LOCALLY VARYING DISSIPATION SCALES IN TURBULENT FLOWS

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ABSTRACT

The classical picture of one mean dissipation scale which is known as the Kolmogorov length can be extended to a continuum of local dissipation scales in order to capture for the intermittent nature of small-scale turbulence. The distribution of these local dissipation scales is found to agree with a recently calculated analytical form. We discuss consequences of this generalized perspective for the decay of energy spectra in the far-dissipation range and the turbulent mixing of passive scalars in the viscous-convective range at Schmidt numbers larger than unity.

MOTIVATION

Turbulence is a real multiscale phenomenon in classical physics. In case of a turbulent bulk flow, the scales are usually divided into an inertial range which is characterized by a constant energy flux from large to small scales and a viscous or dissipation range, respectively. In the classical theory of turbulence, the so-called Kolmogorov dissipation length η_K marks the small-scale end of the inertial range. Nonlinear advection is then balanced by viscous dissipation and the well-known relation for the Kolmogorov dissipation length

$$\eta_K = \left(\frac{\nu^3}{\langle\epsilon\rangle}\right)^{1/4}, \tag{1}$$

follows with $\langle\epsilon\rangle$ being the mean energy dissipation rate and ν the kinematic viscosity of the turbulent fluid (Kolmogorov, 1941). On scales $\ell < \eta_K$, the fluid is dominated by viscous effects and expected to be spatially smooth in contrast to the much more irregular turbulent motion on scales $\ell > \eta_K$ where most studies on turbulence are concentrated. However, there are good reasons for a closer inspection of the dynamics on Kolmogorov and even on sub-Kolmogorov scales.

Definition (1) does not capture for the intermittent nature of the small-scale turbulence, in particular for that of the energy dissipation rate field. The intermittent fluctuations of the dissipation field, which is given as

$$\epsilon(\mathbf{x},t) = \frac{\nu}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 \tag{2}$$

with $i,j = x,y,z$, are however a well-known fundamental building block of our understanding of the small-scale nature of turbulence (Kolmogorov, 1962). This intermittency means that, on one hand, very steep gradients as appearant in thin and stretched vortex tubes can be expected to have diameters that are finer than η_K. Spatial variations across sub-Kolmogorov scales will thus exist. On the other hand, ambient regions will exist with typical spatial variations larger than η_K. It seemed therefore plausible to include

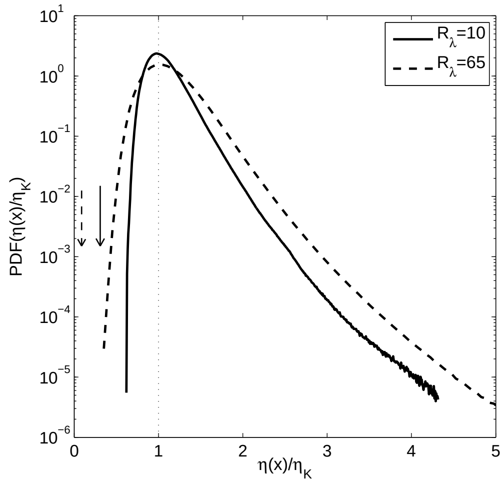


Figure 1: Probability density function of $\eta(\mathbf{x})$ for two different Taylor microscale Reynolds numbers R_λ as indicated in the legend. The vertical arrows mark the grid resolution of the corresponding simulation. Data are from Runs 1 and 4 (see table 1).

these fluctuations and to extend the definition of a single dissipation scale to that of a range of local dissipation scales. This idea was put forward first within the multifractal formalism and resulted in the intermediate dissipation range model by Frisch and Vergassola (1991). In a nutshell, the finest (largest) local dissipation scales correspond there with the roughest (smoothest) subsets of velocity increments in the inertial range. Spatial roughness means nothing else but steep local gradients. The ideas which were outlined in the multifractal approach are based on algebraic scaling properties of velocity subsets in the inertial range which are hard to determine in simulations or experiments.

