DISPERSION OF A VERTICAL JET OF BUOYANT PARTICLES IN A STABLY STRATIFIED WIND-DRIVEN EKMAN LAYER

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ABSTRACT

Dispersion of a buoyant jet of fresh water particles in a salt water thermally stratified environment is investigated. The carrying flow field is a wind driven mid-latitude Ekman layer. The investigation is carried out using Large Eddy Simulation. The dispersed phase is simulated in a Lagrangian way, solving a modified form of the Maxey and Riley equation for each particle of the jet. In order to simulate a large Reynolds number flow, the thin wall layer is not directly resolved and the wall stress and the wall heat flux are directly imposed at the free surface. Results of the simulations show that the presence of incoming heat flux produces a picnocline below the free surface and inhibits turbulent transport. In particular the turbulent penetration depth is strongly reduced by stratification as well as the level of the turbulent fluctuations. The dispersion of the buoyant plume of particles is dramatically modified by stratification. In the neutral case, the plume is spread over the horizontal direction in the free surface region and is driven by the mean Ekman current. In the stratified case, the particles remain entrapped in wavy motion present the region of the picnocline. The horizontal transport is strongly reduced for two reasons: first in the region where the picnocline develops the mean velocity is small compared with the value reached at the free surface; second, the turbulent transport is very small due to the suppression of velocity fluctuations.

INTRODUCTION

An environmental problem that is becoming increasingly relevant nowadays is the prediction of dispersion of polluting particulate in a marine environment. Particles of buoyant fluid in a marine environment may be released for for example when leakage occurs in submarine pipelines or other devices (oil spilling problems). In this kind of applications it is of significant interest the evaluation of the concentration of the released substance as well as the evaluation of the amount of particulate that reaches the free surface of the sea. In literature (see for example Rubin and Atkinson (2001) for a general discussion) Eulerian models are commonly used, in which the plume of particles is described in an Eulerian way as a space-time distribution of their concentration. Although this approach is computational inexpensive, it suffers from empirism in particular when the concentration of the dispersed phase is small and it is mainly composed of an ensemble of separate particles travelling in the carrying fluid. In the present paper we study the dispersion of a buoyant jet of particles of fresh water released into a salty water basin. In particular we study an archetipal problem, representative of the upper part of the ocean forced by a constant wind stress and the release of the buoyant particles in a region below the free surface where turbulent mixing is negligible.

From a fundamental point of view, the top region of the sea can be treated as a wind driven Ekman layer, namely the boundary layer created by the tangential stress supplied by the action of the wind in a rotating environment. The aim of the present work is to understand how a cloud of particles released in the water column is dispersed in the wind-driven Ekman layer subject to different conditions of thermal stratification. The study is performed numerically using a Lagrangian-Eulerian approach, which consists of moving Lagrangian particles in an Eulerian carrier phase.

THE PROBLEM FORMULATION

We use a Lagrangian-Eulerian approach, in which the dispersed phase is treated as an ensemble of Lagrangian particles moving in an Eulerian flow field. In order to evaluate the force field acting over the Lagrangian particles an interpolation of the Eulerian flow field onto the particle position is carried out.

As regards the Eulerian field, we consider a mid-latitude, wind driven Ekman layer, thus including both the vertical and the horizontal components of the rotation vector in the governing equations (see Coleman *et al.* (1990) and Salon *et al.* (2005)). The relevant scales of the problem are: the friction velocity $u_\tau = \sqrt{\tau_w/\rho_0}$ associated to the imposed wind stress τ_w (ρ_0 is the bulk density of the fluid); a time scale $T = 1/f_z$ associated to the Coriolis parameter (the z-direction is vertical upward) $f_z = 2\Omega_H sin\theta$, where Ω_H is the earth rotation frequency and θ the latitude; the penetration length $\delta = u_\tau/f_z$ which gives an estimation of the depth of the turbulent boundary layer. A typical full-scale value of $Re = u_\tau\delta/\nu$ is of the order of 7×10^5 considering the data of wind stress given in Price & Sundermeyer (1999). We consider a moderate value of Reynolds number $Re = 10000$, far from that of practical applications, but large enough to minimize small scale effects on the evolution of the velocity

