NUMERICAL STUDY OF THE STABILISATION OF TOLLMIEN-SCHLICHTING WAVES BY FINITE AMPLITUDE STREAKS

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ABSTRACT

Large-eddy simulations of laminar-turbulent transition due to Tollmien-Schlichting (TS) waves and their control by steady boundary-layer streaks are performed. The streaks are forced at the inflow as optimal solutions to the linear parabolic stability equations (PSE), and the TS-waves are excited via a harmonic volume force (together with small-amplitude noise) within the computational domain . The results show, in agreement with recent experimental and theoretical studies, that significant damping of unstable two-dimensional TS-waves of various frequencies can be obtained. The damping characteristics are mainly dependent on the streak amplitude. A new phenomena is also identified which is characterised by the strong amplification via nonlinear interactions of the second spanwise harmonic of the streak when the streak amplitude is comparable to the TS amplitude. Moreover, visualisations of the flow field are used to highlight the different vortical structures and their interactions that are relevant to this flow case.

INTRODUCTION

The reduction and control of the viscous drag force exerted on thin bodies moving in a fluid is of great technical interest. Several active and passive methods to achieve a delay of laminar-turbulent transition in the boundary layer have been developed in the past. The study by Cossu and Brandt (2002) showed the stabilisation of the Tollmien-Schlichting (TS) waves by steady streaks of finite amplitude in the Blasius boundary layer. In the presence of streaks, *i.e.* spanwise modulation of the two-dimensional bounday-layer flow, the unstable TS-waves evolve from two-dimensional waves to spanwise modulated waves, referred to as streaky TS-waves. They have similar phase speed as their two-dimensional counterpart and are less unstable. The experiments by Fransson *et al.* (2005) confirmed the theoretical predictions and demonstrated that such a stabilising effect can indeed lead to transition delay (Fransson *et al.*, 2006). In this study, we perform a numerical study of such a stabilisation in a realistic framework in order to investigate the effect of streaks of varying amplitude, spacing, and the corresponding sensitivity of the transition delay. In particular, the evolution of perturbations at low streak amplitudes, *i.e.* when the amplitude of both TS-waves and streaks are comparable, is considered.

SIMULATION APPROACH AND VALIDATION

Numerical method

The presented simulation results are obtained using a spectral method to solve the three-dimensional, time-dependent, incompressible Navier-Stokes equations (see Lundbladh *et al.* (1999)). In the streamwise and spanwise directions, Fourier series are used whereas the wall-normal direction is discretised with Chebyshev polynomials. The periodic boundary conditions in streamwise direction are combined with a spatially developing boundary layer by adding a "fringe region" at the end of the domain. In this region, the outflowing fluid is forced via a volume force to the prescribed inflow velocity field, which in this case consists of a Blasius boundary layer profile (zero-pressure gradient) with superimposed optimal streaks (see below). The inflow is located at Reynolds number $Re_{\delta_0^*} = U_\infty \delta_0^*/\nu = 300$ (corresponding to $Re_x = 32000$), where ν is the fluid viscosity, U_∞ the free-stream velocity and δ_0^* the displacement thickness at the inlet. The simulation box has dimensions $L_x \times L_y \times L_z$ equal to $2000 \times 60 \times 180$ in the streamwise, wall-normal and spanwise directions, respectively, made non-dimensional based on δ_0^*. Results are obtained with a resolution $N_x \times N_y \times N_z$ of $512 \times 121 \times 128$ grid points. With this resolution the use of large-eddy simulation (LES) is necessary to obtain accurate results. For this purpose the ADM-RT model is employed (Schlatter *et al.*, 2004). With this model, the effect of the unresolved spatial scales is accounted for by adding to the momentum equations a relaxation term proportional to the high-pass filtered velocity field, *i.e.* $-\chi H_N * \overline{u}_i$. Here, χ is a model coefficient which is set constant in the present work, H_N* symbolises the action of the high-order high-pass filter defined in three dimensions, and $\overline{u}_i$ is the grid-filtered velocity. The relaxation term acts as an energy sink and thereby inhibits the build-up of energy near the numerical cutoff. The ADM-RT model was found to be well suited for the spectral simulations of transitional flows. In particular, the vortical structures during breakdown can be predicted accurately in both the temporal and spatial setting (Schlatter *et al.*, 2006).

