# REYNOLDS-STRESSES MODIFICATION IN PARTICLE-LADEN TURBULENT CHANNEL FLOWS

Maarten J. Bijlard

Multi-Scale Physics Department, Delft University of Technology Prins Bernhardlaan 6, 2628 BW Delft, The Netherlands M.J.Bijlard@tudelft.nl

Luís M. Portela\* Multi-Scale Physics Department, Delft University of Technology Prins Bernhardlaan 6, 2628 BW Delft, The Netherlands L.Portela@tudelft.nl

#### ABSTRACT

In turbulent flows laden with small heavy particles the interaction between the particles and the turbulence can promote a large modification in the turbulence characteristics of the flow. These effects, already significant in homogeneous isotropic turbulence, can become much larger in wall-bounded flows. In this work, we study the Reynoldsstresses modification in a fully-developed turbulent channel flow laden with small heavy spherical particles, using standard point-particle direct numerical simulations with twoway coupling and Stokes drag. Our results indicate that the particles promote a strong increase in the anisotropy of the Reynolds-stresses, with the turbulence becoming more onedimensional, with a modest modification in the streamwise velocity fluctuations, and a large decrease in the normalwise and spanwise velocity fluctuations and in the Reynolds shear-stress. We show that these modifications can be understood in terms of: (i) the kinetic (Reynolds) shear-stress of the particles, which, by a simple balance of momentum in the streamwise direction, leads to a decrease in the Reynolds shear-stress of the fluid, (ii) a large disruption in the Reynolds-stresses budget, with a large decrease in the production in the streamwise direction and in the redistribution between the streamwise and normalwise and spanwise directions, and (iii) the agglomeration of the particles into streamwise-elongated streaks.

# INTRODUCTION

Wall-bounded turbulent flows laden with small heavy particles are important in numerous situations, like: catalytic reactors, risers and downers, coal combustors, pneumatic conveying, etc. (Portela and Oliemans, 2006). In the case of small heavy particles, the interaction with the turbulence is mostly through an exchange of momentum, determined by the particle inertia, and "wake effects", taking explicitly into account the particle size, are not likely to play an important role. Since the particles are heavy, the interaction between the particles and the turbulence can promote a large modification in the turbulence characteristics of the flow (a phenomenon sometimes called "turbulence modulation"), even with a small particle-volume-fraction. These effects, already significant in homogeneous isotropic turbulence (e.g., Squires and Eaton, 1990; Boivin et al., 1998), can become much larger in wall-bounded flows (Kulick et al., 1994). However, the mechanisms involved, and the reasons why these effects can become much stronger in wall-bounded flows are still not well understood.

In this work, we study the Reynolds-stresses modification in a fully-developed turbulent channel flow laden with small heavy spherical particles, using standard point-particle Eulerian-Lagrangian direct numerical simulations with twoway coupling and Stokes drag. Contrary to previous works (e.g., Li et al., 2001; Yamamoto et al., 2001) we consider two-way coupling (i.e., the simultaneous forcing of the particles by the flow and of the flow by the particles) with a simple linear drag force (Stokes drag) and, on purpose, neglect any other coupling mechanisms (inter-particle interactions and gravity). This allows us to focus exclusively on the particle-turbulence interaction in the simplest possible way. We should note, however, that, in an actual flow, for large particle-concentrations inter-particle collisions play an important role (Li et al., 2001).

First, we briefly describe the numerical set-up. Then, we present the results of the Reynolds-stresses modification, and explain how this modification can be understood in terms of: (i) a balance of momentum in the streamwise direction, (ii) the budget of the Reynolds-stresses in the streamwise, spanwise and normalwise directions, and (iii) the agglomeration of the particles into large-scale structures near the wall (streamwise-elongated particle streaks). The combination of these mechanisms can explain why the turbulence modification can be much stronger than in homogeneous isotropic turbulence.

