ANALYTICAL SOLUTIONS FOR STRATIFIED TURBULENT SHEAR FLOW IN A ROTATING FRAME

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ABSTRACT

Rapid distortion theory is applied to stratified homogeneous turbulence which is sheared in a rotating frame. Insight into the stabilizing and destabilizing effects of the combined stratification and frame rotation is gained by considering initial fields that are two-dimensional (but three-componential), with the axis of independence aligned with the direction of the mean shear. For these conditions we derive solutions for the Fourier components of the flow variables, and we compute one-point statistics such as the Reynolds stresses and the structure dimensionality tensor. The results are in very good agreement with the exact numerical solution for initially three-dimensional isotropic homogeneous turbulence, especially regarding the large time behavior, and they could be a reference point for the development of turbulence models. We also study the short time behavior of the 3D initially isotropic case, and we show that it is mainly dependent on the level of stratification.

INTRODUCTION

The effects of system rotation and stratification on turbulent shear flows have received considerable attention because of their relevance to important technological and astrophysical problems. The state of sheared turbulence changes significantly when it is subjected to frame rotation or stratification. Previous work had clearly shown that rotation or stratification can act to either stabilize or destabilize turbulent shear flow. In the present study, we use inviscid Rapid Distortion Theory (RDT) to investigate the evolution of stratified turbulence in a rotating frame as a function of the rotation rate (including stable, neutral and unstable regimes), and examine the sensitivity of the results to the level of the stratification of the flow. Under RDT the non-linear effects resulting from turbulence - turbulence interactions are neglected in the governing equations. RDT is a closed theory for two-point correlations or spectra, but the one-point governing equations are, in general, not closed due to the non-locality of the pressure fluctuations (Townsend, 1976; Savill, 1987; Cambon and Scott, 1999). Simple cases of rapid deformation often admit closed form solutions for individual Fourier coefficients.

The few cases where closed-form solutions can be obtained for one-point statistics, like the Reynolds stresses, offer valuable insight. RDT has been widely used in studying the effects of rotation (Salhi, 2002; Cambon, 1997; Akylas et al., 2006; 2007) or stratification (Hanazaki, 2002; Hanazaki and Hunt, 2004; Galmiche and Hunt, 2002) in sheared turbulence. In this study, we use RDT to examine the combined effects of rotation and stratification in the case of stratified turbulence which is sheared in a rotating frame (Fig. 1).

The case studied is of relevance to turbomachinery flows (for example with internal blade cooling passages) and to geophysical flows (for example the flow over a hill or bump). Most recently a connected work has been studied by Salhi and Cambon (2006), where the rotation was aligned with the vertical direction. Also Kassinos et al. (2007) studied in detail a case similar to the one presented here, but considering the turbulence to be independent of the axis of the mean flow (x_1). The latter explained correctly both the stability criterion and the asymptotic limits for the unstable cases. In this study, we extend the last work in a way that allows for a better comparison with the initially isotropic 3D case. We solve analytically the RDT equations in spectral space, for a two-dimensional (2D) but three-componential (3C) initialization. In this case we take the wave number vector component $k_2 = 0$ initially, which means that the initial turbulent field is independent of the x_2-axis. The investigation covers the development of the Reynolds stresses and the structure dimensionality tensor D_{ij}, introduced by

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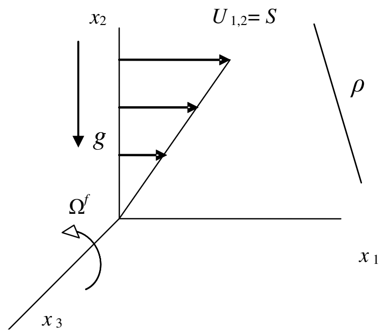

Figure 1: Illustration of the general case of stratified sheared homogeneous turbulence which is examined here.

Kassinos et al. (2001). The combined use of these tensors allows one to distinguish between the componentality of the turbulence (described by the Reynolds stress tensor) and its dimensionality, which has to do with the morphology of the turbulence eddies, and is described by the structure dimensionality tensor D_{ij}. For example if $D_{11} = 0$ then the turbulence is independent of the x_1-axis; that is, it consists of very long structures aligned with the x_1 direction.