Therefore other approaches to locally varying dissipation scales have been suggested. A simple way to extend definition (1) is the substitution of the mean energy dissipation rate by its local fluctuating value (Schumacher et al., 2005), i.e.

$$\eta(\mathbf{x}) = \left(\frac{\nu^3}{\epsilon(\mathbf{x})}\right)^{1/4}. \tag{3}$$

The finest local dissipation scales are then assigned with the maxima of the dissipation field. Figure 1 shows the corresponding scale distributions for two different Reynolds numbers. With increasing value of the Taylor microscale Reynolds number R_λ the tails of the distribution get fatter to both sides. However, this extension (3) still relies on the classical dimensional estimate which combines the viscosity and the dissipation rate to a length scale.

A more recent approach to this issue which stayed close to the Navier-Stokes equations was suggested by Yakhot (2006). He derived a relation for the local dissipation scales, which will be denoted as η in the following, starting from exact equations for the n-th order velocity increment moments. This result is also related to the dissipative anomaly in turbulence. Loosely speaking, this anomaly means that for the limit of vanishing viscosity velocity gradients become infinitely large such that the dissipation rate (which contains basically the product of both) remains finite. The relation which follows from this calculation is

$$\nu \approx \eta \delta_\eta u = \eta |u(x+\eta) - u(x)| \,. \tag{4}$$

We see, that the dissipation scale η is now considered as a field fluctuating in space and time, simply since the velocity increments $\delta_\eta u$ will fluctuate and the r.h.s. of (4) is constant. Starting from (4), it is possible to derive an analytical expression for the probability density function (PDF) of the local dissipation scales, $Q(\eta, Re)$. Furthermore, Eq. (4) suggests an implicit way to determine the local dissipation scales from direct numerical simulations, namely via velocity increments taken over scales which are smaller and larger than the Kolmogorov scale. Figure 2 shows an instantaneous isolevel set of local dissipation scale field η. It underlines the fluctuating character of the local dissipation scales arising from (4) very nicely.

NUMERICAL SIMULATIONS

The Navier-Stokes equations that describe here a homogeneous and locally isotropic turbulent flow are solved by a standard pseudospectral method with a 2/3 de-aliasing. Time advancement is done by a second-order predictor-corrector method. The statistics of the turbulent velocity field is kept in a statistically stationary state by a large-scale volume forcing. More details on the scheme that sustains turbulence are described in Schumacher et al. (2007). The passive scalar fields, which are advected in such flow, are sustained statistically stationary by a constant mean scalar gradient. The magnitude of the gradient remained unchanged throughout the parameter studies. More details on the passive scalar parameters can be found in Schumacher et al. (2005). The superfine numerical mesh which we will apply here limits the Taylor microscale Reynolds numbers to moderate values up to $R_\lambda \sim 100$ even for resolutions of up to $N^3 = 2048^3$ points. We will see later in the text that these resolutions are necessary for the study at hand. It opens however the opportunity to explore systematically Reynolds numbers trends in the dissipation range. Further turbulence parameters of the pseudospectral simulations are summarized in table 1.

THEORETICAL PREDICTION

In the following, we line out the calculation of the distribution of local dissipation scales as suggested by Yakhot (2006). The evaluation of the distribution is based on the Mellin transform (Courant 1989). It allows the calculation of the symmetric part of the PDF of the longitudinal velocity increments across a scale ℓ, denoted as $P_\ell(\delta_\ell u)$, on the basis of the corresponding increment moments of all orders and is given as

$$P_\ell(\delta_\ell u) = \frac{1}{i\pi\delta_\ell u} \int_{-i\infty}^{+i\infty} \mathrm{d}n \; (\delta_\ell u)^{-n} \, \langle (\delta_\ell u)^n \rangle \tag{5}$$

Table 1: DNS parameters: $R_\lambda = \sqrt{15/(\langle\epsilon\rangle\nu)}\sigma_L^2$ is the Taylor-microscale Reynolds number, and Re the large scale Reynolds number with the integral scale L and σ_L. The spectral resolution is indicated by $k_{max}\eta_K$ where $k_{max} = \sqrt{2}N/3$. The mean energy dissipation rate is $\langle\epsilon\rangle = 0.1$.