# NUMERICAL SET-UP

The situation under consideration is sketched in figure 1. The streamwise, spanwise and normalwise directions are denoted by the indexes x, y and z, or 1, 2 and 3 (when using standard Cartesian tensor notation), respectively. The velocities in the the streamwise, spanwise and normalwise directions are also denoted by U, V and W, respectively. Unless otherwise noted, all quantities are expressed in wall-units (i.e., non-dimensionalized using the wall-shear velocity  $u_{\tau}$ , and the kinematic viscosity,  $\nu$ ). The superscript "+" is used to denote a quantity in wall-units. The subscript "p"

<sup>\*</sup>Corresponding author.



Figure 1: Particle-laden channel flow.

is used to denote a quantity associated with the particles. We use a standard Eulerian-Lagrangian point-particle direct numerical simulation, which is adequate provided that the particles are significantly smaller than the relevant flow scales (Portela and Oliemans, 2001). In this approach, the interaction between the particles and the fluid is considered through a force applied at the center of the particle. Since the volume-fraction of particles is very small, the effect of the particles on the continuity equation is neglected. The interaction between the particles and the fluid is felt through an exchange of momentum: the Navier-Stokes equation contains an extra particle-forcing term, and is solved together with the equation of motion of each particle. We used linear (Stokes) drag for the force acting on the particles, and did not consider gravity and inter-particle collisions.

For the numerical solution we use a standard two-step predictor-corrector solver for incompressible flows. The flow is driven by a pressure gradient imposed along the streamwise direction, and we impose periodic boundary conditions both in the streamwise and spanwise directions. The particles are tracked using a modified second-order Runge-Kutta scheme, with elastic bouncing (specular reflection) at the walls. Details can be found in Portela and Oliemans (2003).

We used an uniform grid in the periodic (streamwise and spanwise) directions and an hyperbolic-tangent gridstretching in the normalwise direction. Several simulations were performed with different grids and domain sizes, in order to check the independence of the results. Two Reynolds numbers (based on H and  $u_{\tau}$ ) were considered:  $Re_{\tau} = 360$ and  $Re_{\tau} = 500$ . The particle diameter was always  $D_p =$ (1/1000)H, and two particle-fluid density ratios were considered:  $\rho_p/\rho = 2000$  and  $\rho_p/\rho = 8000$ , corresponding to particle relaxation-times approximately equal to  $\tau_p^+ = 14$ and  $\tau_p^+ = 58$  for  $Re_{\tau} = 360$ , and  $\tau_p^+ = 28$  and  $\tau_p^+ = 111$ for  $Re_{\tau} = 500$ . Apart from the unladen and one-way coupling simulations (i.e., not taking into account the forcing of the flow by the particles), two mass-fractions (mass of the particles divided by the mass of the fluid) were considered :  $\phi_m = 0.16$  and  $\phi_m = 0.65$ .

The simulations were started without particles. After a statistically-steady state was reached, the particles were introduced in the flow with a randomly-uniform distribution. After their introduction, the number of particles remained constant. When one particle leaves the domain, it is reintroduced with the same velocity at the opposite side.

Due to the turbophoresis, the particles are pushed towards the wall, and the concentration there keeps increasing with time. This process is very slow, but eventually a statistically-steady situation is reached (Portela et al., 2002)



Figure 2: Concentration profile (volume-fraction of the dispersed phase,  $\overline{\phi}_v$ ) for  $Re_\tau = 360$ .

with a very high concentration of particles at the wall, as shown in figure 2. As observed previously by several authors (e.g., Eaton and Fessler, 1994; Rouson and Eaton, 2001; Portela and Ferrand, 2005), the concentration very near the wall is far from uniform and the particles organize themselves into streamwise-elongated streaks which are closely related with the low-speed fluid streaks, as shown in figure 3. After the particle-laden flow reached a statisticallysteady situation, the simulation was continued and statistics were obtained using uncorrelated fields extracted during a large interval of time ( $\Delta t^+ > 10^4$ ).