The initially non-isotropic solutions are compared with the 3D initially isotropic case (solved using the Particle Representation Model, PRM, developed by Kassinos and Reynolds, 1994; 1999), in order to assess their potential as a simplified qualitative representation. The findings of this work can be seen as additional information on the trends of the turbulence that is sheared in a rotating frame. In addition to the general interest, a strong motivation for this study arose from our efforts to develop an algebraic structure-based turbulence model (ASBM), which has been successfully used, so far, to compute the characteristics of rotating turbulent channel and boundary layer flow (Kassinos et al., 2005). The model uses the RDT asymptotic limits as targets or guidelines, for determining the anisotropy of the Reynolds stress and structure dimensionality tensors, under strong deformations, aiming to improve the dependability and reliability.

LINEAR EQUATIONS

Under inviscid RDT, and using the Boussinesq approximation, the transport equations for the fluctuating velocity and density components u_i and ρ, become (Kassinos et al., 2007)

$$\begin{aligned} \dot{u_i} + Sx_2 u_{i,1} &= -Su_2\delta_{i1} - p_{,i}/\rho_0 - 2\rho g\delta_{i2}/\rho_0 + 2\epsilon_{ij3}\Omega^f u_j \\ \dot{\rho} &= u_2\rho_0 N^2/g = u_2\rho_0 S^2 Ri/g \end{aligned} \quad (1)$$

where $S = dU_1/dx_2$ is the mean velocity gradient, Ω^f is the frame rotation rate, g is the gravitational constant, and ρ_0 is the reference density. The Brunt-Vaisala frequency $N^2 = -Hg/\rho_0$, and the dimensionless Richardson number $Ri = N^2/S^2 = -Hg/S^2\rho_0$, are functions of the mean density gradient $H = d\overline{\rho}/dx_2$ (Figure 1). Using the Rogallo (1981) transformation we set

$$\xi_1 = x_1 - x_2 St, \quad \xi_2 = x_2, \quad \xi_3 = x_3, \quad \tau = t, \quad (2)$$

and (1) transforms to

$$\begin{aligned} \partial u_i/\partial\tau &= -\delta_{i1}Su_2 - (\partial p/\partial\xi_i - \delta_{i2}S\tau\partial p/\partial\xi_1)/\rho_0 \\ &\quad - \delta_{i2}\rho g/\rho_0 + 2\epsilon_{ij3}\Omega^f u_j \\ \partial\rho/\partial\tau &= u_2\rho_0 S^2 Ri/g \end{aligned} \quad (3)$$

Through (3), the Fourier transformed variables (denoted with ˆ) read

$$\begin{aligned} d\hat{u}_i/d\tau &= -\delta_{i1}S\hat{u}_2 + i\hat{p}k_i/\rho_0 - \delta_{i2}\hat{\rho}g/\rho_0 + 2\epsilon_{ij3}\Omega^f\hat{u}_j \\ d\hat{\rho}/d\tau &= \hat{u}_2\rho_0 S^2 Ri/g \end{aligned} \quad (4)$$

where the wave numbers evolve as $k_i = k_i^0 - \delta_{i2}S\tau k_1$, (the superscript 0 denotes initial values). Applying the Fourier transformed continuity equation $k_i\hat{u}_i = 0$ in (4), we solve for the pressure

$$ik^2\hat{p}/\rho_0 = (2Sk_1\hat{u}_2 - 2\Omega^f k_1\hat{u}_2 + k_2\hat{\rho}g/\rho_0 + 2\Omega^f k_2\hat{u}_1) \quad (5)$$

and by substituting into the system (4), this simplifies to

$$\begin{aligned} \frac{d\hat{u}_i}{d\beta} &= \left[\frac{(2-n)k_1k_i}{k^2} - \delta_{i1}\right]\hat{u}_2 + \frac{\eta k_2 k_i\hat{u}_1}{k^2} + \eta\epsilon_{ij3}\hat{u}_j \\ &\quad + \left(\frac{k_2k_i}{k^2} - \delta_{i2}\right)\frac{\hat{\rho}g}{S\rho_0} \\ \frac{d\hat{\rho}}{d\beta} &= \frac{\rho_0 SRi}{g}\hat{u}_2 \end{aligned} \quad (6)$$