Run	N	ν	L	R_λ	Re	$k_{max}\eta_K$
1	512	1/30	1.02	10	12	33.6
2	1024	1/75	0.92	24	32	33.6
3	1024	1/200	0.76	42	74	15.9
4	1024	1/400	0.69	65	143	9.6
5	2048	1/400	0.69	64	140	19.2
6	2048	1/1000	0.66	107	339	9.7

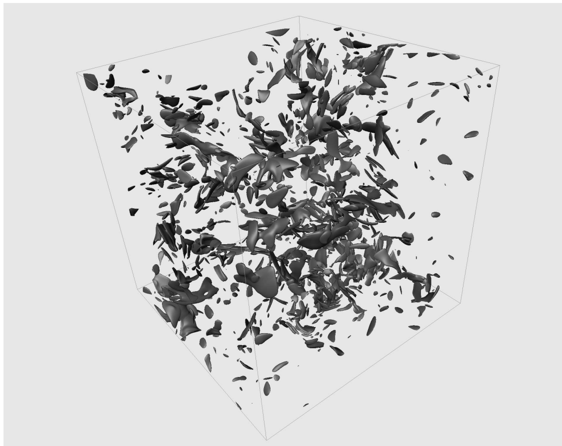


Figure 2: Spatial distribution of local dissipation scales. An instantaneous snapshot of a simulation with 1024^3 grid points resolution is shown (see Run 4 in table 1). Isosurfaces are plotted at the level $\eta = 4\eta_K/3$.

By using the result that the increment moments are Gaussian distributed at the outer scale of turbulence, L, we can write down the following expression for the increment moments in the inertial range of turbulence, $\eta_K \le \ell \le L$,

$$\langle (\delta_\ell u)^{2p} \rangle = (2p-1)!! \, \sigma_L^{2p} \left(\frac{\ell}{L} \right)^{\xi_{2p}} , \tag{6}$$

where $\sigma_L = \langle (\delta_L u)^2 \rangle^{1/2}$. It is then used that for $p > 0$ the factorial can be substituted by an Gaussian integral to

$$(2p-1)!! = \frac{2^p}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \mathrm{d}x \, \exp(-x^2) x^{2p} \,. \tag{7}$$

The large scale Reynolds number will be defined as

$$Re = \frac{\sigma_L L}{\nu} \,. \tag{8}$$

The inertial range scaling exponents ξ_{2p} of the longitudinal structure functions (see equation (6)) can be approximated well by the following polynomial for the lowest moment orders ($p > 0$)

$$\xi_{2p} = 2ap - 4bp^2 \,, \tag{9}$$

with $a \approx 0.383$ and $b = (3a-1)/9 \approx 0.0166$. Constants a and b have been chosen such that $\xi_0 = 0$ and $\xi_3 = 1$ hold. The latter is nothing else but the famous Kolmogorov 4/5-th law (1941). The increment moments are evaluated at the crossover region between inertial and viscous range, i.e. exactly for scales η. Recall that at the lower end of the inertial range relation (6) still holds and we have

$$\langle(\delta_\eta u)^{2p}\rangle = (2p-1)!!\,\sigma_L^{2p}\left(\frac{\eta}{L}\right)^{2ap-4bp^2}\,, \qquad (10)$$

Equations (7) and (10) are inserted into the Mellin transform for $n = 2p$ and it follows

$$P_\eta(\delta_\eta u) \;=\; \frac{2}{i\sqrt{\pi^3}\delta_\eta u}\int_{-i\infty}^{+i\infty}\mathrm{d}p\int_{-\infty}^{+\infty}\mathrm{d}x\,\exp(-x^2)\,\times$$
$$\times 2^p x^{2p}(\delta_\eta u)^{-2p}\sigma_L^{2p}\left(\frac{\eta}{L}\right)^{2ap-4bp^2}. \qquad (11)$$

