HYBRID V2F RANS/LES MODEL AND SYNTHETIC INLET TURBULENCE APPLIED TO A TRAILING EDGE FLOW

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ABSTRACT

The flow over a trailing edge is computed using two different techniques to reduce the computational costs of LES. A hybrid method designed to split the contributions of the averaged and fluctuating velocity fields is used in order to relax the near-wall mesh requirements. A synthetic method for turbulence generation is used at the inlet in order to avoid a costly precursor simulation. The methodology has been first tested on channel flows at high Reynolds numbers on coarse meshes. The results at different Reynolds numbers up to Re_τ are presented. They agree well with DNS data available in terms of mean velocities and stresses. The results for the trailing-edge flow are compared with the full LES with inlet boundary conditions from a precursor boundary layer simulation. Two cases are presented, one with a precursor simulation and one with the synthetic eddy method. The predictions of mean velocities and turbulent content agree well with the reference LES simulation.

INTRODUCTION

Large Eddy Simulation has been successfully applied to many different kinds of flows, but its use in industry has remained scarce, mainly due to the large constraint present in the mesh requirements of wall bounded flows, especially at high Reynolds numbers. For such flows, the size of the energy containing structures scales with $Re_\tau = 4000$ and hence the number of grid points required to resolve accurately the near wall eddies scales with $Re^{1.76}$ at least (unstructured grids). To circumvent this severe near wall requirement, LES can be restricted to the simulation of the outer flow eddies with a RANS like eddy viscosity model used to model the dynamics of the near wall eddies. In recent years, such hybrid methods combining RANS and LES have received increased attention from groups around the world. In an attempt to ease such computational requirements in wall bounded flows, many approaches have been suggested. One method is to use so-called "wall functions" to bridge the viscous sublayer and provide a suitable boundary condition for the wall cells (Piomelli and Balaras, 2002). This can range from a log-law approximation (Schumann, 1975) to a solution of a system of simplified equations in the near wall region (Balaras et al., 1996).

Another approach is the use of RANS equations near the wall to provide the outer layer with correct information. The main problem of this type of approach is how to connect a statistically averaged flow (RANS) with the instantaneous filtered field (LES). A way to couple the two types of flows is the Detached Eddy Simulation (DES) (Spalart et al., 1997; Travin et al., 1999) in which the turbulent lengthscale in the RANS equation is switched to a lengthscale based on the mesh filter width in order to reduce the viscosity in the separated region. Other approaches are 'zonal', in which a part of the domain is set to be computed using RANS equations and the rest is computed with LES. Examples of such types of models can be found in Davidson and Peng (2003), Temmerman et al. (2005) or Hamba (2003). In the zonal approach, the treatment of the interface has always been of importance for the success of the method since the RANS information does not provide correct turbulent fluctuations. Some ways to deal with this issue are the introduction of backscatter (Piomelli et al., 2003), damping the modelled stresses (Temmerman and Leschziner, 2002), the addition of fluctuations at the LES side of the interface (Davidson and Dahlström, 2005) or the use numerical smoothing (Tucker and Davidson, 2004).

A second constraint of the LES technique has been the problem of boundary conditions, which ideally need to be time and space dependent. To reduce the cost incurred by retaining boundary conditions from precursor calculations, the development of synthetic turbulence generation methods has been the focus of many studies in recent years (Klein et al., 2003; Keating et al., 2004). These synthetic methods are able to reproduce spectra or moments of real turbulence but do not produce turbulence eddies with neither correct shape nor dynamics. Therefore the flow downstream of the inlet undergoes an adjustment as the synthetic fluctuations evolve until the correct phase information is retrieved.

In this paper we address both of these issues. First by introducing a hybrid model that uses the elliptic relaxation approach described in Laurence et al. (2004) as a RANS baseline model to be combined with the standard Smagorinsky sub-grid scale model for LES. Results for channel flows up to $Re_\tau = 4000$ in coarse meshes are presented. Secondly the case of flow over a aerofoil is computed using the a synthetic eddy method of Jarrin et al. (2006) at the inlet. In this way we show that the flow can be computed with significant reduction in the amount of CPU resources used compared to traditional LES, without a significant loss in performance.

