LAMINAR-TO-TURBULENT TRANSITION OF PIPE FLOWS THROUGH SLUGS AND PUFFS

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ABSTRACT

Laminar-to-turbulent transition of pipe flows occurs, for sufficiently high Reynolds numbers, in the form of slugs. These are initiated as intermittent disturbances in the entrance region of a pipe flow, and grow in length in the axial direction as they move downstream, initially as sequences of slugs. These merge at some distance from the pipe inlet to form finally the state of fully developed turbulent pipe flow. This formation process is generally known but, as shown in this paper, the randomness of slug formation does not permit detailed studies of slug flows. For this reason, a special pipe flow test rig was developed and built for detailed investigations of deterministically generated slugs. The triggering device for this is outlined. It is also employed to generate puff flows that develop out of disturbances at lower Reynolds number. With increasing Reynolds number, "puff splitting" is observed and the split-up puff develops into a slug. Thereafter, the laminar-to-turbulent transition occurs in the same way as found for slug flows.

INTRODUCTION AND AIM OF WORK

Since the famous work of Reynolds (1883), it is well known that all phenomena connected to the laminar-to-turbulent transition of pipe flows are intermittent. If the Reynolds number is high enough so that slugs form, a finite development length in the pipe is required for the flow to go from its laminar into its turbulent state. Over this length, the flow loses its strongly intermittent nature and turns into a fully developed turbulent pipe flow. For a constant pipe length, one observes that the flow intermittency goes from 0 to 1 over a small but finite Reynolds number range.

After the initial work by Reynolds (1883), there were numerous experimental studies, e.g. carried out by Rotta (1956), Lindgren (1969), Wygnanski and Champagne (1973), Wygnanski et al. (1975), Rubin et al. (1980), Darbyshire and Mullin (1995), Draad et al. (1998) and Hof et al. (2003), showing that two distinct turbulent flow structures determine the intermittent laminar-to-turbulent transition process. Whether one or the other structure occurs is dependent on the Reynolds number and on the flow disturbance introduced. These flow structures are known as puffs (a low Reynolds number phenomenon) and slugs (a high Reynolds number phenomenon).

The above-mentioned studies support the contention that the laminar-to-turbulent transition of pipe flows takes place in form of puff structures for $Re \lesssim 2300$ and slug structures for $Re \gtrsim 3000$, e.g. as reported by Wygnanski and Champagne (1973). The results of their experimental studies showed that puff structures travel with a velocity close to the cross-sectional area averaged flow and their front edge does not have a clear boundary. As experiments show, the front edge velocity of a puff-like flow structure changes gradually in time but its back-edge velocity changes suddenly, showing a well-defined change from the laminar to the turbulent region of the flow. The above investigations clearly show that slug structures have different velocities at their front and back edges. Their front edge travels faster than the cross-sectional area averaged flow velocity and also faster than their back edge, so that the lengths of slugs increase as they move downstream in a pipe. Furthermore, slug structures have, within themselves, turbulence properties similar to those of fully developed turbulent flow. All this information can be extracted from the investigations of Wygnanski and Champagne (1973), Wygnanski et al. (1975), Darbyshire and Mullin (1995) and partly Durst and Ünsal (2006).

In spite of the already existing knowledge on slug- and puff-like flows, there are still open questions regarding the laminar-to-turbulent flow transition in pipes. Some of these relate to the development of puffs and slugs from the pipe inlet to the pipe outlet. Hence the development of these flows needs further experimental studies to understand physically the related flow processes. These studies need to be carried out with the aim of providing some basic knowledge for advanced theoretical and numerical treatments of the laminar-to-turbulent transition of pipe flows.

The present work is a continuation of the investigations of Durst and Ünsal (2006) using an extension of their test rig. They performed hot-wire velocity measurements at the end of an $L/D = 666.7$ long pipe and carried out pressure measurements over the entire pipe length. In their experiments, the laminar-to-turbulent transition occurred naturally at $Re \approx 13000$. Through wall fence-type obstacles of different heights, placed at the pipe inlet, the transitional Reynolds number was decreased from 13000 to approximately 2300. The transitional flow structures were formed as puffs for $2300 \lesssim Re \lesssim 3000$ and in form of slugs for $3000 \lesssim Re \lesssim 13000$.