The distribution $P_\eta(\delta_\eta u)$ is now transformed into the distribution of the local dissipation scales $Q(\eta, Re)$. The relation that connects velocity increments with scales is given by equation (4) which is made an equation now by introducing an additional constant c of order one,

$$\eta\delta_\eta u = c\nu\,,. \qquad (12)$$

Recall that relation (4) was an approximate result. Consequently, $\delta_\eta u = c\nu/\eta$ can be substituted in (11). With $P_\eta(\delta_\eta u)\mathrm{d}\delta_\eta u = Q(\eta, Re)\mathrm{d}\eta$ one gets the PDF for the local dissipation scales via

$$Q(\eta, Re) = -\frac{c\nu}{\eta^2}P_\eta\left(\frac{c\nu}{\eta}\right). \qquad (13)$$

The integral resulting from (10) and (12) is solved by a saddle point approximation for small p. One gets the final expression

$$Q(\eta, Re) = \frac{1}{\pi\eta\sqrt{b\log\left(\frac{L}{\eta}\right)}}\int_{-\infty}^{+\infty}\mathrm{d}x\,\times$$
$$\times\exp\left[-x^2 - \frac{\left(\log\left(\frac{\sqrt{2}xRe}{c}\left(\frac{\eta}{L}\right)^{a+1}\right)\right)^2}{4b\log\left(\frac{L}{\eta}\right)}\right], \qquad (14)$$

which will be evaluated by numerical quadrature for a direct comparison with the DNS results. The PDF of the local dissipation scales is supported on scales $0 \le \eta \le L$ only.

SIMULATION RESULTS

Dissipation scale distribution

The calculation of $Q(\eta, Re)$ from the simulation data works as follows. The scale ℓ is fixed as an integer multiple of the grid spacing, $\ell = n\Delta$. The longitudinal velocity increments with respect to ℓ are determined at each grid vertex in all three directions. If the relation $\ell u_\ell/\nu \approx 1$ holds, the grid site is counted for $Q(\ell, Re)$. The resulting distributions are shown in figure 3. The inset of this figure underlines the necessity for the large spectral resolution applied here. We compare the distributions of a standard resolution case with the present one. All other parameters were left the same. It can be seen, that the whole left tail of the distribution cannot be resolved in the standard resolution case. It is noted that the number of data points for the standard- and high-resolution cases was about the same order of magnitude such that statistical convergence issues can be ruled out.

The main picture of the figure compares our data for Runs 1,4 and 6 with the theoretical prediction from (14). We see that the distributions coincide quite well in the core and right tail of the PDF with the analytical shape. The nearly algebraic decay for scales $\eta > \eta_K$ does not vary significantly with the Reynolds number, neither for the data nor for the theory. Deviations arise in the left tail, i.e. for the finest local dissipation scales. Since we substitute $\eta \sim 1/\delta_\eta u$, high-order moments of the local dissipation scale do not exist. They correspond with large negative increment moments of the velocity field and thus with the left tail.

It can be observed that the maximum of the distribution gets shifted from 1.2 η_K at $R_\lambda = 10$ to 1.4 η_K at $R_\lambda = 107$. The right tail of the distributions $Q(\eta, Re)$ which corresponds with increments over larger distances remains nearly insensitive to the increase of the Reynolds number. All data collapse. They have been shifted in this figure for a better visibility only, the data for $R_\lambda = 65$ by one order of magnitude and the data for $R_\lambda = 107$ by two orders, respectively. The left tail however becomes slightly fatter with increasing Reynolds number which indicates an increasing probability of very fine scales to appear. Increasingly finer scales go in line with increasing degree of the small-scale intermittency in turbulence, i.e. with larger breakouts of the velocity gradients magnitudes to very large magnitudes (see Schumacher et al., 2007). The small-scale end of the support of the PDFs is denoted as the scale η_{min}. It can be considered as the lower end of the intermediate dissipation range that was suggested by Frisch and Vergassola (1991) within the multifractal model. Exactly these scales will be assigned with the steepest velocity gradients. The theoretical model by Yakhot (2006) predicts an estimate for this scale,