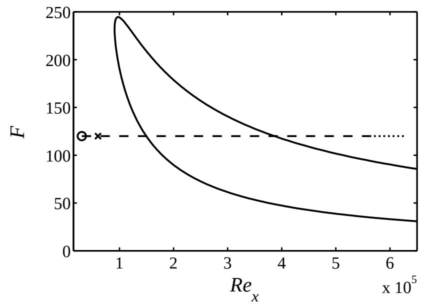

Figure 1: Stability diagram showing the neutral curve for two-dimensional TS waves. The dashed line signifies the computational domain with inlet at $Re_x = 32000$ ($\circ$), and the cross $\times$ indicates the streamwise position where the volume forcing is applied. The dotted line is the fringe region downstream of $Re_x = 590000$.

TS-wave generation

The TS-waves are forced at $Re_x = 60000$ by a harmonic volume force acting in the wall-normal direction at a non-dimensional frequency $F = 120$, corresponding to $\omega_0 = 0.036$, see Figure 1. The rms-TS amplitude at branch I ($Re_x \approx 150000$) is approximately 0.76%. Small-amplitude steady, spanwise random noise is also introduced. This will trigger secondary instability of the two-dimensional waves leading to K-type transition shortly after branch II if no control is applied. The validation of the forcing of the TS-waves is presented in Figure 2, which displays the perturbation streamwise velocity profile slightly after branch I in comparison with linear stability theory (LST). Figure 3 shows the growth rate of the wall-normal maximum of the streamwise velocity fluctuation compared to results from solving the (linear) parabolic stability equations (PSE). The TS-waves evolve nonlinearly into a saturated state, therefore also a comparison with lower amplitude forcing is provided in the figure. Good agreement with LST and PSE is obtained by the current LES for both the velocity profile and the growth rate of the TS-waves.

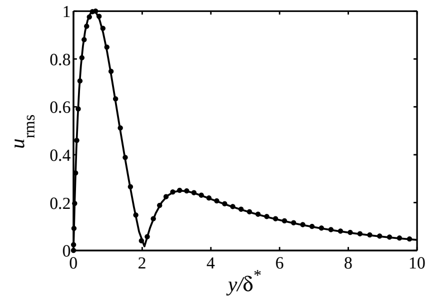

Figure 2: Comparison of TS-waves obtained by LES with LST. $u_{\rm rms}$ at $Re_x = 216000$, ——— LES, • LST.

The nonlinear evolution and breakdown of the TS-waves is also correctly captured by the current LES, as shown in Figure 4 where the classic subharmonic scenario by Herbert (1993) is reproduced. In the plot, the evolution of the relevant Fourier components of the perturbation fields is displayed and compared to PSE.

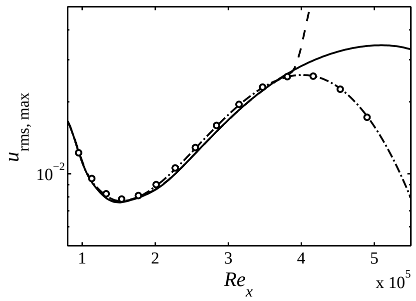

Figure 3: Comparison of TS-waves obtained by LES with LST and PSE. Wall-normal maximum of $u_{\rm rms}$ for - - - - uncontrolled case with random 3D disturbances, ——— 2D nonlinearly-saturated TS-wave, —·— linear low-amplitude TS-wave (rescaled), $\circ$ PSE.

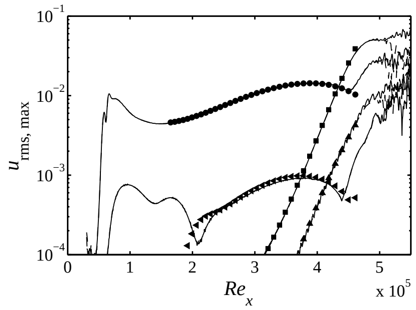

Figure 4: Evolution of various Fourier modes for H-type (subharmonic) transition (Herbert, 1993). ——— current LES; symbols: PSE. • mode $(\omega_0, 0)$, ■ $(\omega_0/2, \beta_{\rm crit})$, ◀ $(2\omega_0, 0)$, ▲ $(3\omega_0/2, \beta_{\rm crit})$.