## **RESULTS AND DISCUSSION**

The modification in the Reynolds-stresses promoted by the particles is shown in figure 4. We can notice that the particles promote a strong increase in the anisotropy of the Reynolds-stresses, with a small change in the streamwise velocity fluctuations, and a large decrease in the normalwise and spanwise velocity fluctuations and in the Reynolds shear-stress. Clearly, the particle mass-fraction has a major influence on the magnitude of these effects, which can be very strong even at moderate loadings. Even though less pronounced than the effect of the mass-fraction, the flow Reynolds number and the particle relaxation-time also affect the magnitude of the turbulence modification. This modification is smaller at a higher Reynolds number, and, for the same mass-fraction, is smaller for higher values of the particle-relaxation time.

The change in the Reynolds shear-stress can be understood using a simple balance of momentum in the streamwise direction. In wall-units the total shear-stress acting on the fluid is given by:

$$\tau_{\rm tot} = 1 - \frac{2z^+}{Re_\tau} = \frac{\partial \overline{U}}{\partial z^+} - \overline{u'w'} + \int_0^{z^+} \overline{F}_x \, dz \qquad (1)$$

where  $\overline{U}$ ,  $\overline{u'w'}$  and denote the mean streamwise-velocity and Reynolds shear-stress. From a balance of momentum for the particles, we have that the mean back-forcing of the particles on the fluid,  $\overline{F}_x$ , is given by:

$$\overline{F}_x = -\frac{\partial}{\partial z} \left( \phi_m \overline{u'_p w'_p} \right) \tag{2}$$

where  $\overline{u'_p v'_p}$  denotes the Reynolds-averaged particle kineticshear-stress (particle Reynolds-shear-stress). Combining equations 1 and 2 we have:

$$\tau_{\rm tot} = 1 - \frac{2z^+}{Re_\tau} = \frac{\partial \overline{U}}{\partial z^+} - \left(\overline{u'w'} + \phi_m \overline{u'_p w'_p}\right) \qquad (3)$$



Figure 3: Streaky pattern in a plane close to the wall ( $z^+ \approx 5$ ), for  $Re_{\tau} = 360$ ,  $\tau_p^+ = 58$  and  $\phi_m = 0.65$ . Left: snapshot of the streamwise velocity fluctuation. Right: snapshot of the position of the particles (sheet thickness:  $\Delta z^+ \approx 2$ ).

From the previous equations, it is clear that, when the viscous stresses are not dominant and/or the changes in the mean streamwise velocity of the fluid are small, the existence of the particle Reynolds-shear-stress leads to a decrease in the fluid Reynolds shear-stress. From previous studies with one-way coupling, it is known that there exists a good correlation between the particle and the fluid Reynolds-stresses. Actually, as explained by Portela et al. (2002), this is closely related with the well-known "local-equilibrium assumption" of Tchen (Hinze, 1975). Therefore, the presence of the particles has to promote a decrease in the shear-stress of the fluid. The contribution of the several terms in equation 1 to the total shear-stress of the fluid is shown in figure 5. We can notice that most of the changes in the Reynolds shear-stress of the fluid can be attributed directly to the back-forcing of the fluid; i.e., the increase in the total shear-stress due to the particle Reynolds-shear-stress is mostly compensated by the decrease in the Reynolds shear-stress of the fluid. It is interesting to also note that, as observed by other authors (e.g., Kulick et al., 1994), the particles do not promote a large change in the streamwise velocity of the fluid. Clearly, this is consistent with the fact that the viscous stresses of the fluid do not play an important role, except very close to the wall.