To trigger single puff and slug structures, Durst and Ünsal (2006) also employed an electrically driven iris diaphragm, which created instantaneous small flow blockages in the pipe inlet for closing durations of down to 10 ms. This triggering technique permitted the repeatable generation of single puff- and slug-like flow structures in the Reynolds number region where natural transition did not occur. Hence, in this way, the device of Durst and Ünsal (2006) greatly facilitated accurate measurements of the front and back edge velocities of puffs and slugs through the installed pressure sensors. Phase-resolved velocity measurements with a hot-wire sensor, located at the end of the pipe, were also performed to yield a deeper insight into the flow structures of slugs and puffs. These latter measurements were demonstrative in nature in the work of Durst and Ünsal

(2006) and did not aim to give detailed characteristics of the single structures.

In the present work, the "iris diaphragm single flow structure triggering technique" was used to investigate the dynamics of the puff- and slug-structure developments, from their generation at the pipe inlet over the entire pipe length. The measurements were conducted for different pipe lengths starting from $L = 0.5$ m and extending to 8 m. The pipe diameter was kept constant at 15 mm. For every pipe length, the occurrences of flow structures and their developments were investigated by hot-wire velocity measurements. The change in the puff structures with increasing Reynolds number to slug structures could be shown through phase-averaged velocity measurements. The splitting of puff structures, starting at a certain Reynolds number and at various pipe lengths, are shown from velocity signals obtained at the center of the pipes. This phenomenon occurred at Reynolds numbers that were not high enough to produce slugs directly. The investigations showed that the split puffs propagate without changing their forms and durations, if the Reynolds number is not sufficiently higher than the Reynolds number when puffs first form. This and other findings are described in detail below. They are based on different possible definitions of the start of transition, which is by the changes of U_{rms} values (Durst and Ünsal (2006)), and by the occurrence of first puffs (this paper).

TEST RIG, TRIGGERING DEVICE AND MEASUREMENT EQUIPMENT

The experimental set-up, including the mass flow rate control unit (see Fig. 1), used in the present investigations, was explained in detail by Durst et al. (2003) and Durst and Ünsal (2006). Therefore, only a brief description will be given here for the sake of completeness and to provide a clear understanding of the experimental equipment employed.

The main parts of the test rig, shown in Fig. 1, were the mass flow rate control unit (MFCU), the pipe flow test section shown together with a flow conditioner (plenum chamber with honeycomb structures and an inlet nozzle) at the inlet of the pipe, the hot-wire anemometer system and the computer operated with a special data acquisition system. In the set of experiments, a mass flow rate controller unit was employed, which had the task of supplying constant mass flow rates, for each set of Reynolds numbers, with an accuracy of $\pm 1\%$ for any downstream flow condition. This allowed different Reynolds numbers to be set and the preset flowrate to be maintained for each of the attempted investigations. Due to the critical valve operation principle of the MFCU (see Durst et al. (2003)), the pressure changes in the pipe, induced by the turbulence structures during an intermittent transition process, did not affect the set mass flow rate value. Hence the employment of the MFCU ensured constant Reynolds number operation of the test rig in spite of the occurence of the laminar-to-turbulent transition of the investigated pipe flows.

To trigger single puff and slug structures in pipe flows, an electrically driven iris diaphragm unit, shown in Fig. 2, was employed. Two magnets were used to achieve independently controlled closings and openings of the iris diaphragm. The height of the iris diaphragm after closing ("wall fence") was adjusted to yield predefined wall fence heights at the wall to trigger the flow in accordance with the findings of Durst and Ünsal (2006) regarding the critical wall fence height.