Streak generation

The complete velocity vector field obtained with the linear code developed by Levin and Henningson (2003) is used to force the desired streaky perturbation at the inflow of the computational domain. These streaks are introduced in the fringe region by adding them to the laminar Blasius profile U_B. The streaks considered are optimally growing perturbations, *i.e.* solution of the linearised boundary-layer equations, and are characterised by the spanwise wavenumber $\beta_{\rm st} = 2\pi 10/L_z$ and the streamwise location of their maximum amplitude ($Re_x \approx 185000$). The latter values are chosen to approximatively match the streaks in the experiments by Fransson *et al.* (2005). The different values of the streak amplitudes considered are reported in Table 1. The streak amplitude $A_{\rm st}$ is defined as:

$$A_{\rm st}(x) = \left[\max_{y,z}(U - U_B) - \min_{y,z}(U - U_B)\right] / 2U_\infty$$

Note that the streaks of largest amplitude are susceptible to secondary inviscid instability (Andersson *et al.*, 2001).

RESULTS

Two-dimensional waves

Several LES using the above setup have been performed. The results presented are obtained by averaging in time and in the spanwise direction and by performing Fourier analysis on the velocity fields saved during one or two periods of the

Table 1: Amplitudes $A_{\rm st}$ of the streaks used for the various simulations.

Streak	$A_{\rm st}$ at inlet	$A_{\rm st,max}$	Re_x of maximum
A	29%	39%	120000
B	20%	32%	150000
C	10%	19%	170000
D	5%	10%	195000
E	2.6%	5.1%	190000
F	1.7%	3.4%	185000
G	0.6%	1.2%	185000
N	0%	0%	n/a

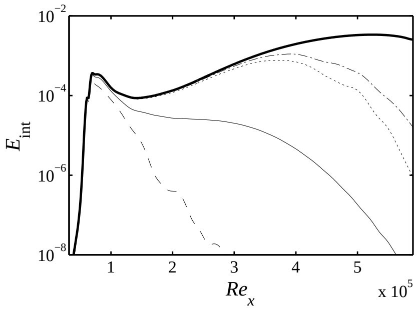

Figure 5: Energy integrated in the wall-normal direction contained in mode $(\omega_0, 0)$ of the two-dimensional TS waves at $F = 120$ in the presence of streaks. ‐ ‐ ‐ ‐ streak B, —— streak C, ········ streak E, –·– streak F, —— no streak (case N).

TS-waves. Analysis of the spectral content of the velocity fields has been performed by Fourier transforming a number of full velocity fields in time and in the spanwise direction, where modes are denoted by $(\omega_0, \beta_{\rm st})$-pairs in the following. The linear evolution of the TS-waves in the streaky boundary layer is considered first. The results confirm the finding by Cossu and Brandt (2002) and Fransson *et al.* (2005); streaks of increasing amplitude have a stronger quenching effect on the unstable waves (Fig. 5). Figure 6 displays the behaviour of TS-waves of lower frequency in a longer domain. By comparing with the streak amplitude in the lower plot it is evident that the observed stabilisation is related to the local streak amplitude. In the following we will therefore consider only waves with frequency $F = 120$, with the assumption that the effect of the streak amplitude can be related to that of the wave frequency. Lower frequencies will be amplified further dowmstream and will ride on streaks of lower amplitudes; in other words, what is found at lower streak amplitudes can also be observed for stronger streaks, only further downstream and with waves of lower F.

The wall-normal disturbance profiles of the streamwise velocity belonging to the streaky TS-waves averaged in the spanwise direction are reported in Figure 7. The typical M-shaped structure, *i.e.* featuring two local maxima of the rms values close to the wall, observed in previous numerical and experimental studies, is well captured by the current LES.