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field. The stratification is considered as a variation of the density field with respect to a reference value ρ_0. Specifically the density field is $\rho_{tot,d}(x, y, z, t) = \rho_0 + \rho_d(x, y, z, t)$ with $\rho_d << \rho_0$ (hereafter the index d denotes dimensional quantities). In our numerical experiment the stratification comes from the imposition of a heat flux at the free surface, as in Taylor *et al.* (2005); physically this corresponds to heating a fluid column by an incoming heat flux. Here we discuss two cases, respectively the case of neutral flow $Ri \to 0$ and the case of strongly stratified flow $Ri = 40$ where the Richardson number is $Ri = g/\rho_0 \ d\rho/dz|_{fs} \ \delta^2/u_\tau^2$. In the present case, the Richardson number is defined using a density scale $d\rho/dz|_{fs}\delta$ which is related to the free surface heat flux as follows $d\rho/dz|_{fs} = -\rho_0 \alpha dT/dz|_{fs}$. As regards the Eulerian field the wind stress acts in the direction south-north (x axis), the y -axis is directed from east to west and the z-axis is vertical upward. A mid-latitude case ($\theta = 45^o$) is considered, and thus both the vertical component of the rotation vector f_z and the horizontal component $f_x = 2\Omega_H cos\theta$ are considered in the governing equations. A schematic of the problem under investigation is in Fig. 1.

The equations of the Eulerian flow are solved using Large Eddy Simulation (LES). We use the Boussinesq approximation which holds in cases where the density variations in the flow field are small compared to a reference density ρ_0. This approximation is commonly used in the analysis of thermally stratified water. The non-dimensional filtered equations are:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_j \, \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} + -\epsilon_{ijk}\frac{f_j}{|f_3|}\bar{u}_k - Ri\rho\,\delta_{i3} - \frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{u}_j \, \bar{\rho}}{\partial x_j} = \frac{1}{Re\ Pr}\frac{\partial^2 \bar{\rho}}{\partial x_j \partial x_j} - \frac{\partial \lambda_i}{\partial x_i} \tag{3}$$

where the symbol $\bar{\cdot}$ denotes a filtered variable. In Eqs.1, 2 t is the time made non dimensional with $1/f_z$, u_i is the velocity component in the i-direction made non dimensional with u_τ, x_i is the i-coordinate made non dimensional with δ, p is the pressure made non dimensional with $p_d/\rho_0 u_\tau^2$, and ρ is the density made non dimensional with the free-surface density gradient $d\rho/dz|_{fs}$ already defined and δ In the present paper directions 1,2,3 respectively correspond to x,y,z. The non-dimensional groups Re and Ri have been already defined, the Prandtl number is $Pr = \nu/k$ with k the thermal diffusivity of the medium.

The quantities τ_{ij} and λ_i are respectively the subgrid-scale stresses and the subgrid-scale buoyancy fluxes. The dispersed phase is treated in a Lagrangian way using a simplified form of the Maxey and Riley equation (1983). Following Armenio and Fiorotto (2001) the Stokes drag only is considered in the particle motion equation together with the gravity term. In the present work we consider the effect of the variation of density of the Eulerian field in the particle motion equation. The dimensional equation of the particle motion is:

$$\frac{dV_{p,i}}{dt_d} = \left(\frac{\rho_p - \rho_{tot,d}}{\rho_p}\right) g\delta_{i3} + \frac{3}{4\,D_p}C_D\,|U_{p,i} - V_{p,i}|\,(U_{p,i} - V_{p,i}) - 2\epsilon_{ijk}\Omega_j V_{p,k} \tag{4}$$