The duration of operation of the iris diaphragm was controlled electrically through a driving signal imposed on the

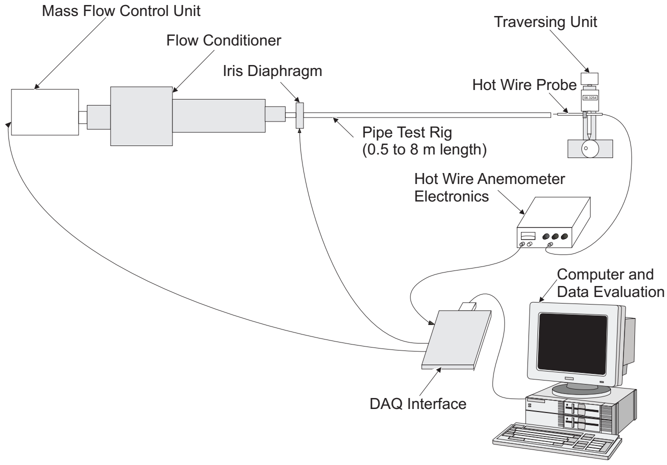

Figure 1: Schematic representation of the experimental test rig, triggering device and measuring equipment

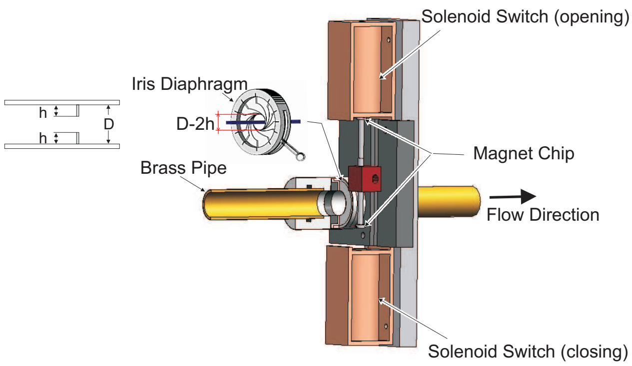

Figure 2: Three-dimensioned presentation of the iris diaphragm triggering system

triggering device by the computer employed. The smallest closing and opening duration that could be realized with the new magnets was 30 ms, which was small enough to generate repeatable single puff and slug structures in all experiments.

The main experimental investigations were carried out for a pipe of $D = 15$ mm diameter and for nine different pipe lengths ($L = 0.5$, 1, 2, 3, 4, 5, 6, 7 and 8 m). Hot-wire velocity measurements were conducted at each pipe outlet, i.e. for each pipe length, over the entire Reynolds number range $Re \approx 2300$ to $Re \approx 11500$. ($Re \approx 11500$ was the critical Reynolds number ($(Re)_{crit}$) at which the laminar-to-turbulent transition occurred naturally with the present pipe flow.) For each pipe length, the laminar velocity profiles were measured to ensure flow symmetry. The actual velocity measurements for studying the occurrence of slug- and puff-like flow structures were conducted at the centerline of the pipe. Puff and slug structures, generated through periodical closing and opening of the iris diaphragm, were investigated as functions of time by hot-wire velocity measurements. Approximately 100 puffs and slugs were investigated in each experiment. In this way, statistically averaged velocity values, such as the mean velocity and different turbulence quantities, could be measured for each of the Reynolds numbers investigated. The test rig was also employed for transitional flow investigations with a fixed height of the wall fence produced by ring-type obstacles mounted as described by Durst and Ünsal (2006). These experiments are described separately in Section 6.

The hot-wire probe employed was mounted on a traversing unit. A single wire probe, connected to DISA 55 M01 constant-temperature hot-wire anemometer electronics, was used in all experiments. To obtain velocity profile measurements, the vertical motion of the traversing system was activated and the wire position was controlled through the computer. The hot-wire anemometer outputs was connected to a 16-channel, 16-bit, 333 kHz data acquisition card for simultaneous measurements of velocities. The input flow rate of the mass flow rate controller was also set by the computer and a special software program ensured that the entire measurements were carried out in a well-controlled manner. Various sub-programs within the data acquisition software were written to carry out the processing of all data to yield the flow information provided in the present paper.