Influence of the spanwise scale of the streak

Simulations of transition featuring TS-waves and streaks under controlled conditions have been performed to investigate the effect of the streak spanwise scale on the stabilisation and possible transition delay. It is observed that if the TS-waves reach sufficiently high amplitudes, *i.e.* of the order of 0.5% of the free-stream velocity, and the streaks are characterised by the spanwise scale of the unstable secondary instability modes, turbulent breakdown is indeed promoted by the presence of the steady streaks. If, on the other hand, the streak spacing is chosen too narrow, the coupling between TS-wave and streak is low leading to a reduced damping effect. Therefore, the streak spanwise wavenumber used in the following results is chosen about three times larger than that of the most unstable wavenumber of secondary instability. This scale roughly corresponds to that already used in the experiments by Fransson *et al.* (2005).

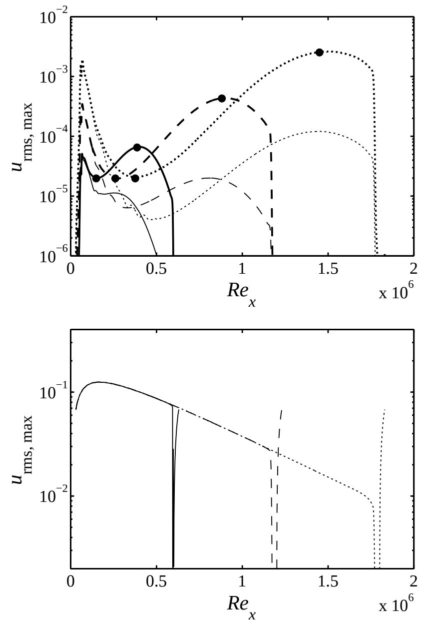

Figure 6: Streak C and 2D TS-waves of different amplitude and frequency. —— $F = 120$, $A_{\rm TS} = 2.575 \cdot 10^{-5}$, ‐ ‐ ‐ ‐ $F = 70$, $A_{\rm TS} = 1.692 \cdot 10^{-4}$, ········ $F = 50$, $A_{\rm TS} = 1.000 \cdot 10^{-3}$. *a)* Mode $(\omega_0, 0)$, thick lines are only TS-waves, thin lines controlled with streak C. • position of branch I and II according to LST. *b)* Streak amplitude $u_{\rm rms,max}$.

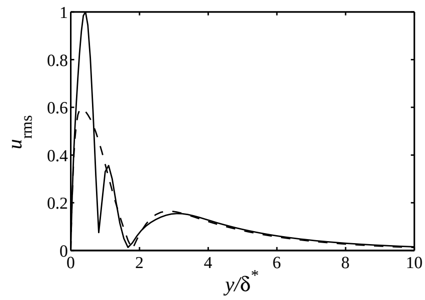

Figure 7: Waveforms of the —— streaky TS-waves compared to the corresponding normal TS waves (‐ ‐ ‐ ‐) at $Re_x = 482000$. The amplitude of the streaky waves are rescaled by 110 to account for the lower growth rate.

2D waves with small-amplitude noise

The transition delay obtained in the presence of the steady streaks is displayed in Figure 8. The uncontrolled reference case is given by the two-dimensional forcing at $F = 120$ (exciting two-dimensional TS-waves) and a steady forcing random in the spanwise direction. The amplitude of the random forcing is more than one order of magnitude lower than that for the TS-wave. The described setup will lead to K-type breakdown characterised by an aligned pattern of Λ-vortices which subsequently break down to turbulence. When increasing the streak amplitude, the transition location moves monotonically downstream. The skin-friction coefficient c_f (Fig. 8*a)*)remains at the laminar value for the two cases with streaks of largest amplitude (streaks C and D), an increase of c_f by a few percent being only observable where they reach their peak amplitude. The explanation for the observed stabilisation is provided in Figure 8*b)* where the shape factor H_{12} associated to the base flows under consideration is reported. The presence of the streaks progressively reduces the value of H_{12} in the initial laminar region thus stabilising the flow. Figure 8*c)* shows the level of streamwise velocity perturbation in the boundary layer, accounting for the presence of the streaks. For large amplitudes of the latter, $u_{\mathrm{rms,max}}$ is dominated by the steady contribution, so the curves basically display the streamwise streak development. Conversely, in the absence of streaks the pertubation consists mainly of TS-waves and the breakdown can be identified by the sharp rise in the fluctuation level at higher Re_x. For intermediate values of the streak amplitudes, both the initial amplitude of these spanwise modulation and the breakdown further downstream can be seen.