$$\eta_{min} = LRe^{-1}. \qquad (15)$$

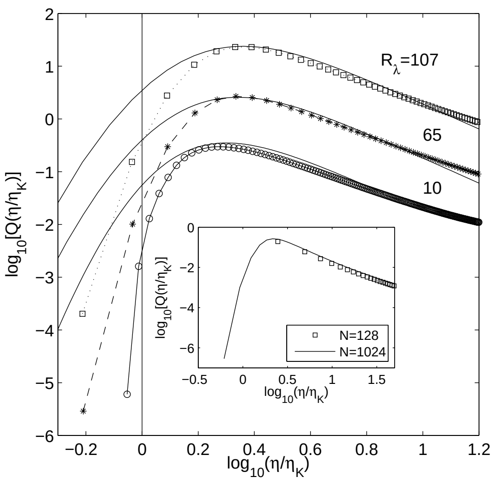
Note that relation (15) puts a stronger constraint on direct numerical simulations in terms of resolution. The standard requirement, that the Kolmogorov length is the smallest scale which has to be resolved, results in a weaker Reynolds number dependence as given by

$$\eta_K = LRe^{-3/4}. \qquad (16)$$

We have compared the smallest scale of the support of $Q(\eta, Re)$ with the prediction (15) from the theory. It was found that η_{min}^{DNS} decreases more slowly with respect to the Reynolds number. The value of η_{min}^{DNS} reached $0.6\eta_K$ for the largest Reynolds number in Run 6. To conclude, ever finer sub-Kolmogorov scales are clearly excited for increasing Re, but not as strong as predicted by Yakhot (2006).

Far-dissipation range energy spectra

One resulting question from our studies above is if the increasing degree of small-scale intermittency in physical space manifests as well for the energy spectra in the crossover to the dissipation range or even in the far-dissipation range. We recall here that Kolmogorov himself postulated a universal form of the energy spectrum $E(k)$ reaching from the inertial range deep down to the viscous scales (Kolmogorov, 1941). Since then several analytical attempts have been made to determine the form of the decay of the energy spectrum in the dissipation range. They left however unspecified constants (Heisenberg, 1948; Kraichnan, 1959; Foias et al., 1990; Gagne and Castaing 1991) or considered an infinitely extended range of excited scales which is not present in real-life flow (Sirovich et al., 1994; Lohse and Müller-Groehling 1995). The generally accepted form of this exponential de-



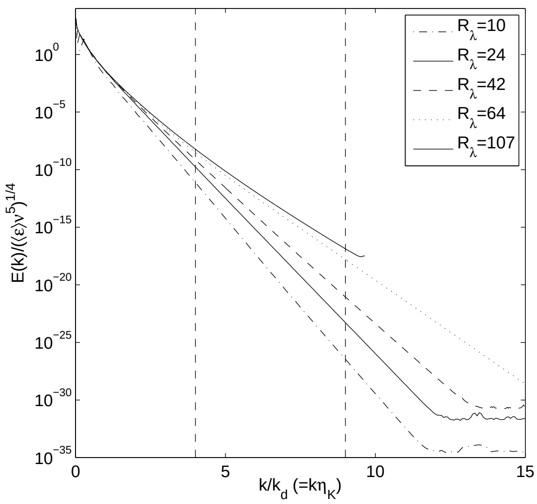


Figure 3: Comparison of numerical and theoretical results for the probability density function (PDF) of the local dissipation scale field η at a given Reynolds number, $Q(\eta, Re)$. Data are for Runs 1 (circles), 4 (asterisks) and 6 (squares) as given in Tab. 1. The values for the constant c (see equation (12)) are 2.6 (Run1), 4.0 (Run4), and 4.3 (Run6) respectively. All other parameters in DNS and (14) are identical. The data for Runs 4 and 6 are shifted upwards in the diagram for a better visibility. The inset shows a comparison of $Q(\eta, Re)$ for the standard grid resolution with $k_{max}\eta_K = 1.2$ and $N = 128$ (red asterisks) and the present very-high-resolution case (Run 4). It demonstrates clearly the necessity of the high resolution taken here.