where $k^2 = k_1^2+k_2^2+k_3^2$, $\beta = St$ (total shear) and $\eta = 2\Omega^f/S$. The above system (6) can be solved analytically for simplified cases where either k_1^0 or k_2^0 or k_3^0 equal zero, that is the turbulence is initially independent on one direction. This is equivalent with the derivation of 2D information from the one-dimensional energy spectra when multiplied by proper length scales (see Townsend, 1976). As mentioned, Kassinos et al. (2007), have solved the above system for a two dimensional (2D) turbulence, independent of the flow direction x_1, with $u_i = u_i(x_2, x_3)$ for $i = 1, 2, 3$, that is for $k_1 = 0$. That solution explained accurately the stability criterion for the TKE evolution, through the sign of the combined stability parameter

$$Z = B - Ri \quad (7)$$

where $B = \eta(1 - \eta)$ is the Bradshaw(1969)-Pedley(1969) stability parameter, and $Ri = -Hg/S^2\rho_0$ is the Richardson number. Furthermore, that solution described well the asymptotic behavior of the unstable (corresponding to $Z > 0$) cases. However, the stable and neutral regimes have not been described accurately so far.

In this study, we consider a more appropriate, initially three-component (3C) but two-dimensional (2D) turbulence, independent of the scalar gradient direction x_2, with $u_i = u_i(x_1, x_3)$ for $i = 1, 2, 3$. Note that for this initialization, the 3-dimensionality is recovered since there is formation of the wave number component $k_2 = -k_1\beta$ which becomes dominant at large times. As will be shown this solution coincides at large times with the behavior of the 3D initially isotropic case. In order to complete the picture, we also calculate the approximate behavior of the 3D initially isotropic case expanding the Fourier components over β, for short times, as follows.

3D EXPANSION FOR SHORT TIMES

The Fourier components of the velocity and density fields in (6) are expanded as follows

$$\hat{u}_i(\beta) = \sum_{p=0}^{\infty} \hat{u}_{ip}\beta^p \qquad \hat{\rho}(\beta) = \sum_{p=0}^{\infty} \hat{\rho}_p \beta^p \tag{8}$$

Solving (6) to order 3 in β, we analytically compute the $\hat{u}_{ip}$ and $\hat{\rho}_p$ for $p = 1, 2$ and 3, and then we integrate the 3D spectra

$$E_{ij}(k,\beta) = \frac{1}{2}\overline{\hat{u}_i\hat{u}_j^* + \hat{u}_j\hat{u}_i^*} \quad \Phi_j^\rho(k,\beta) = \frac{1}{2}\overline{\hat{\rho}\hat{u}_j^* + \hat{u}_j\hat{\rho}^*} \tag{9}$$

over all the wave numbers in order to compute the stresses and the density fluxes. The initial 3D isotropic velocity spectrum is given by

$$E_{ij}(\mathbf{k},0) = \frac{E(k_0,0)}{4\pi k_0^2}\left(\delta_{ij} - \frac{k_i^0 k_j^0}{k_2^0}\right) \tag{10}$$

while, for convenience, we consider in this study that the initial density perturbations are zero. We should note here, however, that this is a major point since the presence of initial density fluctuations can modify the behavior at short times significantly. The investigation of the dependence of the solution on the initial ratio of the TKE to the potential energy is a future task. By setting $\hat{\rho}(0) = 0$, the solutions for the normal stress components become

$$\begin{aligned} \frac{R_{11}}{q_0^2} &= \frac{1}{3} - \frac{(28\eta + 7Ri - 20)}{210}\beta^2 + O(\beta^4) \\ \frac{R_{22}}{q_0^2} &= \frac{1}{3} + \frac{2(7\eta - 14Ri - 2)}{105}\beta^2 + O(\beta^4) \\ \frac{R_{33}}{q_0^2} &= \frac{1}{3} - \frac{(7Ri - 16)}{210}\beta^2 + O(\beta^4) \end{aligned} \tag{11}$$