The large decrease in the Reynolds shear-stress of the fluid (together with the small change in the mean streamwise-velocity of the fluid) leads to a large decrease in the production of the streamwise Reynolds-stress (the production of the streamwise Reynolds-stress is the product of the Reynolds shear-stress and the gradient of the mean streamwise-velocity of the fluid), which in turn leads to a large decrease in the normal Reynolds-stresses in the spanwise and normalwise directions. This can be understood in terms of the budget of the normal Reynolds-stresses. The transport equation for the Reynolds-stresses of the fluid can be written symbolically as (Li et al., 2001):

$$\frac{D\tau_{ij}}{Dt} = \mathcal{P}_{ij} + \Pi_{ij} + \mathcal{T}_{ij} + \mathcal{D}_{ij} - \epsilon_{ij} + \epsilon_{ij}^{(p)} \qquad (4)$$

where  $\mathcal{P}_{ij}$  is the production,  $\Pi_{ij}$  is the pressure-strain,  $\mathcal{T}_{ij}$  is the turbulent diffusion,  $\mathcal{D}_{ij}$  is the viscous diffusion,  $\epsilon_{ij}$  is the viscous dissipation, and  $\epsilon_{ij}^{(p)}$  is the "direct forcing" of the particles. The traceless pressure-strain tensor re-distributes energy from the normal Reynolds-stress in the streamwise direction to the normal Reynolds-stresses in the spanwise and normalwise directions. In a fully-developed channel flow, the material derivative of the Reynolds-stresses and the production of the spanwise and normalwise Reynolds-stresses are equal to zero, and  $\Pi_{ij}$  acts essentially as a sink in the streamwise direction and a source in the spanwise and normalwise directions. Therefore, the large decrease in the production of the streamwise Reynolds-stress, associated with the large decrease in the Reynolds shear-stress, leads to a large decrease in  $\Pi_{11}$ , which leads to a large decrease in the source term for the spanwise and normalwise directions ( $\Pi_{22}$  and  $\Pi_{33}$ ), as can be observed in figures 6 and 7, which results in a large decrease in the spanwise and normalwise Revnolds-stresses. Note that, except close to the wall, the streamwise budget is dominated by the production, pressure-strain and dissipation, whereas the spanwise and normalwise budgets are dominated by the pressure-strain and dissipation (figure 7). It is quite remarkable that the magnitude of the "directforcing term" is mostly negligible; i.e., the Reynolds-stresses modifications are due to "indirect effects", associated with the dynamics of the particle-turbulence interaction.

The budget of the normal Reynolds-stresses, together with the large decrease in the Reynolds shear-stress, can explain the large decrease in the spanwise and normalwise Reynolds-stresses, however, it is not directly related with the behavior of the streamwise Reynolds-stress, which is basically determined by the transport of fluid and particles in the direction normal to the wall, which, due to the large gradients in the normal direction leads to large velocity fluctuations in the streamwise direction. Essentially, for both the fluid and the particles, this is a "mixing-length mechanism", which is not necessarily directly connected with an increase or decrease in the different terms of the energy budget or with "local equilibrium" considerations (Portela et al., 2002).

A closer look at the streamwise Reynolds-stress, shows that very close to the wall (roughly,  $z^+ < 20$ ) occurs a small decrease in the streamwise Reynolds-stress, whereas further away from the wall occurs a small increase. The modest modifications in the streamwise Reynolds-stress are not due to the (small) "direct-forcing term" of the particles. Similarly to the spanwise and normalwise directions, the modifications in the streamwise Reynolds-stress are also due to "indirect effects", associated with the dynamics of the particle-turbulence interaction, however, these "indirect effects" are different from the ones that promote the large changes in the spanwise and normalwise Reynolds-stresses.

The small increase in the streamwise Reynolds-stress further away from the wall can be explained by the normalwise gradient in the particle concentration and the streamwiseelongated particle-streaks. Essentially, when the particle-



Figure 4: Turbulence intensity modification. (a) streamwise  $u'_{rms}$ , (b) shear-stress  $-\overline{u'w'}$ , (c) spanwise  $v'_{rms}$ , (d) normalwise  $w'_{rms}$ .

streaks are lifted from the wall they carry a large inertia together with a pattern that contains a significant variation in the streamwise velocity (the particles are preferentially concentrated in low streamwise-velocity regions), which, in turn, leads to an increase in the fluctuation of the streamwise velocity of the fluid. This is consistent with the one-way coupling results of Portela et al. (2002), which also observed a small increase in the streamwise Reynolds-stress of the particles. The association between the particle-streaks and the increase in the streamwise Reynolds-stress is also consistent with the two-way coupling simulations with interparticle collisions of Li et al. (2001). For large particleconcentrations, collisions play a crucial role, and since they smear the particle-streaks, Li et al. (2001) observed a small increase in the streamwise Reynolds-stresses (similar to the one described above) only at the smaller particleconcentrations.