In Eq. 4 t_d is the dimensional time, $V_{p,i}$ and $U_{p,i}$ are respectively are the i-component of the dimensional particle velocity and of the dimensional fluid velocity at the particle location, ρ_p and D_p are respectively the density and the diameter of the particle, $C_D = 24/Re_p(1 + 0.15 Re_p^{0.687})$ where $Re_p = |U_{p,i} - V_{p,i}|\,D_p/\nu$ is the particle Reynolds number. If we replace the total density $\rho_{tot,d}$ with $\rho_0 + \rho_d$, and we make Eq. 4 non dimensional we obtain:

$$\frac{dv_{p,i}}{dt} = \left(1 - \frac{1}{\Delta\rho}\right)\frac{1}{Fr^2}\delta_{i3} - \frac{Ri}{\Delta\rho}\rho\delta_{i3} + \frac{3}{4\,d_p}C_D\,|u_{p,i} - v_{p,i}|\,(u_{p,i} - v_{p,i}) - \epsilon_{ijk}\frac{\Omega_j}{\Omega_3}v_{p,k} \tag{5}$$

In Eq. 5 $\Delta\rho = \rho_p/\rho_0$, $v_{p,i} = V_{p,i}/u_\tau$, $u_{p,i} = U_{p,i}/u_\tau$, $d_p = D_p/\delta$ and the buoyancy contribution appears split into two terms: the first one is the usual contribution proportional to the Froude number $Fr = u_\tau/\sqrt{g\ \delta}$; the second one takes into account the density variation in the Eulerian field, and is proportional to the Richardson number. Note that $Fr^2 Ri = \rho_s/\rho_0$ where ρ_s is a typical density scale of the problem under investigation, *i.e.* $\delta d\rho/dz|_{fs}$ in our case. Under the Boussinesq approximation $Fr^2 Ri << 1$. The second term (II) on the RHS of Eq. 5 gets important when it is comparable with the first term (I) on the RHS. It can be easily shown that $II/I = (\rho_s/\rho_0)\,/\,((\rho_p - \rho_0)/\rho_0)$, thus II influences the particle motion when the particle density is very similar to the reference density of the carrying fluid. This is the case of particles of fresh water released in a salty water environment.

Finally the position of the particles is advanced in time as follows:

$$\frac{dx_{p,i}}{dt} = v_{p,i} \tag{6}$$

with $x_{p,i}$ the i-component of the particle position made non dimensional with δ.

In the LES approach herein used, the subgrid stresses are modeled by using the dynamic mixed model of Armenio & Piomelli (2000), whereas the subgrid density fluxes are modeled using a dynamic eddy diffusivity model. Details on the model are in Armenio & Sarkar (2002). The Reynolds number of the simulations is large enough to make unfeasible the use of *resolved LES* (which denotes a simulation in which the viscous sub-layer is directly resolved). As in Zikanov *et al.* (2003) our strategy is to use a wall-modeling approach at the free surface, directly imposing the wall stress and the heat flux at the surface avoiding the direct resolution of the near surface viscous sublayer.

The Lagrangian-Eulerian model herein employed works as follows: The LES field is first obtained through integration of the filtered Navier-Stokes equations; the LES field is then reconstructed using an approximate deconvolution technique (Kuerten, 2006) in order to recover the subgrid-scale contribution to the particle motion; the reconstructed field is later on interpolated onto the particle position using a technique recently developed by Marchioli *et al.* (2007). The interpolated velocity is finally considered in the particle motion equation and the particle position is thus advanced in time. The governing equations of the Eulerian field are solved using the finite-difference fractional-step technique of Zang *et al.* (1994). The particle motion equations are advanced in time using a second-order accurate Adams-Bashforth technique.

The mathematical model herein employed has been validated in a number of cases. As regards the Eulerian field, a direct comparison between our results and literature data is not possible. However we have ran a polar, neutrally-stratified simulation and compared our results with the $Re = \infty$ LES correspondent simulation of Zikanov *et al.* (2003) (not shown here). Although the difference in the value of Re, the agreement between our results and the reference ones is pretty good, due to the fact that our Re number is large enough to minimize its own effect on the re-scaled velocity field. The accuracy of the Lagrangian-Eulerian model has been verified in the contest of an international test-case aimed at the validation of Lagrangian-Eulerian (LES) solvers (see the webpage http://www.wtb.tue.nl/woc/ptc/benchmark.pdf).