cay for $\tilde{k} = k/\eta_K \geq 1$ is

$$\tilde{E}(k) = \frac{E(k)}{\nu^{5/4}\langle\epsilon\rangle^{1/4}} = F(\tilde{k}) = \tilde{k}^{\alpha}\exp(-\beta\tilde{k})\,. \tag{17}$$

We take $k_d = \eta_K^{-1}$ as the dissipation wavenumber and α and β are Reynolds-number-dependent dimensionless constants. It is immediately clear, that investigations of such rapidly decaying spectral intensities are extremely challenging since small amplitudes have to be advanced in the direct numerical simulations (DNS) (Chen et al., 1993; Martinez et al., 1997; Ishihara et al., 2005). The rapid decay of the spectra is shown in figure 4. Table 2 lists the fit results for both coefficients (see equation (17)) in the range $4 \leq \tilde{k} \leq 9$. While a direct comparison with the results of Ishihara et al. (2005) cannot be made since their studies were focussed on the near-dissipation range, the magnitude of the slope β is nevetheless consistent with the results of Chen et al. (1993), Martinez et al. (1997) and Ishihara et al. (2005). The table displays systematic trends of both constants with Reynolds number and suggests a saturation of the far-dissipation range coefficient β to a non-zero asymptotic value for larger Reynolds numbers (Kraichnan, 1959). The decreasing slope of decay of the spectra in the far-dissipation range can be interpreted as a growing excitation of sub-Kolmogorov fluctuations of turbulence. Our observations seem therefore to be directly connected with the increasing degree of small-scale intermittency which is observed for the velocity gradients (Schumacher et al., 2007). The larger amplitudes seem also to be connected with the excitation of ever finer local dissipation scales as seen in the last chapter.

Figure 4: Decay of energy spectra $E(k)$ in the far-dissipation range of turbulence. The corresponding Reynolds numbers are indicated in the legend. The vertical dashed lines mark the range for which the spectra were fitted to relation (17).

Table 2: Constants α and β as obtained by a least square fit of equation (17) to the energy spectra $E(k)$. The fit range was $4 \leq k\eta_K \leq 9$ for all simulations.

Run	R_λ	α	β
1	10	-2.83	-6.60
2	24	-0.58	-6.12
3	42	-2.16	-5.13
5	64	-4.08	-3.62
6	107	-6.96	-2.85

Possible impact on high-Schmidt number scalar mixing

If these intermittent excitations at sub-Kolmogorov scales increase with Reynolds number they should have an impact on the mixing of scalar fields $\theta(\mathbf{x}, t)$ at Schmidt numbers larger than unity. The Schmidt number is defined as

$$Sc = \frac{\nu}{\kappa}\,. \tag{18}$$

It relates the kinematic viscosity of the fluid, ν, to the diffusivity of the scalar, κ. When this dimensionless parameter is larger unity, a significant fraction of the scalar filaments will be mixed on scales that are smaller than the Kolmogorov length. This regime was studied first by Batchelor (1959) and one of the main assumptions in his model is the spatial smoothness of the flow field. Corresponding to Batchelor, the full information on the flow is contained in the local principal rates of strain which are nothing else but the first order Taylor expansion coefficients of the velocity, describing a local pure-strain flow. In the light of our findings, the conclusion follows that this basic assumption of Batchelors model might not be vaild, at least not in the intermediate dissipation range. In other words, as deep down as filaments and patches from the inertial range can sweep into the sub-Kolmogorov scale range the scalar will be stirred by a partially rough flow. Furthermore, the obtained Reynolds-number dependent decay of the spectra in the far-dissipation range will introduce a Reynolds number dependence for the