Far from the wall, the budget of the streamwise Reynolds-stress is dominated by the production, dissipation and pressure-strain, and an increase in the streamwise velocity-fluctuation does not have a significant effect on this balance. However, at the wall, the production and pressurestrain are equal to zero, and very close to the wall the budget of the streamwise Reynolds-stress is dominated by the balance between the viscous diffusion and the viscous dissipation (the only non-zero terms at the wall). Due to the overall large decrease in the viscous dissipation (consequence of the large overall decrease in the production), there exists also a decrease in the viscous dissipation close to the wall, which, in turn, must lead to a decrease in the viscous diffusion very close to the wall. Since the viscous diffusion is proportional to the normalwise gradient of the streamwise Reynolds-stresses, this results in a decrease in the streamwise Reynolds-stress very close to the wall.

#### CONCLUSION

In turbulent flows laden with small heavy particles, the interaction between the particles and the turbulence can promote a large modification in the turbulence characteristics of the flow. These effects, already significant in homogeneous isotropic turbulence, can become much larger in wall-bounded flows. Our results indicate that the particles promote a strong increase in the anisotropy of the Reynolds-stresses, with a modest modification in the streamwise velocity fluctuations (a small decrease very close to the wall and a small increase further away from the wall), and a large decrease in the spanwise and normalwise velocity fluctuations and in the Reynolds shear-stress.

The Reynolds-stresses modifications are not associated with the "direct-forcing term" of the particles, which is mostly negligible, they are due to "indirect effects", associated with the dynamics of the particle-turbulence interaction. The Reynolds-stresses modifications are essentially associated with three major effects: (i) the kinetic (Reynolds) shear-stress of the particles, which, by a simple balance of momentum in the streamwise direction leads to a decrease in the Reynolds shear-stress of the fluid, (ii) a large disruption in the Reynolds-stresses budget, with a large decrease in the production in the streamwise direction and in the re-distribution between the streamwise and spanwise and normalwise directions, and (iii) the agglomeration of the particles into large-scale structures near the wall (streamwise-elongated particle streaks). The decrease



Figure 5: Individual shear-stress contributions for  $Re_{\tau} = 360$ , for unladen and laden flow ( $\phi_m = 0.65, \tau_p^+ = 58$ ); •: laden.

in the production in the streamwise direction is due mostly to the first effect, and combined with a large decrease in the re-distribution between the streamwise and spanwise and normalwise directions leads to a large decrease in the velocity fluctuations in the normalwise and spanwise directions The modification of the streamwise velocity fluctuation is more subtle and its slight increase (except for a slight decrease very close to the wall) appears to be connected with the very-large inhomogeneities in the particle distribution.

We should note that all these mechanisms are intrinsically wall-effects and/or shear-flow effects, which are not present in homogeneous isotropic turbulence. This indicates that it might not be appropriate to try to understand and/or model the turbulence modification in particle-laden wall-bounded flows using homogeneous isotropic turbulence concepts.

## REFERENCES

Boivin, M., Simonin, O., and Squires, K. D., 1998, "Direct Numerical Simulation of Turbulence Modulation by Particles in Isotropic Turbulence", *Journal of Fluid Mechanics*, Vol. 375, pp. 235-263.

Eaton, J. K., and Fessler, J. R., 1994, "Preferential Concentration of Particles by Turbulence", *International Journal of Multiphase Flow*, Vol. 20 (supplement), pp. 169-209.

Hinze, J. O., 1975, *Turbulence*, second edition, McGraw-Hill, New York.