VERIFICATION EXPERIMENTS

Prior to carrying out studies of puff and slug flow developments as a function of pipe length, verification experiments were performed to ensure the correct functioning of the entire test rig, especially the triggering device and the measuring equipment employed. These measurements clearly showed that the test rig permitted the pipe flow to remain laminar up to $Re \approx 11500$ and over the length of $L = 8$ m corresponding to $L/D = 533.3$ with $D = 15$ mm. For different pipe lengths, the laminar flow development was checked, yielding detailed cross-sectional velocity profile information. The developments of the velocity profiles with pipe length agreed with corresponding flow predictions and confirmed the development length correlation proposed by Durst et al. (2005):

$$\frac{L}{D} = \left[(0.619)^{1.6} + (0.0567Re)^{1.6}\right]^{\frac{1}{1.6}} \qquad (1)$$

Hence, for every investigated flow Reynolds number, the development length required, to yield the parabolic velocity profile could be computed. This information permitted the state of the flow to be assessed for the investigated pipe lengths, i.e. the state that the flow would have without the introduced intermittent flow disturbances.

The triggering system of the iris diaphragm permitted controlled flow disturbances to be introduced that resulted in puff- or slug-like flow structures for $2300 \lesssim Re \lesssim 11500$. To test the triggering system, ring-type obstacles were also employed to trigger the flow. For small Reynolds numbers and large triggering wall fence heights h (see Fig. 3), puff-like flow structures resulted. For higher Reynolds numbers, only slug-like flow structures were found. This is shown in Fig. 3 providing information on the required triggering height h as a function of Reynolds number to yield the laminar-to-turbulent transition of pipe flows.

The results in Fig. 3 agree fairly well with those presented by Durst and Ünsal (2006), who also performed measurements with carefully manufactured wall fence obstacles. The small differences recorded for the region of puff formation are not of any importance for the studies described in this paper.

To demonstrate that the iris diaphragm tripping device is able to provide the change of intermittency of the laminar-to-turbulent transition, the frequency of slug and opening of the iris diaphragm was changed 0 to a maximum value, defined by the pipe length. In this way, the transition, usually observed for the natural laminar-to-turbulent transition could be studied at every chosen Reynolds number ($Re < (Re)_{crit}$) by adjusting the wall fence height accordingly. This readily suggests that the test rig with the iris diaphragm triggering device is very well suited for studying laminar-to-turbulent transition in pipe flows. The control provided is well suited for detailed experimental flow investigations of otherwise uncontrolled flow phenomena.

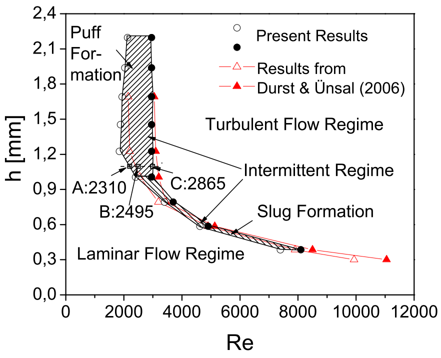

Figure 3: Dependence of critical Reynolds number on the normalized wall fence height

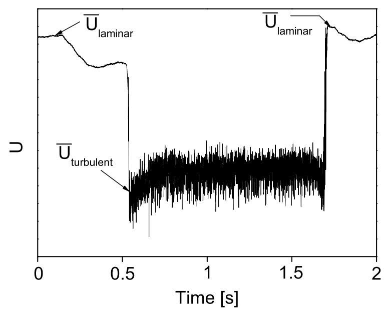

Figure 4: Hot-wire velocity signal of a single slug during a natural transition at $Re = 11500$

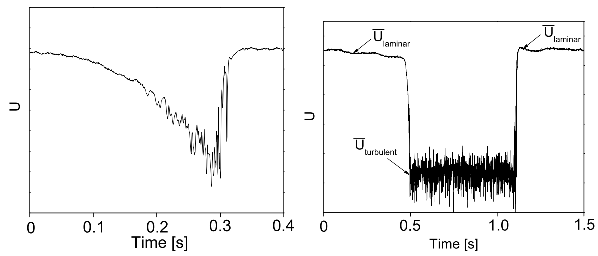

Figure 5: Hot-wire velocity signal of puff (left) and slug (right) initiated by the iris diaphragm triggering device

For Reynolds numbers of approximately $Re \approx 11500$, naturally occurring slug flows were recorded for the present test rig and examples are shown in Fig. 4, which illustrates the typical features already pointed out by Wygnanski and Champagne (1973). Typical flow structures for $Re \lesssim 2300$ puff flows and for $Re \gtrsim 3000$ slug flows are indicated in Fig. 5, which were initiated by the iris diaphragm triggering device. A comparison of literature data with the traces in Fig. 5 shows that there is no difference in the forms of puffs and slugs compared with the signals reported by Wygnanski and Champagne (1973).