where the dependence on both Ri and η is present, although not through the combined parameter Z. After the summation of the above relations we find that the initial evolution of the turbulent kinetic energy is mainly driven by the level of the stratification, since it is described by

$$\frac{q^2}{q_0^2} = 1 - \frac{(5Ri - 2)}{15}\beta^2 + O(\beta^4) \tag{12}$$

This result determines a critical value of Ri that characterizes a strongly stable regime (inside the generally stable cases for $Z < 0$), where the turbulent kinetic energy shows a diminishing stable behavior from the initial times. More specifically, for $Ri > 0.4$ the initially isotropic case starts with a decreasing kinetic energy, while for $Ri < 0.4$, independently on the value of η, the turbulent kinetic energy shows an initial growth (unstable case). Continuing with the shear stresses and the buoyancy term, we find that they equal

$$\frac{R_{12}}{q_0^2} = \frac{-2}{15}\beta + \frac{(22\eta^2 - 16\eta - 21\eta Ri + 14Ri)}{315}\beta^3 + O(\beta^4) \tag{13}$$

$$\frac{g}{\rho_0 SRi}\frac{\overline{\rho u_2}}{q_0^2} = \frac{1}{3}\beta - \frac{(7\eta^2 - 28\eta + 56Ri + 12)}{315}\beta^3 + O(\beta^4)$$

Coupling the above terms with the TKE evolution equation

$$\frac{dq^2}{d\beta} = -2R_{12} - 2\frac{g}{\rho_0 S}\overline{\rho u_2} \tag{14}$$

we find that the TKE evolves as

$$\frac{q^2}{q_0^2} = 1 - \frac{5Ri - 2}{15}\beta^2 - \left(\frac{\eta^2(22 - 7Ri) - \eta(16 - 7Ri)}{630} + \frac{2Ri - 56Ri^2}{630}\right)\beta^4 + O(\beta^5) \tag{15}$$

As a result, we still notice that even though a secondary dependence on η appears in (15), the main dependence still is on the value of Ri, as described above. At the limit of $Ri = 0.4$, however, the value of η determines the initial destabilization through the sign of the term $[\eta^2(22 - 7Ri) - \eta(16 - 7Ri) + 2Ri - 56Ri^2]$. Thus, as shown in Fig. 2, for values of $-0.39 < \eta < 1.08$, the TKE initially grows (although Z is less than zero).

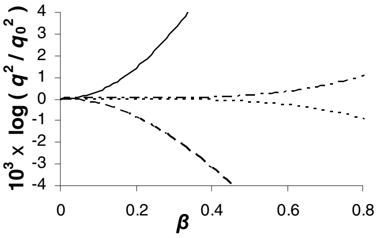

Figure 2: Short time evolution of the TKE for the 3D initially isotropic case, with $\eta = -0.5$ and $Ri = 0.15$ (solid line), $\eta = 0.5$ and $Ri = 0.55$ (long dashed), $\eta = -0.5$ and $Ri = 0.4$ (short dashed), and $\eta = -0.25$ and $Ri = 0.4$ (dotted dashed).

ANALYTICAL SOLUTION FOR $k_2(0) = 0$

By setting $k_2 = 0$ we introduce a 2D (but 3C) initialization, where the turbulence is initially independent of the x_2 direction. In this case the turbulence establishes the dependence on the third direction due to the formation of $k_2 = -k_1\beta$, which dominates the solution for large times. For this specific choice of $k_2(0) = 0$, the 3D system (6) simplifies to

$$\begin{aligned} \frac{d\hat{u}_1}{d\beta} &= \frac{(2-\eta)k_1^2 - (1-\eta)k^2}{k^2}\hat{u}_2 - \frac{\eta k_1^2\beta}{k^2}\hat{u}_1 - \frac{k_1^2\beta}{k^2}\frac{g\hat{\rho}}{S\rho_0} \\ \frac{d\hat{u}_2}{d\beta} &= \frac{(\eta-2)k_1^2\beta}{k^2}\hat{u}_2 - \eta\frac{k_1^2 + k_3^2}{k^2}\hat{u}_1 - \frac{k_1^2 + k_3^2}{k^2}\frac{g\hat{\rho}}{S\rho_0} \\ \frac{d\hat{u}_3}{d\beta} &= \frac{(2-\eta)k_1k_3}{k^2}\hat{u}_2 - \frac{\eta k_1 k_3\beta}{k^2}\hat{u}_1 - \frac{k_1k_3\beta}{k^2}\frac{g\hat{\rho}}{S\rho_0} \\ \frac{d\hat{\rho}}{d\beta} &= \frac{\rho_0 SRi}{g}\hat{u}_2 \end{aligned} \tag{16}$$