a)

b)

c)

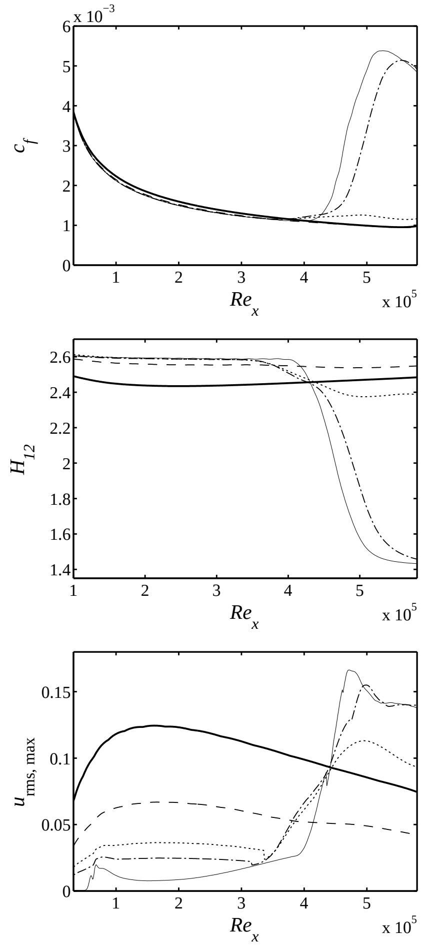

Figure 8: *a)* Skin friction coefficient c_f, *b)* shape factor H_{12}, and *c)* $u_{\mathrm{rms,max}}$ averaged in time and spanwise direction. —— Streak C, – – – – streak D, ········ streak E, –·– streak F, —— case N (uncontrolled).

Results of the Fourier analysis are presented in Figure 9 for streaks C, D and E. For the largest streak amplitude, streak C considered in Figure 9*a)*, the fundamental steady streak can be seen as the only dominant mode $(0, \beta_{\mathrm{st}})$.

a)

b)

c)

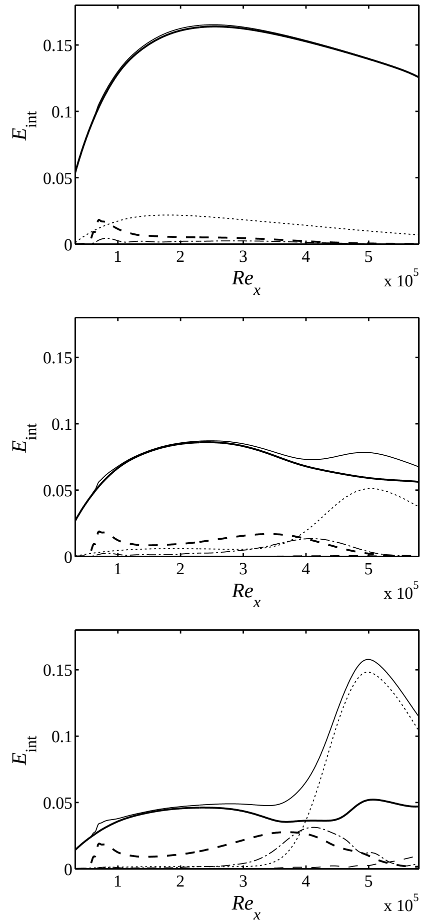

Figure 9: Energy integrated in the wall-normal direction for selected From *a)* to *c)*: Streaks C, D, E, respectively. —— Steady streak $(0, \beta_{\mathrm{st}})$, – – – – TS-wave $(\omega_0, 0)$, ········ first harmonic of streak $(0, 2\beta_{\mathrm{st}})$, –·– first harmonic of streaky TS-wave $(\omega_0, \beta_{\mathrm{st}})$, – – – – oblique mode $(\omega_0, \beta_{\mathrm{st}}/3)$, —— sum of all displayed modes.