Furthermore, the velocity records along the pipe permitted the front and back edge propagation velocities of the triggered slugs to be compared with corresponding results available in the literature. Fig. 6 shows good agreement

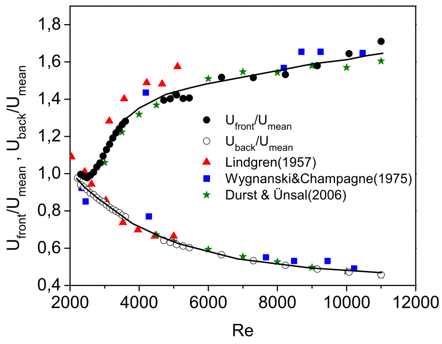

Figure 6: Comparison of the authors' non dimensional front and back edge propagation velocity of puffs and slugs with former results

between triggered and naturally occurring transitional flow propagation velocities. The comparison was made with the data recorded by both Lindgren (1969) and Wygnanski and Champagne (1973), who measured the propagation velocities of naturally occurring transitional flow structures together with the results from Durst and Ünsal (2006). The comparisons in Fig. 6 confirmed that the present puff and slug structures were similar to those which occur at Reynolds numbers where the laminar-to-turbulent transition occurs naturally as it is briefly mentioned in Durst and Ünsal (2006).

STUDIES OF SLUG DEVELOPMENTS

The test rig described in Section 2 and applied in Section 3 for first experimental studies of flow phenomena occurring in the region of laminar-to-turbulent transition of pipe flows was then extensively applied to study slug flows. Performing hot-wire measurements at the pipe axis at different downstream locations showed that slugs form basically in the same way, at different Reynolds numbers, if the generation mechanism is kept the same. Hence only their development when they move downwards in the pipe is dependent on Reynolds number.

Examples of hot-wire velocity measurements at pipe lengths of $L = 0.5$, 3 and 8 m are shown in Fig. 7 for $Re = 6250$. Each of the time records shown is composed of 11 individual slugs. The individual slugs in Fig. 7 cannot be identified any longer, which shows the high degree of reproducibility with which the iris diaphragm triggering device is able to produce slugs.

The shortest time (30 ms) achievable to close partially and reopen the iris diaphragm was employed for each Reynolds number. Each closing produced a wall fence disturbance with a height of h mm, in accordance with results plotted in Fig. 3 to ensure that the initial disturbances had sufficient strength to form a slug and to provide the basis for its study as it moves downstream in the pipe. During this motion, the length of the slugs is increasing. This is shown in Fig. 8 for nine downstream measuring locations and four Reynolds numbers. The figure clearly indicates the different speeds at which the front and the back edges of the slug move. A higher front edge velocity, which is also indicated in Fig. 9. The propagation velocities in Fig. 9 were calculated from the time which the front or back edge of the slug has reached to the point where the hot-wire is located. Therefore it is pipe length averaged propagation velocity, though the difference between front and back edge propagation ve-