where the magnitude of the wave number reduces to $k^2 = k_1^2(1+\beta^2) + k_3^2$. Setting in cylindrical coordinates $k_1 = k_0\cos\theta$, $k_3 = k_0\sin\theta$ (where $k_0 = \sqrt{k_1^2 + k_3^2}$), and assuming zero initial density fluctuations $\hat{\rho}^0 = 0$, we find the homogenized form

$$\begin{aligned} (1 + \beta^2\cos^2\theta)\frac{d^2\hat{u}_2}{d\beta^2} &= -4\beta\cos^2\theta\frac{d\hat{u}_2}{d\beta} \\ &\quad + [Z - (\eta+1)(2-\eta)\cos^2\theta]\hat{u}_2 \\ \left.\frac{d\hat{u}_2}{d\beta}\right|_{\beta=0} &= -\eta\hat{u}_1^0 \end{aligned} \tag{17}$$

where the superscript 0 denotes initial values. Clearly the solution of (16-17) depends on the value of Z. As shown by Kassinos et al. (2007) from their 2D analysis with $k_1 = 0$, positive values of this parameter correspond to unstable cases, resulting in an exponential turbulent kinetic energy TKE growth, while negative values cause a stabilizing behavior. Regarding this stability criterion, a similar result can also be correctly obtained using a simplified 1D pressureless analysis with $k_1 = k_2 = 0$ (Speziale and Mac Giolla Mhuiris, 1989, Kassinos et al., 2007). However in this study, we obtain solutions which maintain the 3D character of the turbulence and thus, they are more accurate, especially regarding the behavior of the turbulence at neutral and stable cases ($Z \leq 0$). Solving (16-17), we obtain the evolution of the Fourier coefficients of the density and the velocity components

$$
\begin{aligned}
\hat{u}_1 &= \beta(1 + (2-\eta)\cos^2\theta) \\
&\quad \times F[3/4-\alpha, 3/4+\alpha; 3/2; -\beta^2\cos^2\theta]\hat{u}_2^0 \\
&\quad + (1 - \beta^2\eta\cos^2\theta F[5/4-a, 5/4+a; 3/2; -\beta^2\cos^2\theta])\hat{u}_1^0 \\
&\quad + [3/4 + 8/3\alpha^2(-5+8\alpha^2)]\beta^3\cos^4\theta \\
&\quad \times \frac{F[7/4-\alpha, 7/4+\alpha; 5/2; -\beta^2\cos^2\theta]}{-1+16\alpha^2}\hat{u}_2^0 \\
&\quad + 4(\eta-\eta^2)\tan^2\theta \\
&\quad \times \frac{-1 + F[1/4-\alpha, 1/4+\alpha, 1/2, -\beta^2\cos^2\theta]}{-1+16\alpha^2}\hat{u}_1^0 \\
\hat{u}_2 &= F[3/4-\alpha, 3/4+\alpha; 1/2; -\beta^2\cos^2\theta]\hat{u}_2 \qquad (18) \\
&\quad - \eta\beta F[5/4-\alpha, 5/4+\alpha; 3/2; -\beta^2\cos^2\theta]\hat{u}_1 \\
\hat{u}_3 &= \cot\theta(\beta\hat{u}_2 - \hat{u}_1) \\
\frac{g}{Ri\rho_0 S}\hat{\rho} &= \beta F[3/4-\alpha, 3/4+\alpha; 3/2; -\beta^2\cos^2\theta]\hat{u}_2 \\
&\quad - 4\eta\sec^2\theta\frac{F[1/4-\alpha, 1/4+\alpha; 1/2; -\beta^2\cos^2\theta]}{-1+16\alpha^2}\hat{u}_1
\end{aligned}
$$

