ASSESSMENT OF LES AND RANS PREDICTIONS OF IMPINGING FLOWS

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ABSTRACT

RANS modelling and LES of a plane air jet impinging orthogonally on a flat surface is described. Comparisons of predictions are made with the data of Yoshida et al. (1990), with good agreement found in both the free and wall jet regions. Some under-prediction, particularly by the RANS approach, of turbulence quantities is apparent, and overall the LES is in closer agreement with data, particularly in zones where large turbulent structures dominate. Differences in LES and RANS for the flow considered are not large, however, and this may be attributed to the effort already expended in the development of second-moment turbulence closures with wall reflection effects for impinging flow application. The results presented go some way towards validating LES for application to high Reynolds number impinging flows, although uncertainties in the data set used for comparison purposes demonstrate a requirement for further experimental work.

INTRODUCTION

The impingement of a jet on a flat surface is a flow configuration of interest in many engineering applications. Because of the high heat transfer rates that occur, it is used extensively in process engineering applications that involve cooling, heating and drying operations. It is also encountered in many mass transfer applications, including paint spraying and cavitation drilling, and is a generic flow used in assessments of the consequences of accidental releases of flammable and toxic materials from chemical and process plant. Impinging flows are also used extensively in the nuclear industry to re-suspend particles that have formed beds within storage equipment or ponds prior to the transport and processing of waste as a liquid-solid particle slurry.

Reynolds-averaged turbulence modelling remains the principal approach for predicting turbulence for engineering application, and the impinging jet has been the subject of considerable interest in recent years because of its practical relevance and since it represents an important test case for the development and validation of Reynolds-averaged Navier–Stokes (RANS) approaches. Detailed experimental data, e.g. (Cooper et al., 1993), has been used to improve understanding and permit the formulation and evaluation of the turbulence closures used within these models. In particular, further development of second-moment (Craft et al., 1993; Dianat et al., 1996a) and non-linear eddy viscosity (Craft et al., 1995) turbulence closures has been pursued to allow the more accurate prediction of these flows, and work in these areas continues.

Direct numerical simulation (DNS) and large eddy simulation (LES) of impinging jets has also been performed to elucidate the detailed turbulence structure of these complex flows, although the simulations are generally limited to low or moderate Reynolds numbers. Studies have focussed mainly on impinging plane jets (e.g. Voke and Gao, 1998; Tsubokura et al., 2003), although round impinging jets have also been considered (e.g. Satake and Kunugi, 1998; Tsubokura et al., 2003). These studies have improved our understanding of these flows, with differences between the turbulence structure of round and plane impinging jets identified as a consequence.

DNS and LES are also being used to improve the formulation and predictive accuracy of turbulence models embodied within RANS codes. Despite this, however, there are few studies that compare the accuracy of simulation and modelling techniques for particular impinging flows, nor indeed validate the predictions of simulations, be they DNS or LES, against data at Reynolds numbers representative of practical flows. The present work therefore considers the modelling and simulation of the plane turbulent impinging air flow, Re = 10,000, examined experimentally by Yoshida et al. (1990), with the aim being to assess the ability of both RANS and LES to accurately predict an impinging flow with a practically relevant Reynolds number.

This work has also been performed as a forerunner to using LES to aid the formulation and validation of RANS approaches for multi-phase impinging flows. In the short term, and in the absence of experimental data on such flows, this approach remains one of the few ways forward for the further development of the RANS models of multi-phase flows used almost exclusively in industry. Impinging two-phase jets are of importance in a wide variety of industries handling solid particles, and there is a requirement for techniques capable of predicting the complex flows encountered in practice. The validation of LES, and the sub-grid scale models embodied within it, for predicting single-phase impinging flows therefore represents a first step in this process, with the work of Yoshida et al. (1990) having the added benefit that, to the authors' knowledge, it remains the only comprehensive experimental study of both impinging single-phase and two-phase, particle-laden flows. As noted, however, the focus of the work reported herein remains the single-phase case.

RANS MODELLING

Predictions were based on solutions of the Reynolds-averaged Navier-Stokes equations, assuming an incompressible Newtonian fluid with constant properties.