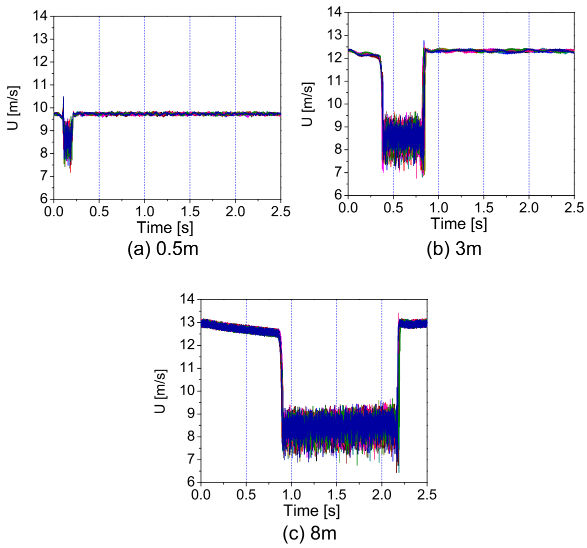

Figure 7: Successive hot-wire velocity signals of triggered slug structure development along the pipe for $Re = 6250$ measured at $L = 0.5$, 3 and 8 m showing good repeatability of slugs.

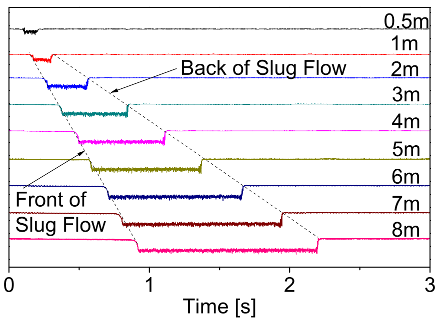

Figure 8: Triggered slug structure development at $Re = 6380$ with increasing pipe lengths $L = 0.5$ to 8 m

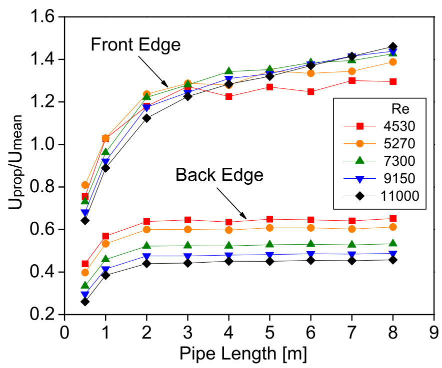

Figure 9: Normalized front and back edge propagation velocities of slugs for different Reynolds numbers

locities are clearly shown. The former is increasing with the pipe length L and the latter remains constant after $L = 2$ m where the flows reach fully developed state.

The test rig also permitted sequences of slugs to be produced with time intervals between them, as indicated in Fig. 10, which indicates that the iris diaphragm permitted the

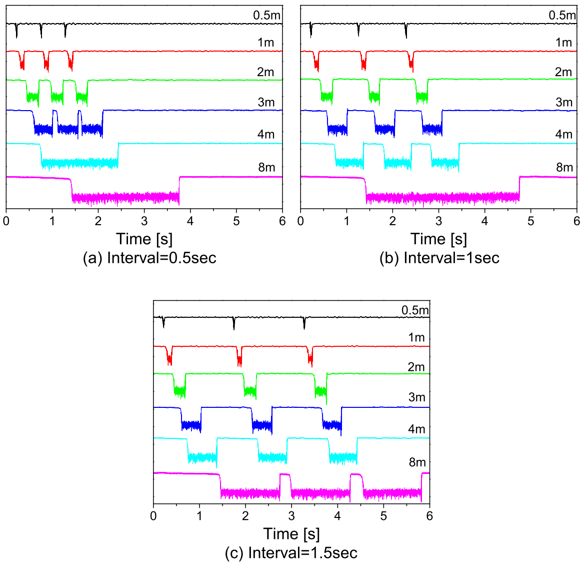

Figure 10: Three successive trigger causing three successive slug structures and their merging at $Re = 4200$

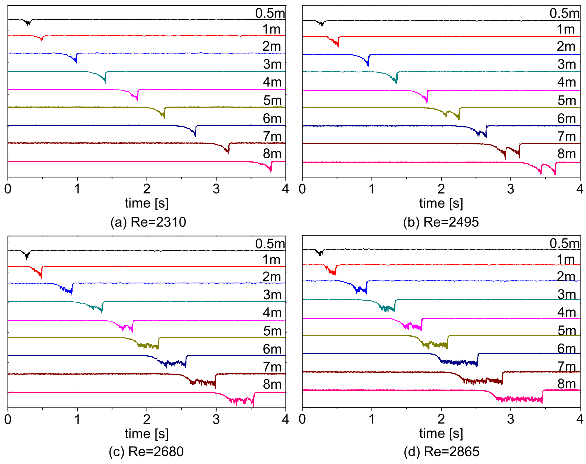

Figure 11: Triggered puff structure development with increasing Reynolds number measured at different locations from $L = 0.5$ to 8 m for $Re = 2310$, 2495, 2680 and 2865