where F is the generalized hypergeometric function, with the parameter α given by

$$\alpha = \frac{\sqrt{4Z\sec^2\theta + (2\eta-1)^2}}{4} \qquad (19)$$

From (19) it becomes clear that for positive values of Z the parameter α is a real number, resulting in exponential increase of the TKE, while when Z is negative α is a complex number resulting in a stabilizing oscillatory behavior. For the neutral cases when $Z = 0$ then $\alpha = \frac{2\eta-1}{4}$, and the Fourier components show algebraic behavior on β, depending on the value of η. The limit of the above solutions (18) when $\theta \to \pi/2$, is identical to the pressureless analysis limit ($d_{33} = 1$), where the solutions become proportional to $\exp(Z\beta)$. However the contribution of the whole range of θ must be taken into account. More specifically, as the solution departs from the most energetic mode at $\pi/2$ the values of the Fourier coefficients are distributed symmetrically around $\pi/2$ towards $\theta = 0$ or $\theta = 2\pi$, where $k_3 = 0$. The solution there becomes independent of the rotation rate, and depends only on Ri in agreement with the principle of material indifference (Speziale, 1981) for 2D turbulence independent of the direction of the frame rotation ($d_{33} = 0$).

EVOLUTION OF THE STRUCTURE TENSORS AND THE TKE

From equations (18) we calculate the evolution of the velocity spectra $E_{ij} \sim \overline{\hat{u}_i\hat{u}_j}$ and we integrate over all the wave number space to find the stresses, the structure dimensionality tensor components, and the TKE. We make use of two different initializations, namely a vortical and a jetal initial velocity spectrum (Kassinos et al., 2001; Akylas et al. 2007). More specifically, in the vortical case the componentality of the initially 2D turbulence is isotropic in planes perpendicular to the axis of independence (x_2 in our case). In this case, the vortical 2D-2C spectrum is given by (see also Cambon et al., 1997)

$$E_{ij}^{\text{vor}} = \frac{E(k,0)}{2\pi k}\delta(k_2)\left(\delta_{ij} - \frac{k_i k_j}{k^2} - \delta_{i2}\delta_{j2}\right) \qquad (20)$$

In contrast, when the initial turbulence is completely jetal, all the velocity fluctuations are in the direction of the axis of independence. The corresponding initial 2D-1C jetal spectrum is

$$E_{ij}^{\text{jet}} = \frac{E(k,0)}{2\pi k}\delta_{i2}\delta_{j2} \qquad (21)$$

In the relations (20) and (21) the initial turbulent kinetic energy spectrum satisfies

$$\int_{k=0}^{\infty} E(k,0)dk = \frac{q_0^2}{2} = \frac{R_{nn}}{2} \qquad (22)$$

Because of the linearity of the governing equations, the solutions for the initially jetal 2D-1C and the vortical 2D-2C cases can be superposed to produce R_{ij} and D_{ij} for various 2D-3C initial fields, consisting of uncorrelated jets and vortices (Kassinos et al., 2001; Akylas et al., 2007).

In Fig. 3 we present a comparison between the TKE evolution calculated from the spectral solutions derived here, for a $(2/3 - 1/3)$ weighted superposition between the vortical and the jetal initializations, and the numerical solution of the 3D-3C initially isotropic case. The latter has been calculated using the PRM (Kassinos and Reynolds, 1994, 1999), with large enough number of particles to ensure the accuracy of the solution. As shown in Fig.3 the evolution of the TKE from the initially 2D approach, coincides fairly well with the 3D initially isotropic case. The 2D initialization explains accurately the type (algebraic or exponential) of the TKE growth, identifying Z as the principal parameter for the determination of the stability of the turbulent flow. Starting with the unstable case with $\eta = 0.2$ and $Ri = 0.08$, we notice the profound exponential evolution with time. An increase of Ri to 0.16, resulting in $Z = 0$, causes a departure from exponential towards a polynomial growth, while a further increase of Ri to 0.24 stabilizes the TKE. Also, a combination of $\eta = -0.1$ (which tends to stabilize) and $Ri = -0.11$ (which tends to destabilize the TKE) resulting in $Z = 0$, again produces a neutral, polynomial growth of the TKE with respect to time.