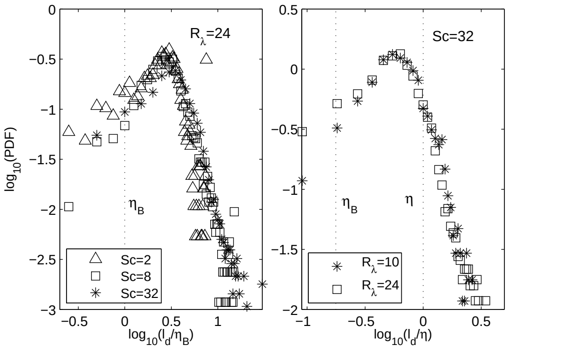


Figure 5: Distribution of the local cross-section thickness l_d of the scalar dissipation rate filaments for level sets that exceed four times the mean scalar dissipation rate. Left panel: Probability density function (PDF) $p(l_d/\eta_B)$ for three different Schmidt numbers at $R_\lambda = 24$ (Run2). The flow corresponds to Run 2. Right panel: PDF $p(l_d/\eta)$ for two different Reynolds numbers at $Sc = 32$. Data correspond to Runs 1 and 2.

scalar mixing. This might be one reason why it is so difficult to observe a Batchelor scaling of the variance spectrum, i.e. $E_\theta(k) \sim k^{-1}$. Numerical studies in this range are currently out of reach, since they would require both, large Reynolds and Schmidt numbers.

This circumstance does however not prevent us from a study of local dissipation scales in scalar mixing. First steps in this direction were done experimentally by Buch and Dahm (1996) in low-Reynolds-number jet flow geometries at $Sc \approx 1900$. Schumacher et al. (2005) discussed this issue in direct numerical simulations of passive scalar mixing for Sc between 2 and 32. The relation (3) was simply applied for the Batchelor diffusion length which is the relevant mean dissipation scale in the case of passive scalar mixing at $Sc \gg 1$t. This results to

$$\eta_B = \frac{\eta_K}{\sqrt{Sc}} \quad\longrightarrow\quad \eta_B(\mathbf{x}) = \frac{\eta(\mathbf{x})}{\sqrt{Sc}}\,. \tag{19}$$

Due to the linearity of the passive scalar dynamics, the distribution of the local dissipation scales of the flow will determine the distribution of the local diffusion scales of the advected passive scalar.

Kushnir et al. (2006) proceeded directly to the determination of the local diffusion scale distribution. A fast multiscale algorithm was applied to quantify the cross-section thickness l_d of sheet-like maxima of the scalar dissipation rate field which is defined

$$\epsilon_\theta(\mathbf{x}, t) = \kappa \left(\frac{\partial \theta}{\partial x_i}\right)^2 . \tag{20}$$

The locally varying thickness was interpreted as the local diffusion scale. The result of this procedure is shown in figure 5 where we compare local diffusion scale distributions as a function of the Schmidt number for a flow at fixed Reynolds number (Run 2) in the left panel. The right panel shows the distributions at two Reynolds numbers (Runs 1 and 2). The good collapse of the data in figure 5 suggests that this transformation of the dissipation scale to the diffusion scale picture, as suggested by relation (19), is appropriate. We observe in the right panel a slightly less steep decay of the left tail which seems to be consistent with our findings on the ever finer scales in the local dissipation scale distribution which are excited.

In conclusion, we see that the whole concept of local dissipation scales of the turbulent velocity can be carried over to local diffusion scales for the passive scalars for the case of high-Schmidt-number mixing.

SUMMARY

The present work discussed ideas of how one mean dissipation scale arising from classical turbulence theory can be extended to a whole range of local dissipation scales. This generalization incorporates the strongly intermittent nature of the gradients fields in turbulence which has been measured in many experiments and simulations in the past. The finest dissipation scales will be associated with the steepest gradients.

It was demonstrated that local dissipation scales can be implicitely calculated via velocity increments. The theoretical prediction for the distribution fits well to the results of the numerical simulations. With increasing Reynolds number the excitation of ever finer scales becomes more and more probable. A second way to determine the growing sub-Kolmogorov fluctuations was done here by the study of the decay of the energy spectra in the far-dissipation range. The slope of the exponential spectral decay is found to be less steep with growing Reynolds number.

The concept of local dissipation scales was finally carried over to that of local diffusion scales in case of passive scalar mixing at high Schmidt numbers. As a consequence, one can expect a Reynolds number dependence of the fine-scale statistics of the scalar fields. This approach can be of interest for non-premixed combustion processes and will be discussed elsewhere.

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