Computational parameters

Following Zikanov *et al.* (2003) the simulations are carried out over a rectangular box whose dimensions are $L_x = \delta$ and $L_y = \delta$ in the horizontal directions, whereas $L_z = 1.5\delta$; the computational grid is uniform and has $64 \times 64 \times 200$ cells respectively in the x,y and z directions. The grid spacing in wall units (made non dimensional with ν/u_τ) is $\Delta x^+ = \Delta y^+ = 156$ and $\Delta z^+ = 75$. The use of such a coarse grid is justified by the fact that the free surface forcing terms (stress and heat flux) are directly imposed as boundary conditions and we avoid solving the thin viscous layer present beneath the free surface region. The molecular Prandtl number is chosen equal to 5 which corresponds to thermally stratified water. Periodic boundary conditions are imposed over the horizontal directions, whereas at the bottom boundary of the computational domain we set $\rho = u_3 = \partial u_1/\partial x_3 = \partial u_2/\partial x_3 = 0$. This choice is justified by the fact that the the domain depth is much larger than the turbulent penetration length and modifications of the flow variables are not expected at the bottom boundary. The *a posteriori* analysis of the numerical results confirmed the effectiveness of such a choice. We consider two cases, a case of neutral flow ($Ri = 0$) and a case of stable stratified flow ($Ri = 40$). As we will discuss later in the paper, the latter corresponds to a case of strong stratification. In both cases, a statistically steady state is first obtained, and later on the particles are released. In the two cases analyzed, we have released 50.400 buoyant particles, with density ratio $\Delta\rho = \rho_p/\rho_0 = 0.976$, corresponding to fresh water particles released in a salt water environment and radius equal to $10^{-3}\delta$. The vertical plume is released continuously in time, up to $t = 1.1$, from a disc of radius 0.05δ, located at 0.5δ below the free surface and containing 900 particles. The particles have non zero initial vertical velocity $v_{p,3} = u_\tau$. A sketch of the problem herein discussed is in Fig.1.

RESULTS

Hereafter we show some results of the simulations, discussing first the Eulerian field and successively the motion of the particles.

The Eulerian field

Figure 2 shows the vertical profiles of the horizontal components of the mean velocity field in the two cases analyzed.

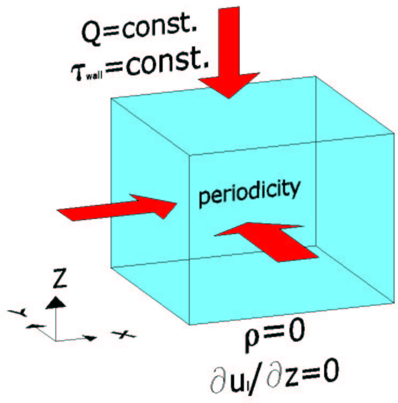

Figure 1: Schematic of the physical problem herein investigated. The quantity Q denotes the free surface heat flux.

The notation $< . >$ denotes Reynolds average quantities. In the neutral case the penetration length of the turbulent field is of the order of δ and, as expected, large gradients are present in the free surface region. The transversal velocity

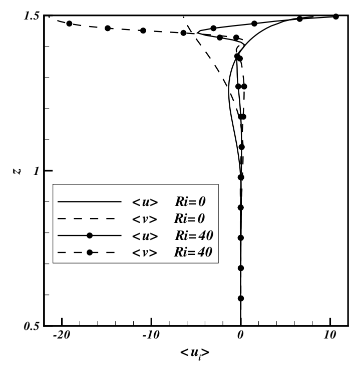

Figure 2: Vertical profile of the non dimensional mean horizontal velocity components.

arising from the rotational motion is large and negative in the free surface region, indicating a rotation of the velocity vector toward the right direction as expected in the northern hemisphere. In the stratified case, the velocity profiles are dramatically affected by the presence of large incoming free surface heat flux. In the stratified case (Fig. 2) we observe a strong reduction of the penetration length of the boundary layer, a small reduction of the streamwise velocity at the free surface and a noticeable increase of the spanwise component. Moreover, large velocity gradients are observed beneath the free surface. As a result, the angle between the wind stress and the velocity vector increases and, in agreement with relevant literature (see for instance Price and Sundermeyer (1999), and Coleman *et al.* 1992) the Ekman spiral appears strongly modified (Fig. 3).