THE HYBRID METHOD

The hybrid method splits the residual stress tensor into two parts, the "locally isotropic" part and the "inhomogeneous" part in the same lines of Schumann (1975)

$$\tau_{ij}^r - \frac{2}{3}\tau_{kk}\delta_{ij} = -\underbrace{2f_b\nu_r(\overline{S}_{ij} - \langle\overline{S}_{ij}\rangle)}_{\text{locally isotropic}} - \underbrace{2(1-f_b)\nu_a\langle\overline{S}_{ij}\rangle}_{\text{inhomogeneous}} \tag{1}$$

The isotropic part, which controls the dissipation of turbulent energy, is treated with a standard SGS viscosity and a fluctuating strain rate. The inhomogeneous part, which

Table 1: Parameters for the channel flow calculations.

Re_τ	Cells	Δx^+	Δz^+
395	40x40x30	59	39
590	40x40x30	88	59
1100	50x50x40	140	88
2000	50x40x50	256	160
4000	64x80x64	400	200

affects the flow directly has a RANS viscosity and a mean strain rate. The two components are joined together via a blending function which relates the length scales provided by the RANS model and the LES.

$$f_b = \tanh\left(\left(C_L \frac{L_t}{L_\Delta}\right)^n\right) \tag{2}$$

where L_t is the turbulent length scale provided by the RANS model and L_Δ is the LES filtered length scale (using $L_\Delta = C_s\Delta$). $C_L = 1$ and $n = 1.5$ are empirical constants chosen to match velocity and stress profiles at $Re_\tau = 395$ on a coarse mesh with $\Delta x^+ = 59$ and $\Delta z^+ = 39$. The subgrid-scale viscosity is calculated using the Smagorinsky (1963) model:

$$\nu_r = (C_s\Delta)^2\sqrt{2s'_{ij}s'_{ij}} \tag{3}$$

$$s'_{ij} = \overline{S}_{ij} - \langle\overline{S}_{ij}\rangle \tag{4}$$

The RANS viscosity is calculated from the averaged velocity field using the elliptic relaxation model of Durbin (1991) modified as in Laurence et al. (2004):

$$\nu_a = C_\mu \varphi k T \tag{5}$$

where $\varphi = \overline{v^2}/k$ and T is the timescale given by

$$T = \max\left(\frac{k}{\varepsilon}, C_T\sqrt{\frac{\nu}{\varepsilon}}\right) \tag{6}$$

The lengthscale L_t is computed as

$$L_t = \varphi\frac{k^{3/2}}{\varepsilon} \tag{7}$$

The effects of the wall are introduced via elliptic relaxation on the variable f which acts as the source term in the equation of φ. The transport equations (k,ε,φ and f, see Laurence et al. (2004)) are solved using the averaged velocity field obtained from a moving average of the instantaneous field computed with equation 1.

The method has been validated for channel flows with Reynolds numbers up to $Re_\tau = 4000$ (see Figure 1). The parameters used for these calculations are listed in table 1. The RANS to LES blending is very smooth since the velocity profiles follow quite closely the log law whereas other hybrid RANS-LES methods are known to introduce an increase of velocity at the interface (see Nikkitin et al. (2000)). The meshes are too coarse for a wall resolved LES. In figure 2 the results at $Re_\tau = 395$ on the same mesh are compared with the standard Smagorinsky LES (with Van Driest damping), which overpredicts the velocity due to the underresolved prediction of the shear stress. The hybrid model improves the shear stress by adding the modelled stress based on the averaged velocity as it is shown in figure 3. The normal stresses are also improved by the hybrid method as it can be seen from figure 4 where the resulting Smagorinsky LES stresses using the same mesh are

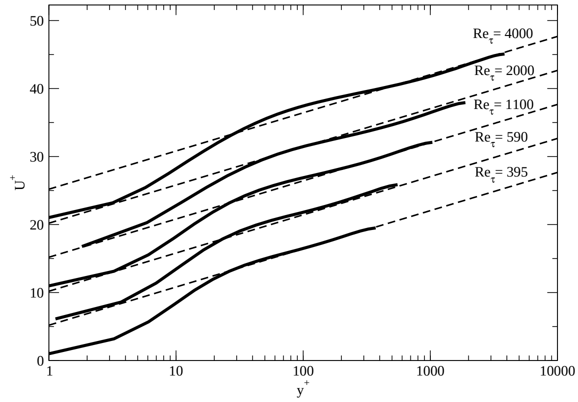

Figure 1: Velocity profiles at different Reynolds numbers with the hybrid model. (dotted line represents the log law)