Both the two-dimensional and oblique TS-waves are quickly damped, and the first harmonic of the streak $(0, 2\beta_{\mathrm{st}})$ remains as the second largest mode. For this the mode associated with the TS-wave $(\omega_0, 0)$ does not experience any significant growth. In this case, the flow is well described by the linear evolution of TS-waves in a spanwise modulated boundary layer as in the analysis by Cossu and Brandt (2004).

On the other hand, the simulations with streaks of lower amplitudes, cases D and E, $A_{\mathrm{st}} = 5\%$ and 2.6% respectively, (Figs. 9*b)* and *c)*) highlight a new physical phenomenon observed at those low streak amplitudes when streaks and TS-waves have similar strength. Thus, more complex nonlinear interactions between both disturbances can take place. Initially, the $(0, \beta_{\mathrm{st}})$ streaky mode is dominating, reducing the growth of the streaky TS-waves $(\omega_0, 0 \dots 2\beta_{\mathrm{st}})$. (Comparison with the uncontrolled case is not shown here due to lack of space.) Further downstream, $Re_x > 4 \cdot 10^5$, however, a significant growth of oblique modes $(\omega_0, \beta_{\mathrm{st}})$ is seen. This induces, by nonlinear interactions, a strong amplification of the steady $(0, 2\beta_{\mathrm{st}})$ mode, *i.e.* a doubling of the initial streaks is observed (see also the visualisation in Figure 11*b)*). Towards the end of the domain, also a growth of the mode $(\omega_0, \beta_{\mathrm{st}}/3)$ can be observed, eventually leading to breakdown further downstream, as partially shown in Figure 11*b)*). Note that the streak doubling occurs also without the presence of the noise (see below), but in that case it is not followed by turbulent breakdown. The latter is associated to the growth of oblique modes of subharmonic frequency and wavenumber, *i.e.* the mode $(\omega_0, \beta_{st}/3)$.

Flow visualisations

Instantaneous visualisations of the flow field are shown in Figure 10 for five streak amplitudes, streaks A, C, D, F and N, and with the same boundary layer excitation (TS-waves and three-dimensional steady noise). In these top views of the three-dimensional flow, light grey isosurfaces indicate vortical structures identified by using the λ_2 criterion, the second largest eigenvalue of the Hessian of the pressure, whereas isosurfaces in lighter and darker grey visualise low- and high-speed streaks, respectively. In the case of streak A, Fig. 10*a)*, breakdown occurs well within the computational domain. Streak A is indeed unstable to linear perturbations (Andersson *et al.*, 2001). The relevant vortical structures at the late stage of transition are quasi-streamwise vortices aligned in a staggered pattern and following the spanwise oscillations of the low speed streak. The scenario observed is the same as that arising from the sinuous secondary instability of a steady streak examined by Brandt and Henningson (2002). This scenario has been identified by the latter authors as the most likey to occur in the case of steady streaks of amplitudes A_{st} larger than approximately 26% of the free-stream velocity. Conversely, streak C is not strong enough to undergo direct secondary instability leading to transition. However, it is strong enough to substantially quench the growth of the TS-waves. In agreement with the experimental findings, a clean spanwise-modulated base flow can therefore be seen in Figure 10*b)*; the slow downstream decay of the streak amplitude is also evident. Further decreasing the streak amplitude, nonlinear development of the TS-waves is observed. For the case in Figure 10*c)* using streak D with steady random noise, this leads to the formation of aligned Λ-structures, associated with the streak doubling discussed above. Considering streak E in Figure 11*b)* amplification of oblique modes is observed at the end of the computational domain, triggering transition of the new streaky base flow, dominated by the doubled mode $(0, 2\beta_{st})$, close to the outflow. For streaks C and D, however, breakdown is not occurring within the computational domain. The streak of lowest amplitude (streak F), conversely, is not able to reduce the TS-waves enough to prevent transition: turbulent flow can indeed be seen at the end of the computational domain in Figure 10*d)*. The last plot, Fig. 10*e)*, displays the uncontrolled case. Aligned Λ-structures, typical of the K-type scenario, can clearly be observed. In summary, Figure 10 depicts the various possible transition mechanisms in a boundary layer with streamwise streaks: From the classic K-type scenario (at low streak amplitude) to bypass transition of high-amplitude streaks. In between these limiting cases, stabilisation and transition delay is achieved by means of spanwise modulations of the base flow by means of moderate amplitude streaks.