Stratification also affects the elements of the Reynolds stress tensor. Figure 4 shows the non dimensional normal Reynolds stresses. As already observed by Armenio and Sarkar (2000) in the analysis of a strongly stratified turbulent channel flow, stratification affects the normal Reynolds stresses in an anisotropic way. Specifically, the horizontal Reynolds stresses are enhanced whereas the vertical Reynolds stress is strongly inhibited. This is due to the fact that stratification suppresses directly vertical Reynolds stresses as well as the pressure-strain correlation which

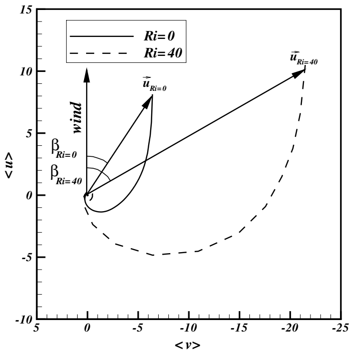

Figure 3: Hodograph of the horizontal components of the mean velocity.

transfers turbulent kinetic energy (TKE) from the horizontal plane to the vertical direction. As a consequence the TKE produced in the horizontal directions tends to remain confined in the horizontal plane.

The non-dimensional Reynolds shear stresses are shown in Fig. 5 for the two cases studied. We observe that stratification inhibits the amount of vertical mixing of momentum and reduces the thickness of the water column where significant mixing is present. In particular the spanwise-vertical Reynolds stress appears dramatically affected by stratification and this explains the strong increase of the mean spanwise velocity observed in Fig. 2.

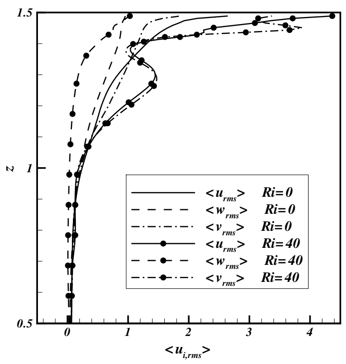

Figure 4: Vertical profile of the normal Reynolds stresses.

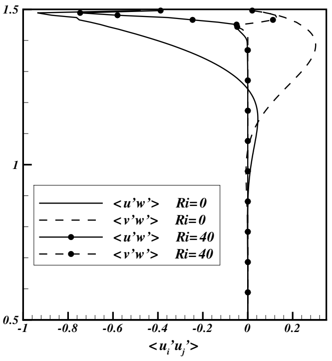

Figure 5: Vertical profile of the Reynolds shear stresses.

The vertical profiles of the mean density and of the density *rms* are shown in Figs. 6 and 7 respectively. In case of neutral flow, the density is to be considered as a passive scalar. From a physical point of view this represents a case where stratification is weak enough that it does not affect the velocity field $Ri \rightarrow 0$. In the $Ri \rightarrow 0$ case we observe

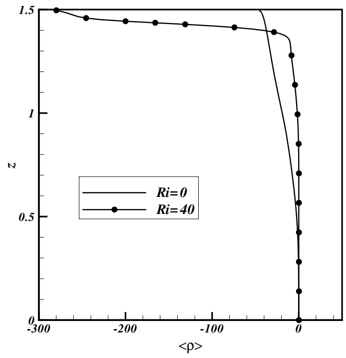

Figure 6: Vertical profiles of the mean density.

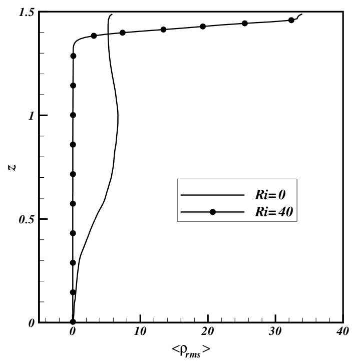

Figure 7: Vertical profiles of the density root mean square.