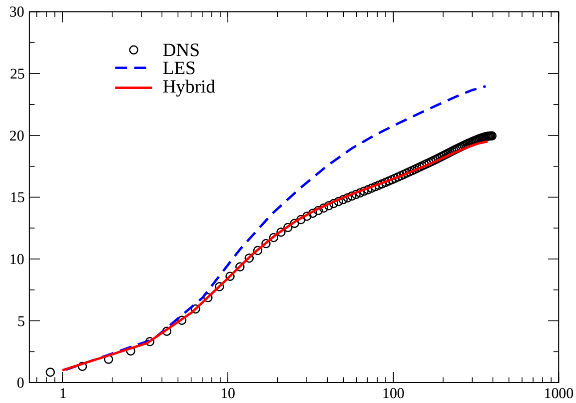

Figure 2: Velocity profile at $Re_\tau = 395$

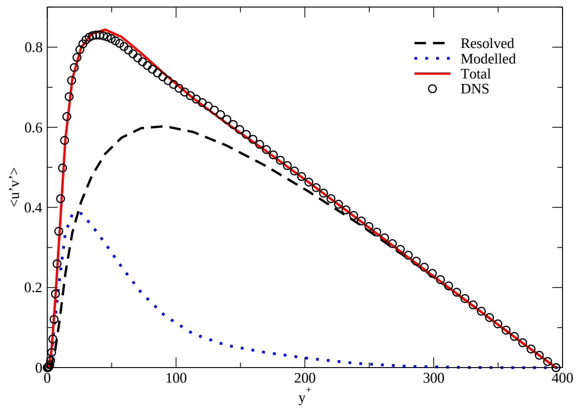

Figure 3: Shear stress at $Re_\tau = 395$

also shown. Results at a $Re_\tau = 2000$ are compared with the DNS data of Hoyas and Jimenez (2006) (figure 5). Despite the very coarse mesh (about 10^5 cells), the turbulent stresses are in good agreement with the DNS (10^{10} cells) for $y^+ > 250$. Below this value of y^+, the blending function is less than 50% hence the simulation is operating mostly in RANS mode which imposes the correct mean velocity profile in the near wall region as seen from 1.

SYNTHETIC EDDY METHOD

Since in LES or hybrid RANS/LES, the unsteady motions of energy-carrying turbulent structures are resolved, the velocity fluctuations imposed at the inflow of the com-

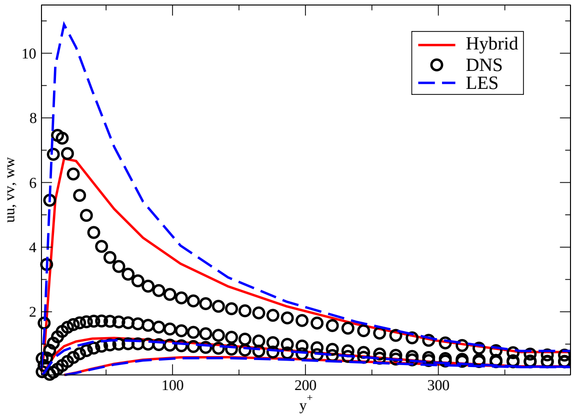

Figure 4: Normal stresses at $Re_\tau = 395$

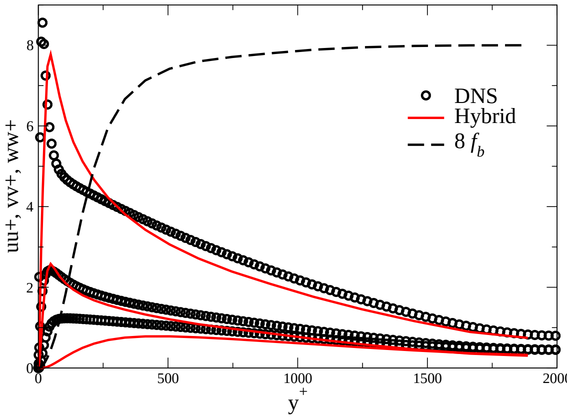

Figure 5: Normal Reynolds stresses for $Re_\tau = 2000$ with the Hybrid model.