Kulick, J. D., Fessler, J. R., and Eaton, J. K., 1994, "Particle Response and Turbulence Modification in Fully-Developed Channel Flow", *Journal of Fluid Mechanics*, Vol. 277, pp. 109-134.

Li, Y. M., McLaughlin, J. B., Kontomaris, K., and Portela, L., 2001, "Numerical Simulation of Particle-Laden Turbulent Channel Flow", *Physics of Fluids*, Vol. 13 (10), pp. 2957-2967.

Portela, L. M., Cota, P., and Oliemans, R. V. A., 2002, "Numerical Study of the Near-Wall Behavior of Particles in Turbulent Pipe Flows", *Powder Technology*, Vol. 125, pp. 149-157.

Portela, L.M., and Ferrand, V., 2005, "Formation of Near-Wall Particle-Streaks in Particle-Laden Wall-Bounded Turbulent Flows", *Proceedings of the Fourth International Symposium on Turbulence and Shear Flow Phenomena*, June 27-29, Williamsburg VA, USA, J.A.C. Humphrey et al., ed., Vol. 1, pp. 57-62.

Portela, L. M., and Oliemans, R. V. A., 2001, "Direct



Figure 6: Effect of the mass-loading on the the redistribution of energy term between the normal Reynoldsstresses  $(\Pi_{\alpha\alpha})$ ; —: unladen; •:  $\phi_m = 0.16$ ;  $\diamond: \phi_m = 0.65$ .

and Large-Eddy Simulation of Particle-Laden Flows Using the Point-Particle Approach", in: Geurts, B. J., Friedrich, R., and Métais, O. (eds.), *Direct and Large-Eddy Simulation IV*, pp. 453-460, Kluwer, Dordrecht.

Portela, L. M., and Oliemans, R. V. A., 2003, "Eulerian-Lagrangian DNS/LES of Particle-Turbulence Interactions in Wall-Bounded Flows", *International Journal for Numerical Methods in Fluids*, Vol. 43 (9), pp. 1045-1065.

Portela, L. M., and Oliemans, R. V. A., 2006, "Possibilities and Limitations of Computer Simulations of Industrial Turbulent Dispersed Multiphase Flows", *Flow Turbulence and Combustion*, Vol. 77, pp. 381-403.

Rouson, D. W. I., and Eaton, J. K., 2001, "On the Preferential Concentration of Solid Particles in Turbulent Channel Flow", *Journal of Fluid Mechanics*, Vol. 428, pp. 149-169.

Squires, K. D., and Eaton, J. K., 1990, Particle Response and Turbulence Modification in Isotropic Turbulence", *Physics of Fluids A*, Vol. 2 (7), pp. 1191-1201.

Yamamoto, Y., Pothoff, M., Tanaka, T., Kajishima, T., and Tsuji, Y., 2001, "Large-Eddy Simulation of Turbulent Gas-Particle Flow in a Vertical Channel: Effect of Considering Inter-Particle Collisions", *Journal of Fluid Mechanics*, Vol. 442, pp. 303-334.



 $\phi_m \simeq 0.16, \tau_p^+ \simeq \tilde{P}^8_{13}$  pure 7: Normal Reynolds-stresses budget modification (equation 4), for  $Re_{\tau} = 360$ . Left: unladen. Right: laden. Top: streamwise. Center: spanwise. Bottom: normalwise. For more clarity, in the w'w' budget (bottom), the two pressure-velocity correlations are lumped together (taken out of the turbulent-diffusion correlation), leading to a pressure-velocity correlation  $\tilde{\Pi}_{33} = 2 \frac{w'}{\rho} \frac{\partial p'}{\partial z}$  and a turbulent velocity-transport correlation  $\tilde{\mathcal{D}}_{33} = -\frac{\partial}{\partial z} \overline{w'w'w'}$ .

(b)

PSfrag replacements

PSfrag replacements

 $\begin{array}{c} 1336\\ \phi_m = 0\\ \textbf{(d)}\\ \phi_m \simeq 0.16, \ \tau_p^+ \simeq 58 \end{array}$