The case of richest nonlinear interaction between streaks and TS-waves of the same order of amplitude, leading to the streak doubling, is further examined in Figure 11. In Fig. 11*a)*, the clean case is considered, *i.e.* only two-dimensional TS-waves and streaks are excited in the numerical simulation without any additional noise. Figure 11*b)* displays the flow when steady three-dimensional noise is also added by a streamwise localised forcing, whereas the data in Fig. 11*c)* pertain to the flow with time-dependent three-dimensional noise. Noise obviously needs to be added to trigger laminar-turbulent transition. The streak doubling is clearly observed in the clean case as well. This feature is therefore neither affected nor triggered by the presence of steady noise. However, in the presence of noise, transition can be observed at the end of the computational domain: Λ and hairpin vortices can be distinguished in the flow. In the case of time-dependent noise transition occurs much earlier, however still later than in the uncontrolled case (not shown here). The streak doubling is present also in this case even though it is not so evident in the present figure, but can clearly be identified via Fourier transform (not shown). The results confirm that for the parameter settings considered here the subharmonic breakdown is the most rapid one. The computations presented have also been performed with unsteady noise and the results give a picture of the effect of the streaks on the subharmonic transition scenario (and associated delay) similar to the effect on the transition induced by the three-dimensional steady noise.

The figure also shows another important finding: The noise forced upstream obviously survives during the streak dubling process and subsequently determines the transition scenario observed further downstream.

CONCLUSIONS

The experimental results presented by Fransson *et al.* (2006) have been successfully reproduced in our numerical simulations using LES. We are indeed able to show the stabilising effect of finite amplitude steady streaks on the evolution of TS-waves and the following transition delay in a realistic numerical setup. The use of LES allowed very large computational domains both in the streamwise and spanwise direction. Further, random background noise of steady and unsteady nature is excited in the flow in addition to the disturbances we wish to consider (streaks and TS-waves), in order to trigger transition in a natural way. The effect of varying the streak amplitude is analysed, and a new phenomena is outlined at low streak amplitudes (or for lower TS frequencies) where more complicated nonlinear interactions are obviously possible. The interaction between TS-waves and streaks of comparable amplitudes leads indeed to a significant growth of oblique modes (ω_0, β_{st}). This induces, by nonlinear interactions, a strong amplification of the steady $(0, 2\beta_{st})$ mode, *i.e.* a doubling of the initial streaks is observed.

Additionally, the effect of the streak spanwise scale is considered. The results indicate that stabilisation and transition delay is possible if the streak wavenumber does not correspond to those of the natural secondary instability of the TS-waves, in the latter case transition promotion is observed. However, if the TS-waves are very weak and the streaks are able to keep their amplitudes to those characteristics of a linear behaviour breakdown is not occurring.

Results presented here are mainly for the case where small-amplitude steady noise was used to trigger natural transition. A series of simulations was also performed with time-dependent three-dimensional random excitations. The results are similar to those presented and are therefore not reported in detail.

The response of the boundary layer to fully three-dimensional excitations, an important study which was not carried out in the experiments, is being performed. The results indicate that the passive control strategy proposed by Cossu and Brandt (2002) and verified by Fransson *et al.* (2006) is going to be successful when the streaks interact a)

b)

c)

d)

e)