putational domain must represent the contributions of these turbulent structures. Although providing accurate inflow boundary conditions, the simulation of the upstream boundary layers requires extra CPU and data storage resources. Synthetic turbulence generation methods, though less accurate, provide the main simulation with inlet boundary conditions for only a fraction of the CPU time needed in the computation of the upstream flow. The main idea of the Synthetic Eddy Method of Jarrin et al. (2006) is to assume that the turbulent inflow is composed of a superposition of coherent structures with particular intensities, shapes and length-scales. Assumptions are made regarding the characteristics of the inflow structures using information provided by the RANS statistics. A random distribution of eddies with prescribed intensities, shapes and sizes is then generated. If x^k, y^k and z^k are the x, y and z coordinate of the centre of eddy k, the velocity signal generated by the SEM reads

$$\mathcal{U}_i(x_j,t) = \sqrt{\frac{V_b}{N}} \sum_{k=1}^{N} \varepsilon_i^k f_L(x_1 - x_1^k) f_L(x_2 - x_2^k) f_L(x_3 - x_3^k) \tag{8}$$

where V_b is the volume of the 'box of eddies' B over which eddies are going to be generated, N is the number of eddies, L is the turbulence lengthscale and f_L is a symmetric function that characterises the decay of the fluctuations generated by each eddy about its centre. In the simulation presented here, the function f_L is a tent function which reads

$$f_L(r) = \sqrt{\frac{3}{2L}}(1 - |r/L|) \quad \text{if } |r| \leq L \tag{9}$$

$$= 0 \quad \text{otherwise} \tag{10}$$

The turbulence lengthscale L is computed from

$$L = \max(k^{3/2}/\varepsilon, \Delta) \tag{11}$$

where $\Delta = \max(\Delta x, \Delta y, \Delta z)$ in order for the synthetic structures generated at the inlet to be discretised on the computational mesh. The intensity of the fluctuations $\varepsilon_i^{(k)}$ are taken from independent normal distribution $N(0,1)$. The initial position of each eddy k is taken from a uniform distribution over a 'box of eddies' B defined by

$$B = \{(x_i) \in \mathbb{R}^3, \quad x_{i,\min} < x_i < x_{i,\max}\}, \tag{12}$$

where

$$x_{i,\min} = \min_{x \in P}(x_i - L) \quad \text{and} \quad x_{i,\max} = \max_{x \in P}(x_i + L) \tag{13}$$

and P is the inlet plane where the velocity fluctuations are computed. In order for the synthetic signal to be correlated in time, the eddies are convected through the inlet plane with the bulk velocity U_b over the boundary layer

$$x_1^k(t + dt) = x_1^k(t) + U_b \, dt. \tag{14}$$

Once an eddy is convected outside of the box, it is regenerated upstream and its intensities $\varepsilon_i^{(k)}$ are drawn again. The signal computed from equation (8) has spatial and temporal correlations and satisfies $\langle u_i \rangle = 0$ and $\langle u_i u_j \rangle = \delta_{ij}$. It can be modified as follow

$$u_i = \langle U_i \rangle + a_{ij} \mathcal{U}_j, \tag{15}$$

where $\langle U_j \rangle$ is a target mean velocity profile and a_{ij} is the Cholesky decomposition of a target Reynolds stress tensor R_{ij}.

TRAILING EDGE FLOW

The efficiency of the hybrid approach for complex turbulent flows is assessed by considering turbulent boundary-layer flows past an asymmetric trailing-edge. The Reynolds number, based on the free stream velocity and the aerofoil chord, is 2.15×10^6. The case has been treated before by Wang and Moin (2000) using a finely resolved LES and by Wang and Moin (2002) and Tessicini et al. (2006) using LES with wall modelling. In order to further reduce the cost of the computation, only the rear-most 38% of the aerofoil chord is computed.

Two cases are presented here in order to test the behaviour of the SEM. In case 1 the inlet boundary conditions are taken from Wang and Moin (2000) using the following procedure. First, an auxiliary RANS calculation is conducted in a domain enclosing the entire strut. The resulting mean velocities, accounting for the flow acceleration and circulation associated with a lifting surface, are used as the inflow profiles outside the boundary layers on both side of the strut. Originally two RANS simulations were performed using the $v^2 - f$ turbulence model of Durbin (1991) and the Menter (1993) SST model. The two turbulence model produce a noticeable difference in the velocity overshoot (undershoot) outside the upper (lower) boundary layer. Within

Table 2: Mesh sizes.

