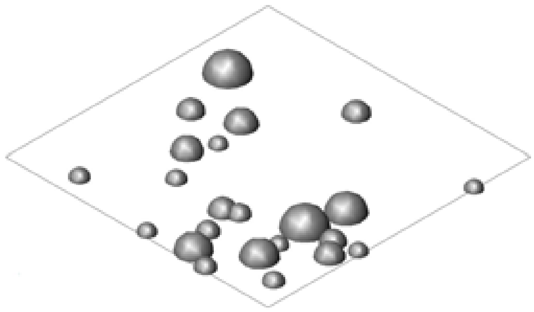

Figure10. Model surface 3 ; Threshold height for tall peaks lowered to $0.1\times h_{rms}^{(0)}$, otherwise kept unchanged from Model 1.

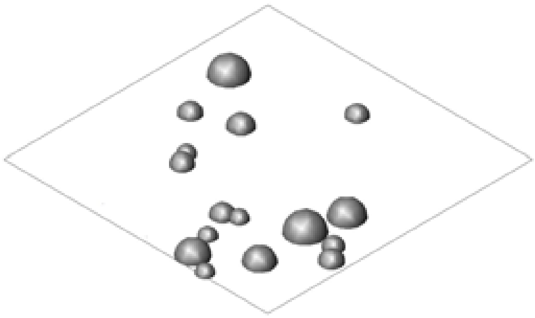

Figure9. Model surface 2 ; Threshold height for tall peaks raised to $0.5\times h_{rms}^{(0)}$, otherwise kept unchanged from Model 1.

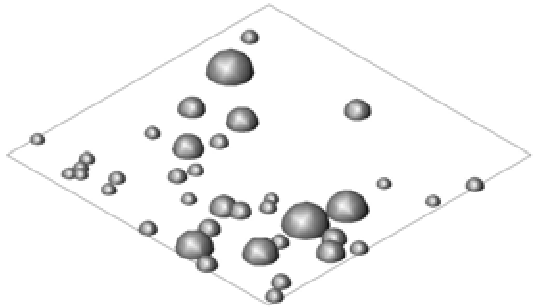

Figure11. Model surface 4, with different level of trimming small hemispheres, otherwise unchanged from Model 1.

Now, how the model surface is modified by the various threshold values in extracting peaks, will be described in the following. In the cases that the first shift of 0-height plane is raised to $0.5\,h_{rms}^{(-3)}$ in place of $0.3\,h_{rms}^{(-3)}$ and otherwise kept same as the prototype model(Fig.8), the model surface becomes as in Fig.9(Model surface 2). Meanwhile, when the first shift of 0-height plane is lowered to 0.1 $h_{rms}^{(-3)}$, it becomes as shown in Fig.10 (Model surface 3). When the threshold of removing small hemispheres in the second trimming process of making up Fig.8 is lowered to 30% in place of 40% of largest hemisphere and otherwise kept unchanged the model surface becomes as shown in Fig.11 (Model surface 4).

Parameters characterizing above-mentioned model surfaces are tabulated in Table.1. It includes N, total number of hemispheres, V_i/V_1 , total volume of hemispheres V_i in the ratio to prototype model(Model 1), $k_{\max}/h_{rms}$, the largest height of hemisphere $k_{\max}$ in the ratio to $h_{rms}^{(0)}$, rms-height of original surface $f^{(0)}(x,y)$ and the minimum height $k_{\min}/h_{rms}^{(0)}$. The threshold values of extracting islands and trimming small hemispheres affect largely on the resulting number of hemispheres, and on total volume as well. The number of hemisphere is reduced in Model 2 but each hemisphere is bigger than in Model 1 and as the consequence, total volume is reduced. While, in Model 3, both number of hemisphere and each hemisphere is larger and consequently, roughness effect is expected to be enhanced.

Simple roughness element other than a hemisphere is also tested. In Table 1, the right-end row is the data for ellipsoid element. In place of hemispheres in making up Fig.8, ellipsoids whose both volume and base area on the 0-height plane are same as the corresponding island are put on a flat plate, otherwise kept unchanged from the prototype model. Roughness elements become lower and more flattened than hemisphere model and consequently, the roughness effect is expected to be suppressed.