time intervals between the slug-generating disturbances to be changed. Fig. 10(a) and (b) show that a length of the pipe of $L = 8$ m was sufficient for the individual slugs to merge with two different intervals. Hence one slug-like flow left at the pipe outlet at $L = 8$ m. In Fig. 10(c), the interval was chosen such that the length of the pipe was insufficient for the slugs to merge. These slug would have merged, however, if the pipe length had been larger. This clearly shows that laminar-to-turbulent transition records show a strong L/D dependence.

STUDIES OF PUFF DEVELOPMENTS

It is apparent that the test facility described in Section 2 is also suited for the introduction of disturbances that result in the formation of puffs. For this purpose, the iris diaphragm was closed to yield disturbances of about $h \approx 1.1$ mm and the Reynolds number of the flow was changed for this set of experiments from $Re \approx 2300$ to 4300. In this

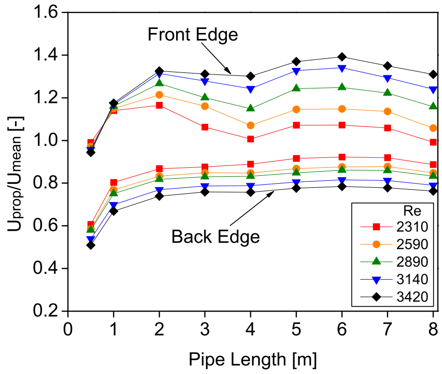

Figure 12: Normalized front and back edge propagation velocities of puffs for different Reynolds numbers

Reynolds number range the changes of the structures of puffs were studied.

The results of the measurements for Reynolds numbers from $Re \approx 2300$ (point A in Fig. 3) to Re ≈ 2900 (point C in Fig. 3) with different pipe lengths are shown in Fig. 11. These show that for the lowest Reynolds number, $Re = 2310$ (Fig. 11(a)), a single puff formed in the entrance region of the pipe. As it moved downstream with the flow, it maintained itself as a single puff. As the Reynolds number was increased to $Re = 2495$ (Fig. 11(b)), corresponding to point B in Fig. 3, a single puff was available only up to about a pipe length of $L = 5$ m. Thereafter puff splitting occurred, yielding basically a sequence of two puffs. For even higher Reynolds numbers (see Fig. 11 (c) and (d)), corresponding to point C in Fig. 3, puff splitting was observed very much earlier, permitting the flow between the front and the back of the disturbed flow to develop and yielding, further downstream, slug-like flows. The front edge of the resulting flow turned out still to have the properties known for puffs. The back edge of the resultant flow structure showed already the steep change from the turbulent to laminar state of the flow, characteristic of slugs.

Fig. 12 is shown for the sake of comparison of the front and back edge propagation velocities between slugs and puffs. The propagation velocities were calculated in the same way as it for slugs thus, it is pipe length averaged propagation velocities. Both front and back edge propagation velocities are increasing with the pipe length up to $L = 2$ m. There is slight increase in front edge propagation velocity at $L = 4$ m which were caused by the puff splitting, though there is no further increase in front edge propagation velocity unlike the case of slugs (see Fig. 9).

For the smallest Reynolds number investigated in this set of experiments, $Re = 2310$, puff averaged velocity information was obtained as shown in Fig. 13(a), showing the velocity on the pipe axis at $L = 8$ m as a function of time. Single puffs with a high repeatability of their velocity fields resulted for this low Reynolds number, showing at their front edge a slow decrease in mean velocity and a sudden velocity change in the back edge. The velocity profiles, provided in Fig. 13(a) as a function of time, are typical for puff-like flow structures in the laminar-to-turbulent transition region of pipe flows at $L = 8$ m.