	Fine LES	Present
Domain size	1536x96x48	512x64x24
Δx^+,Δy^+, Δz^+	62,55,2	206,110,2
Inlet B.L. required	2x(240x96x48)	2x(72x64x24)
Ratio	0.31	0.28

the turbulent boundary layers the time series of inflow velocities are generated from two separate LES calculations of flat-plate boundary layers with zero pressure gradient, using the method described by Lund et al. (1998). The inflow generation LES matches the local boundary layer properties, including the momentum thickness and Reynolds number, with those from the RANS simulation. A no-slip condition is applied on the surface of the strut. The top and bottom boundaries are placed far away from the strut to minimise the impact of the imposed symmetry boundary condition. At the downstream boundary a standard exit boundary condition is applied.

The second calculation (case 2) uses the synthetic eddy method (SEM) of Jarrin et al. (2006) as described in the previous section, which generates the inflow boundary conditions for the hybrid simulation on the truncated domain. This method of generating the inlet conditions is much cheaper than the precursor simulation used in the case 1. In table 2 the size of mesh used in the reference LES and the present calculation are shown. An estimate size of a boundary layer calculation required is also shown. This estimate is based on the rescaling method of Lund et al. (1998). The CPU cost in terms of number of cells of the precursor boundary layer simulation is about 30% of the aerofoil computation. For the SEM the CPU cost is negligible.

In the present case, we used a mean velocity profile computed from a SST (Menter, 1994) simulation of the whole trailing edge and k and ε profiles coming from a log law approximation. Due to the reduced information available, the target Reynolds stress tensor used in the final step equation (15) was approximated by $R_{ij} = 2/3k\ \delta_{ij}$. The number of eddies N was set to 1000 on both the lower and upper boundary layer inlet planes.

Velocity profiles and rms streamwise velocity fluctuations profiles are available from the fine LES (Wang and Moin, 2000) at locations of $x/h = -0.625$, -1.125, -1.625, -2.125, -3.125 on the aerofoil and at $x/h = 0$, 0.5, 1, 2 and 4 in the wake ($x/h = 0$ is located at the trailing edge of the aerofoil).

The absolute velocity profiles ($\sqrt{\langle U\rangle^2 + \langle V\rangle^2}$) on the upper surface of trailing edge are shown in figure 6. and the corresponding levels of u_{rms} in figure 7. The hybrid model predictions agree well with the LES when the same precursor inlet simulation are used. The model is capable of sustaining the turbulence in the inlet boundary layer and predicts separation close to the LES because in the hybrid model the resolved stresses can develop independently from the RANS viscosity, i.e. the model associates the RANS viscosity with the mean flow only and links the resolved scale dissipation to the LES viscosity only.

The results of both cases are very similar but the use of the SEM produces a thicker boundary layer leading to an slightly earlier separation. This can be seen at $x/h = -1.125$ which is a location just after separation. The results obtained at the wake follow a similar trend. In figure 8 the velocity profiles are plotted and in figure 9 the corresponding u_{rms} levels. Here again the velocity profiles are close to the

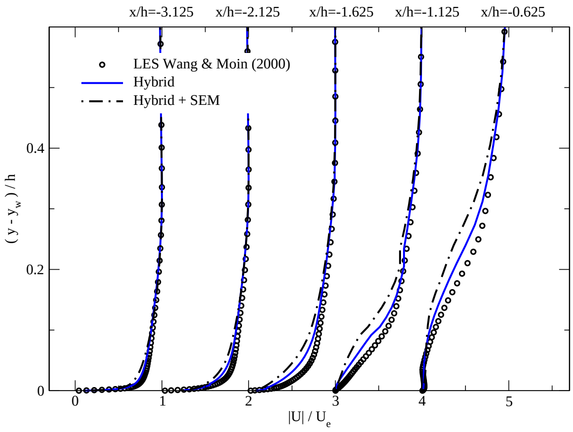

Figure 6: Velocity profiles on the upper surface of the trailing edge.