Other shape such as cubes and/or cones may be available. The present model is hoped to be simplified further, since the hemisphere model requires still heavy computational load. Drag of each modeled roughness element is hopefully represented by shear force on a flat plane in an improved model. For that, the drag of a roughness element is desirable to be less affected by the neighboring elements. Round bodies having no edges has smaller wake area in general and are expected to be less interactive than for example, a hexahedron such as a cube. This is the reason why we choose hemisphere as a roughness element in the model.

The modeling from coarsened surface of different level is also made up. Table 1 includes the case that the surface $f^{(-4)}(x,y)$ is used as the base rough surface and same procedure as for the model surface 1 is applied. Obtained model surface is close to the prototype model surface, though the small hemispheres being located differently but important large ones, almost identical. Size of hemispheres are slightly larger than in the surface 1 and compensates the reduction of the number. While, in the case of $f^{(-5)}(x,y)$,

Table 1. Parameters characterizing model surfaces

	$f^{(-3)}$				$f^{(-4)}$	Ellipsoid
	Model1	Model2	Model3	Model4		
N	20	16	23	39	19	14
V_i/V_1	1.000	0.776	1.260	1.133	1.139	0.925
$k_{\max}/h_{rms}^{(0)}$	5.227	4.953	5.515	5.227	6.162	3.965
$k_{\min j}/h_{rms}^{(0)}$	2.102	2.260	2.207	1.583	1.877	2.645

N : Number of spheres, V_i/V : Total volume of hemispheres compares with that of Model 1.

$k_{\max}/h_{rms}^{(0)}$: Height of the largest hemisphere, $k_{\min}/h_{rms}^{(0)}$: Height of the smallest hemisphere.

$h_{rms}^{(0)}$: rms-height of rough surface $f^{(0)}$.

indentation becomes wavy and too much flattened and the modeled hemispheres are small and low. So, the surface is not suitable as the base surface. Therefore, the coarsened surface of level -3 is best and -4, acceptable.

VALIDATION TEST OF THE MODEL SURFACE

The model surfaces should be tested in that to what extent the flow over them is close to that over the original rough surface. Since the simulation of a flow over rough surface requires huge computer power, reliable validation test is quite limited within low-order parameters. Here, pressure drop at fully rough wall regime is compared between the model surfaces and the coarsened rough wall $f^{(-3)}(x,y)$. Flow simulation over the original wall $f^{(0)}(x,y)$ is not available even with the largest scale engineering work station.

We conducted simulations in a channel flow between two parallel walls as shown in Fig.11. RANS with two-equation model is carried out. Periodic condition is applied in both flow (x) and spanwise (z) directions and pressure drop Δp in the flow direction is calculated. One periodic width L of the computational square is $L/h_{rms}^{(0)}$ =55.5 which is the width of the square reduced from the measured surface by trimming off imperfect portion of edges caused by coarseneng. Channel width H which is between mean planes for original rough surface and between flat plates for model surfaces, is about 250 times $h_{rms}^{(0)}$ for original rough wall and 50 times the largest hemisphere for each model surface. Commercial software SCRYU/Tetra which is provided by Cradle is used for simulation. Total grid number is about 3 million for model surfaces and 8 million for original surface.

Wall friction coefficient c_f is calculated by

$$c_f = \frac{2D_w}{\rho \bar{u}^2} = \frac{\Delta p}{\rho \bar{u}^2}\frac{H}{L} \tag{4}$$

where D is wall drag per unit area on one surface of a channel, ρ is fluid density and $\bar{u}$ is global mean velocity in a channel.

Figure 12 shows wall friction coefficient c_f vs. Reynolds number $\mathrm{Re}_H = \bar{u}H/\nu$. In the figure, broken line is for smooth wall channel

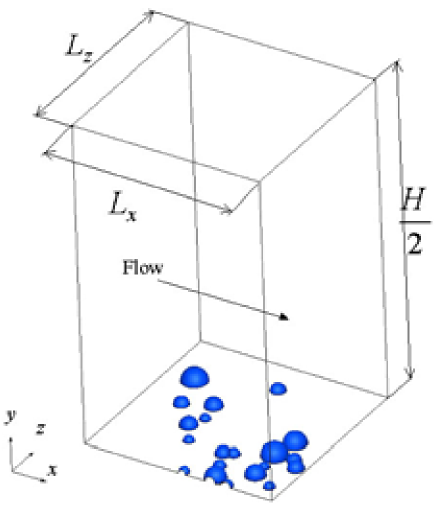

Figure12. Computational box of a periodic channel for flow simulation. $L_x = L_z = 55.5h_{rms}^{(0)}$ and half width $H/2$ ～ $125\times h_{rms}^{(0)}$