In Figs. 4-7 the evolution of the normalized stress components r_{ij} is illustrated for all the above mentioned cases, and compared with the respective 3D-PRM exact numerical solutions. Besides differences at short times, which are due to the different initialization, it is profound that the limiting states reached by the analytical 2D solution are in excellent agreement with the corresponding limiting states obtained numerically (PRM) for initially 3D isotropic turbulence. Especially for the neutral and the stable cases, where the initial 3D behavior is crucial, this agreement was not observed in the case of 2D turbulence independent of x_1 (Kassinos et al., 2007).

The same very good agreement in the limiting states is obtained for d_{ij} as illustrated in Figs. 8-11. For the unstable regime ($Z > 0$), the d_{11} component tends quickly to zero. For such cases the 3C-2D asymptotic states of the turbulence have been explained by Kassinos et al. (2007), with their simplified 2D solution with $d_{11} = 0$. However, for the

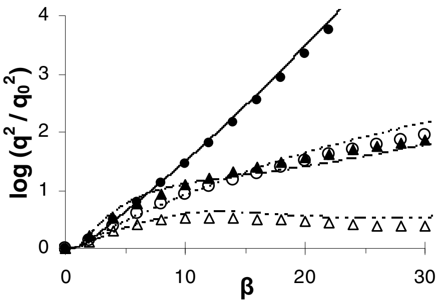

Figure 3: Evolution of the TKE for the cases with $\eta = 0.2$ and for various values of Ri equal to: Ri=0.08 (unstable: thin continuous, solid circles), Ri=0.16 (neutral: short dashed, open circles), Ri=0.24 (stable: short dashed, open triangles), as well as the case with $\eta = -0.1$ and $Ri = -0.11$ (neutral: long dashed, solid triangles) calculated from the 3D initially isotropic exact PRM numerical solution (symbols) and the initially 2D solution with $k_2(0) = 0$ (lines).

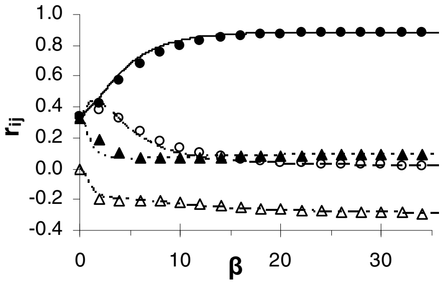

Figure 4: Evolution of the normalized stress components 11 (continuous, solid circles), 22 (short dashed, solid triangles), 33 (long dashed, open circles) and 12 (dotted dashed, open triangles) calculated from the 3D initially isotropic exact PRM numerical solution (symbols) and the 2D analytical solution with $k_1 = 0$ (lines) presented here, for the unstable case with $\eta = 0.2$, $Ri = 0.08$ and $Z = 0.08$.

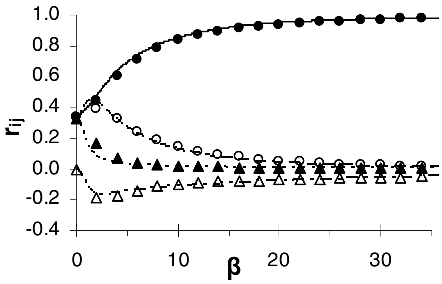

Figure 5: As in figure 4, for the neutral case with $\eta = 0.2$, $Ri = 0.16$ and $Z = 0$.

neutral and unstable cases, when $Z \leq 0$, the initially 3D character of the turbulence becomes more important. For such cases that previous study did not find accurate results. By comparison, the new 2D initialization introduced here (with $k_2 = 0$), provides a very good description of the evolution of the initially isotropic case. The investigation of the velocity spectra in order to derive analytical information on the exact dependence of the stresses and the structure of the turbulence at large values of total shear is a future task.