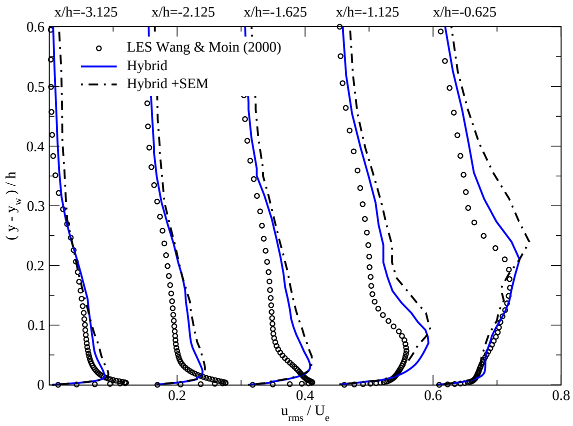

Figure 7: u_{rms} profiles on the upper surface of the trailing edge.

reference LES, but the u_{rms} values are higher when using the SEM, due to the larger separation bubble. The fact that the SEM produces large high energy containing eddies can be attributed to the approximations used for k and ε which are used to calculate an isotropic length scale.

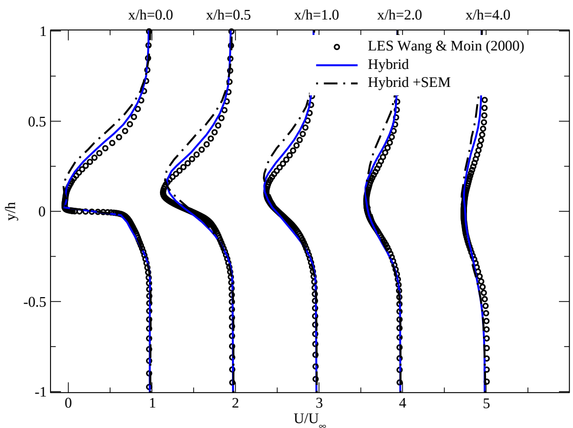

Figure 8: Velocity profiles on the wake of the trailing edge.

To better visualise the effect of the SEM at the inlet, the iso-surfaces of Q are presented in figures 10 and 11 with values of 0.18 (dark) and -0.18 (light). It can be seen that the structures predicted using the SEM are larger than the ones using the precursor simulation. Larger and more energetic eddies lead to a thicker boundary layer and a larger

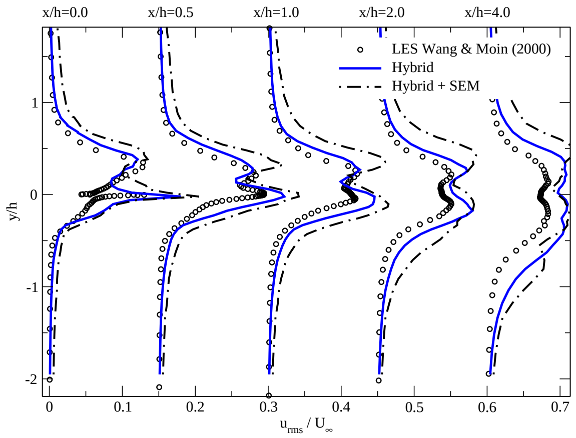

Figure 9: u_{rms} profiles on the wake of the trailing edge.

Figure 10: Q iso-surfaces, precursor simulation at the inlet.

Figure 11: Q iso-surfaces, SEM at the inlet.

recirculation zone which creates greater shedding.

CONCLUSIONS

A new hybrid method for the resolution of instantaneous turbulent field has been presented based on splitting the contributions from the averaged and fluctuating velocity fields. The method performs well in channel flows where the boundary layer is attached and on the separated flow over a trailing edge.The hybrid method is capable of sustaining fluctuating behaviour only limited by the size of the cells. Although the mesh is too coarse to be able to reproduce the small structures, the model successfully includes the near wall effect on mean strain via the mean velocity field, allowing a separation of dissipative effects. This makes the model predict separation and mean quantities reasonably well.

The use of the synthetic eddy method produces a instantaneous inlet field which yields predictions similar to the precursor calculation method. The SEM proved to be a low-resource alternative for the inlet conditions with only mean average quantities as input. Although the synthetic turbulence does not match exactly the precursor characteristics, is close enough to be enable the hybrid model to predict separation only slightly earlier that with the precursor simulation, thus saving a considerable amount of CPU time.

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