that density slowly decreases going up toward the free surface and the density appears to be well mixed in the upper part of the domain. Conversely, in case of strong stratification, the consequence of the confinement of the mixed layer in the very upper region of the domain causes a strong decrease of the fluid density in the free surface region and the development of a very intense picnocline in a thin region below the free surface. A similar behavior is observed for the density fluctuations, quantified by ρ_{rms} (Fig. 7): specifically they are very intense and spread over the fluid column in the neutral case, whereas they are confined within a thin region above the picnocline in the stratified case. The analysis of the vertical profile of the gradient Richardson number $Ri_g = N^2(z)/S^2(z)$, where $N^2(z) = g/\rho_0 d\rho/dz$ and $S = d < u > /dz$, in the stratified case, shows that its value is larger than 0.2 along the whole fluid column. According to Armenio and Sarkar (2000) this indicates that the fluid column is in a *buoyancy dominated* regime, where turbulence is suppressed and internal waves are present in the picnocline region. In Fig. 8 we finally show the vertical buoyancy flux $< \rho' w' >$ which quantifies the attitude to vertical mixing of mass in the fluid column. The $Ri \rightarrow 0$ case shows that large vertical buoyancy fluxes are present in the free surface region where active turbulence is present, however noticeable activity is observable along a depth equal to

1.25δ, even larger than the vertical length scale δ. The presence of strong stratification reduces the vertical buoyancy flux. In particular the maximum value is reduced by more than 20% and, more importantly the region where appreciable values are recognized remains limited in the free surface region. Counter-gradient buoyancy fluxes are recorded in the region where the picnocline intensifies, and this is a typical feature of the *buoyancy dominated* regime described in Armenio and Sarkar (2002).

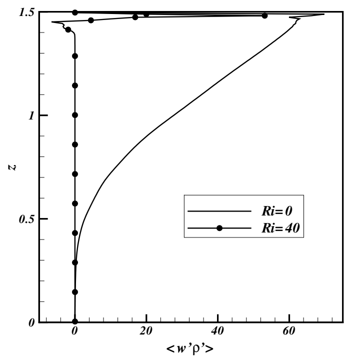

Figure 8: Vertical profiles of the vertical buoyancy flux.

Lagrangian particles

The dynamics above observed has a dramatic impact on the dispersion of the buoyant jet. Figs. 9 and 10 offer a 3D view of the plume of particles at the non dimensional time $t = 1.10$. In case of neutral flow (Fig. 9) the particulate moves upward along a cylindrical path up to $z = 1.25\delta$, that corresponds to a region where the level of turbulent fluctuations is very small; moving upward the particulate enters a region characterized by the presence of non-zero horizontal velocity components and appreciable turbulent fluctuations, that cause the destruction of the cylindrical structure of the jet and the horizontal spreading of the particulate. This effect is enhanced when the particulate continues to move upward and once it reached the free surface it is spread horizontally in the flow field. A very different scenario is observable in the stratified case (Fig. 10). The suppression of turbulence in the region below the picnocline (discussed in the previous section) makes the particulate to travel along a well structured cylindrical path up to the picnocline. This path undergoes a weak distortion in the region $1.2 < z < 1.35$ still maintaining its own organized structure due to the presence of a weak horizontal velocity field and of the Coriolis force. However once the plume reaches the picnocline, it remains confined in that region and oscillates according to a wave-like behavior, due to the presence of internal waves in the fluid column. The confinement of the buoyant jet in the picnocline region has to be attributed to the fact that the particles reach a condition of hydrostatic equilibrium below the free-surface, due to the strong reduction of the ambient fluid density (see Fig. 6). This effect is well known in practical applications (see for example Rubin and Atkinson (2001)) and is well captured by the mathematical model here proposed, which considers an extra-term in the particle motion equation proportional to the Richardson number.

The horizontal displacement and diffusion of the cloud of particulate appear to be inhibited by stratification (see Fig. 11). The horizontal displacement is strongly reduced be-

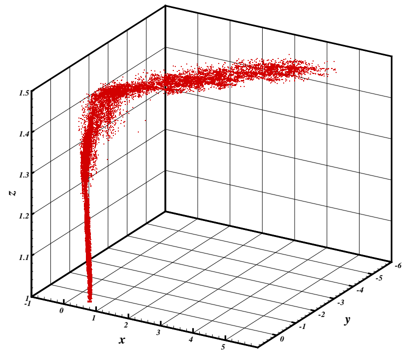

Figure 9: Three dimensional view of the plume of particles at the final time of simulation $t = 1.1$. Case $Ri \rightarrow 0$.