CONCLUSIONS

Analytical solutions have been derived for the evolution of the turbulent spectra in the case of stratified sheared tur-

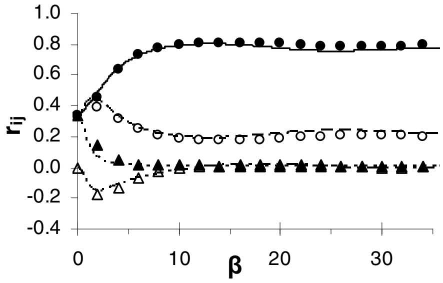

Figure 6: As in figure 4, for the stable case with $\eta = 0.2$, $Ri = 0.24$ and $Z = -0.08$.

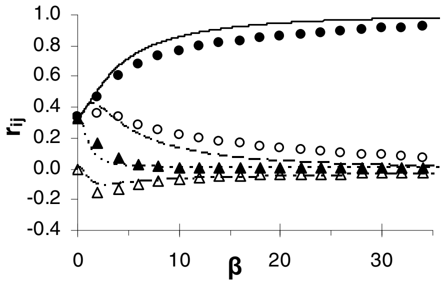

Figure 7: As in figure 4, for the neutral case with $\eta = -0.1$, $Ri = -0.11$ and $Z = 0$.

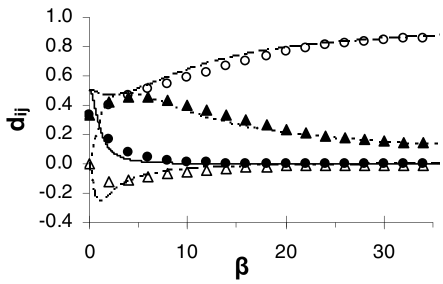

Figure 8: As in figure 4, but for the normalized structure tensor components.

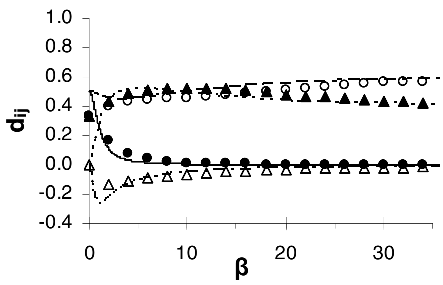

Figure 9: As in figure 5, but for the normalized structure tensor components.

bulence in a rotating frame. We have investigated the short time behavior of the initially 3D isotropic field and we show that it is mainly dependent on the value of the Ri number. For $Ri > 0.4$ the initially isotropic case starts with a decreasing kinetic energy, while for $Ri < 0.4$ the turbulent kinetic energy shows immediate growth, independently of the value of η.

In order to complete the picture, and describe the behavior of turbulence at large times, we also derived solutions for the case where the turbulence is initially independent of the axis of the density gradient. Unlike a more simplified previous work, the new solutions recover the 3D dependence and

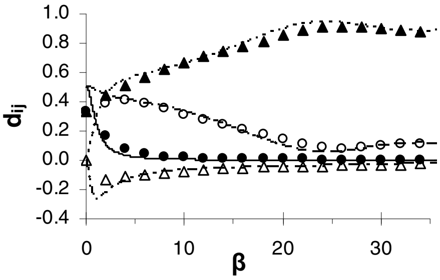

Figure 10: As in figure 6, but for the normalized structure tensor components.

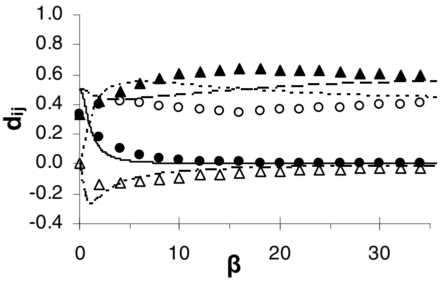

Figure 11: As in figure 7, but for the normalized structure tensor components.

they show impressive agreement with the initially isotropic case, especially in terms of the asymptotic states of the turbulent fields. More specifically, it seems that for the unstable cases the characteristics of the initially isotropic solution are captured quite accurately when the initial turbulence is independent of the flow direction(x_1). For the neutral and the stable regime though, the initial dependence of the turbulence on the axis of the mean flow (x_1) is also crucial, since the 3D-3C character of the turbulence at early times is important for the evolution of the kinetic energy, unlike in the unstable cases.

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