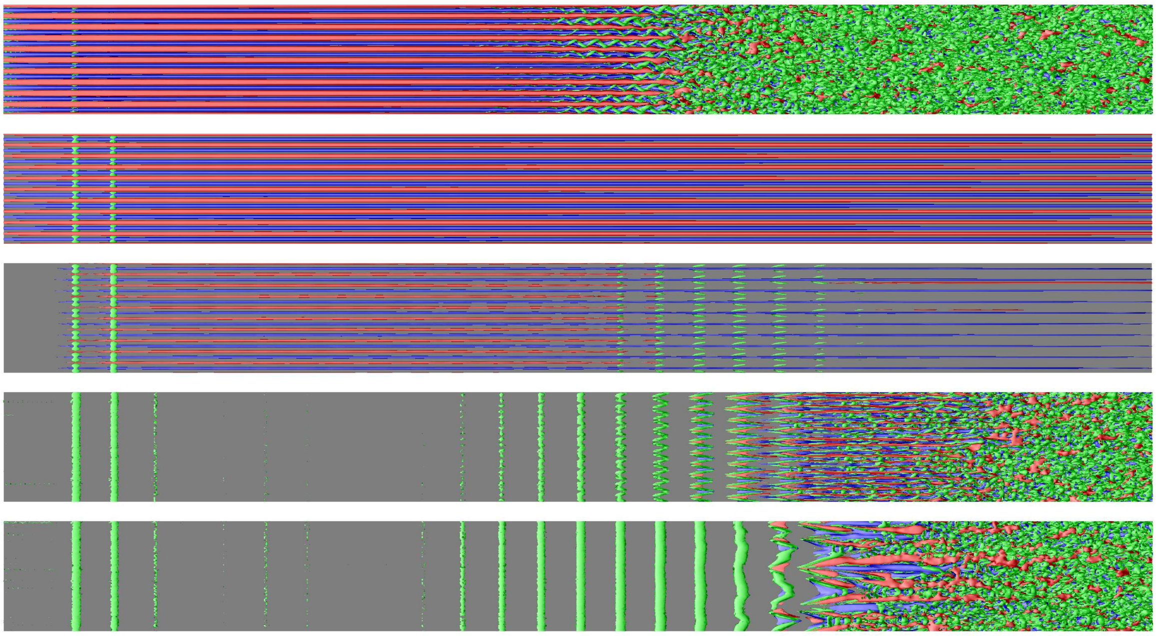

Figure 10: Top view of the three-dimensional flow structures for streaks A, C, D, F, N (uncontrolled), from *a)* to *e)*. Green isocontours represent the $\lambda_2 = -0.00008$ vortex-identification criterion, red and blue isocontours are positive and negative disturbance velocity $u' = \pm 0.07$, respectively. Flow from left to right.

a)

b)

c)

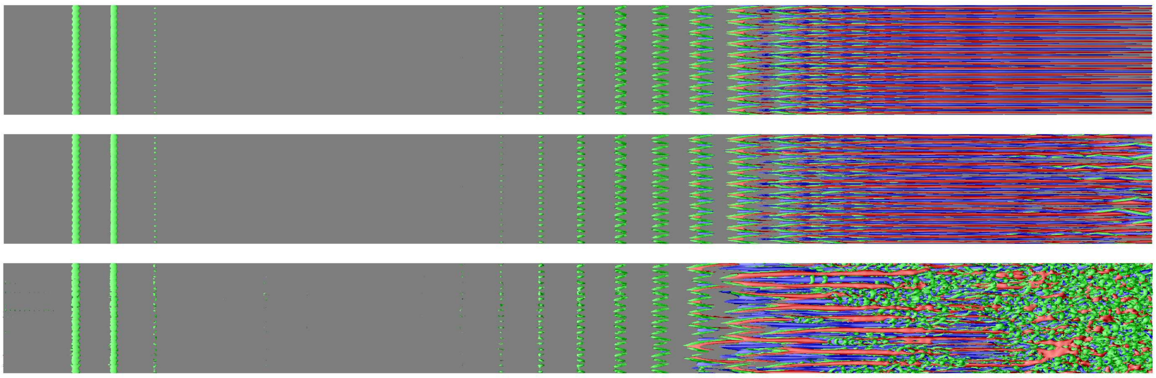

Figure 11: Top view of the three-dimensional flow structures for streak E (same contour levels as in Figure 10). *a)* clean TS-wave, *b)* steady noise, *c)* unsteady noise.

with TS-waves of low amplitudes. If, on the other hand, the transition scenario is not characerised the exponential growth of TS-waves (*e.g.* bypass transition), the control with streamwise streaks appears to be not as successful. However, this will be considered in more detail in future investigations.

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