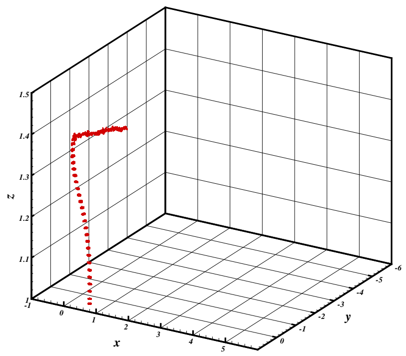

Figure 10: Three dimensional view of the plume of particles at the final time of simulation $t = 1.1$. Case $Ri = 40$.

cause in the stratified case the plume remains entrapped in the picnocline region, well below the free surface, in a region where the horizontal components of the velocity field are small. The horizontal diffusion of the plume is suppressed by stratification due to the inhibition of the turbulent fluctuations in the velocity field.

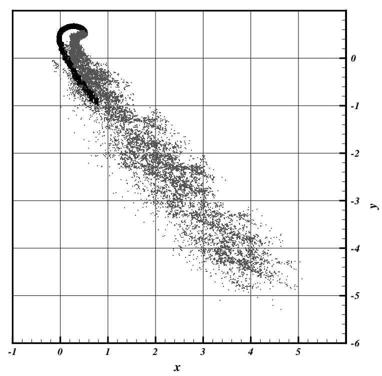

Figure 11: Top view of the plume at $t = 1.1$. Grey particles are Case $Ri \rightarrow 0$; Black particles are Case $Ri = 40$.

A lateral view of the plume of particles (Fig. 12) shows that in the stratified case the jet of particles conserves its shape up to the picnocline; in this region the plume moves horizontally according to the internal waves generated in that region. Conversely, the particles moving in the neutral flow experience a large horizontal spreading well before reaching the free surface.

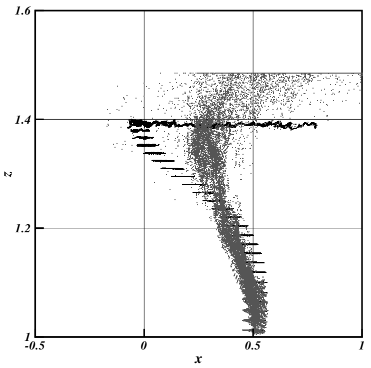

Figure 12: Side view of the plume at $t = 1.1$. Grey particles are Case $Ri \to 0$; Black particles are Case $Ri = 40$.

CONCLUDING REMARKS

In the present paper the dispersion of a plume of buoyant particles in a stratified wind-driven Ekman layer was investigated. The density ratio herein employed is representative of fresh water particles released into a salty water environment. The analysis was carried out in two different conditions, namely a case of neutral flow and a case of strongly stratified flow. Stable stratification was supplied by incoming heat flux at the free surface of the domain. The study was carried out using LES for the Eulerian phase, whereas the dynamics of the plume of particles has been simulated using a Lagrangian technique. In order to make the particles sensitive the actual fluid density, the particle motion equation was improved and an extra buoyancy term, proportional to the Richardson number, has been considered. Due to the Reynolds number used in our numerical experiment, a wall-model approach has been employed, directly imposing the wind stress and the heat flux at the free-surface of the domain, not resolving the near-wall structures. The results of the numerical simulations have shown that under stable stratification, the penetration depth of the boundary layer as well as the vertical mixing in the water column decrease. In case of strong stratification, turbulence is almost completely suppressed in the water column and internal waves are observed in the picnocline region. The dispersion of the buoyant jet of particles is dramatically affected by stratification. In the neutral case, once the plume reach the turbulent region while continuing to go up, it is spread in the horizontal direction by turbulent mixing; afterward, when the particles approach the free surface they are dispersed horizontally according with the local velocity field. In the stratified case, the particles remain entrapped in the picnocline region and are not able to reach the free surface region, characterized by large values of the horizontal velocity component. As a result, both the horizontal displacement and the horizontal spreading of the plume appear strongly inhibited by stratification.

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