If one keeps $h \approx 1.1$ mm and increases the Reynolds number of the pipe flow in small steps, the puff-like flow structure changes, as illustrated in Fig. 13(b) - (d). In Fig. 13(b), typical puff splitting occurs, yielding two velocity peaks. Increasing the Reynolds number further yields

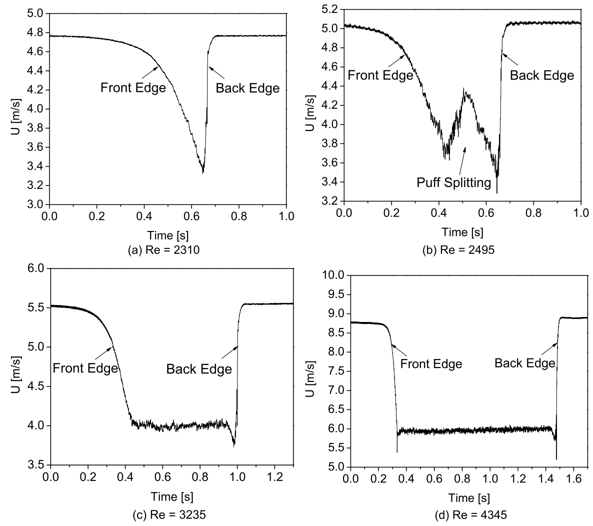

Figure 13: Cycle averaged axial velocity distribution of a center line in a puff at Re = 2310, 2495, 3235 and 4345 measured at $L = 8$ m

velocity distributions over time that are similar to those of slugs (Fig. 13(c) and (d)), at least in the center part of the structure. Only the front and back edges of the signal seem to differ from those of slug flows. The latter form also for high fence heights if the Reynolds number is sufficiently high, i.e. lie higher than the Reynolds number indicated by point C in Fig. 3.

It is interesting that the experimental studies of Wygnanski and Champagne (1973) resulted in two clearly separated regions for the appearance of slugs and puffs in transitional pipe flow. They only observed puffs for very high levels of disturbances and at low Reynolds numbers. This clear separation between slugs and puffs was not found by Darbyshire and Mullin (1995), who detected both in some flow regimes. They also found that their mixed occurrence was not just dependent on the magnitude of the disturbance but also on the type of flow disturbances that they introduced. In the present study, the flow disturbance was introduced at the pipe inlet by a short duration of inserted "wall fences". For this kind of disturbance, the results in Fig. 3 resulted which indicated clear puff formation only for disturbances located to the left of the marked vertical area in Fig. 3, indicating that the introduction of large disturbances was needed. As one moves away from the left-hand line of this area, e.g. forwards from point A, leaving h the same, the puffs that occur split up, yielding a sequence of two puffs initially. As the Reynolds number is increased further, multiple splittings seem to occur that yield, finally, flow structures that have an overall appearance similar to that of slugs. Without any doubt, the present test rig set-up proved to be well suited to studying phenomena of this kind. Its highly reproducible triggering capabilities make the occurrence of puffs and slugs a deterministic rather than a statistical phenomenon.

CONCLUSION, FINAL REMARKS AND OUTLOOK

A special test facility was developed to investigate experimentally the laminar-to-turbulent transition of pipe flows. This facility can deterministically create disturbances which develop into puffs and slugs. This is described clearly in the paper, and also the way in which the experiments and measurements should be carried out to study the properties of slugs and puffs. In particular, the results in this paper show how slugs and puffs develop along the pipe. The information was obtained by measuring the time variation of the longitudinal velocity at the axis of the pipe outlet cross-section using different lengths of pipes. In a first set of experiments, the development of slugs was studied and the results how the slugs expand along the pipe, as a result of differences in the propagation velocities of the front and back edges of the slugs. Second, the development of puffs was investigated with different pipe lengths taking the Reynolds number into account. At lower Reynolds numbers, puffs form and remain nearly unchanged along a pipe. However, when the Reynolds number of the flow is increased, the puffs are characterized by splitting and for sufficiently high Reynolds numbers, the puffs merge into slugs and start to expand as slugs do in the pipe flows.

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