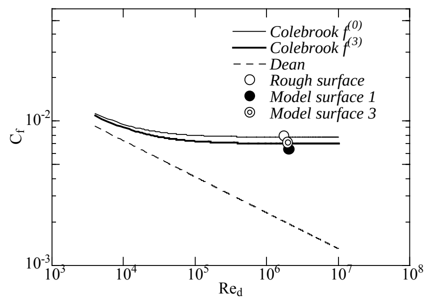

Figure13. Wall drag coefficient c_f. Lines : rough wall by Colebrook, broken line : smooth wall by Dean. Symbols : present simulation, ○ : original rough surface $f^{(-3)}$, ● : model surface 1 and ◎ : model surface 3.

$$c_f = 0.073\,\mathrm{Re}_H^{-1/4} \tag{5}$$

which was given by Dean(1978). Two curves are calculated according to

$$\frac{1}{\sqrt{\lambda}} = -\log\left(\frac{\varepsilon}{3.7H} + \frac{2.51}{\mathrm{Re}_h\sqrt{\lambda}}\right) \tag{6}$$

which is the empirical formula, given by Colebrook(1938-1939) for circular pipe flow. The curves in the figure are converted from λ to c_f by $c_f = \lambda / 4$. The parameter ε representing roughness height is ambiguously defined and Moody(1944) indicated range of its magnitude for various rough walls but no suggestion is given for irregularly indented rough walls. Solid curve is for $\varepsilon = h_{rms}^{(-3)}$ and the fine curve is for $\varepsilon_{rms}^{(0)}$. Since $h_{rms}^{(-3)} / h_{rms}^{(0)}$, as shown in Fig.4, c_f obtained by Colebrook's formula is larger for fine curve than for thick curve. It should be noted that this result does not indicate which h_{rms} is superior to represent random roughness height.

The symbols in Fig.12 are obtained by above-mentioned simulations. The symbol ○ is for original surface $f^{(-3)}$ (Fig.5) as it is without any further modification. The symbol coincide with fine curve which is for $h_{rms}^{(0)}$, suggesting that rms-value of height fluctuation of an irregularly indented surface is a good parameter substituted for ε.

Friction coefficient c_f obtained by model surface 1(Fig.8) is given by the symbol ● (c_f =0.0064). It is observed that the symbol ● is pretty close to ○ (c_f =0.0078) and accordingly, the model surface can be regarded as a good approximation of the original one. However, ● is slightly smaller than ○ and suggests that the roughness effect is modeled too weakly. As for model surface 2(Fig.9), simulation was not performed since the data in Table 1 indicates that its roughness effect is more suppressed than model surface 1. The model surface 3 includes more roughness peaks than model surface 1 and enhanced roughness effect is expected. It gives ◎ (c_f =0.0071), which is closer to ○. The reason for this is that firstly, total volume of retained hemispheres is larger and secondly the height of maximum hemisphere is larger than in other model surfaces. Model surface 4 and ellipsoid model as well, is expected to give smaller drag coefficient than model surface 3 and is not worth while conducting simulation.

Although more careful examinations are necessary with respect to a couple of threshold levels for picking up dominant peaks, the above-mentioned results support strongly our fundamental idea and procedure of the proposed modeling. In particular tall and massive entities are confirmed to be most influential to the flow.

CONCLUSION

Present paper proposes a novel idea to model irregularly indented rough wall to reduce computational road in flow simulations. Large memory size and accordingly small time advancement in computation are avoided by simplified solid surface configuration. Conclusions are as follows.

1. Coarsening of a irregularly indented rough surface by wavelet multi-scale analysis has been proposed and has been confirmed to be a nice tool.
2. A procedure to pick up peaks which are dominantly influential to the flow and to substitute them by a group of hemispheres is proposed.
3. Flow simulations by RANS simulation for channel flows confirmed the validity of the proposed idea.
4. Physics of rough wall flow that tall and massive peaks are vitally important for the flow has been revealed.

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