For solution, the equations were written in a form appropriate to two-dimensional, planar flows. This set of equations is only closed when the Reynolds stress is approximated through the use of a turbulence model.

The Reynolds stress was obtained directly from solutions of modelled partial differential transport equations (Jones and Musonge, 1988). In the model used, the redistributive fluctuating pressure term is modelled as a general linear function of the Reynolds stress tensor, and it is assumed that both the return and the mean strain (or rapid) contribution to the velocity-pressure gradient correlation, normally modelled separately, are directly influenced by mean strain.

Wall reflection effects were incorporated in the second-moment closure of Jones and Musonge (1988) through a correction (Dianat et al., 1996a; 1996b) to the standard redistributive fluctuating pressure term which is included to allow for the influence of pressure reflections from the surface in distorting the fluctuating pressure field away from the wall. The expression used is linear in the Reynolds stress, and hence consistent with the uniqueness arguments invoked by Jones and Musonge (1988) in constructing the linear form of the redistributive fluctuating pressure term. Its basic form is redistributive and involves terms associated with the mean rate of strain, which have been found by Brasseur and Lee (1987) to be of greater significance than terms involving fluctuating velocities alone. The latter work also demonstrated that the return part of the pressure-rate of strain term is associated with much finer scale motions than the rapid component, and, hence, that this component might be expected to be more affected by the presence of any rigid boundary.

These equations were solved in conjunction with a turbulence energy dissipation rate transport equation, specified as in Jones and Musonge (1988). Model constants were taken as standard from Dianat et al. (1996a; 1996b).

The transport equations were solved using a modified version of a computer program described in Fairweather et al. (1988). The numerical solution method used a Cartesian grid and a staggered velocity storage arrangement to prevent uncoupling between the velocity and pressure fields. A linearised, implicit, conservative difference scheme was used, with convection terms approximated by the second-order accurate, and bounded, TVD scheme proposed by Van Leer (1974). Central differencing was used for all other terms, and the resulting quasi-linear algebraic equations were solved using a line Gauss-Seidel method, with solution of the velocity and pressure fields being achieved by a pressure correction method.

The experimental arrangement employed in obtaining the data used for comparison purposes was represented for the computations by a jet issuing vertically downwards from a slot, with the jet impinging orthogonally on a horizontal flat surface. The domain was treated as two-dimensional, since three-dimensional computations of the entire flow revealed no appreciable differences. The boundary conditions applied in the computations assumed symmetry along the jet centre-line. At the solid surface, no-slip conditions were employed, with finite-volume solutions patched onto fully turbulent, local equilibrium wall law profiles, and with the matching point chosen to be at a fixed distance from the wall for all finite-volume grids used. The upper surface of the computational domain was represented by a wall, in line with the experimental configuration used, while the remaining lateral boundary was treated as a constant pressure surface. In performing the calculations, the sensitivity of computed solutions to the positioning of the constant pressure boundary was investigated, and in the results presented below this surface was located at a position which had a negligible influence on the flow. In the absence of appropriate experimental data, initial conditions for the jet were obtained from a separate computation of a developing flow in a slot which used a parabolic marching procedure (Spalding, 1977) that was continued downstream until fully developed conditions were reached. This computation was based on the same equation set described above and used as the basis of the elliptic flow calculations.

Numerical solutions were obtained using expanding finite-volume meshes of up to 160×150 nodes in the horizontal and vertical directions respectively, with the mesh expansion ratio being less than 1.05 in the regions of interest. Results obtained with this grid, and with a mesh containing half this number of nodes, demonstrated that the computations were free of numerical error.

LARGE EDDY SIMULATION

In LES only the largest and most energetic scales of motions are directly computed, whilst the small scales of motions are modelled. Any given function is therefore decomposed using a suitably localised filter function, such that filtered values only retain the variability of the original function over length scales comparable to or larger than that of the filter width. Although several choices are possible, the present work used a top hat filter as this fits naturally into a finite-volume formulation. This decomposition is then applied to the Navier–Stokes equations under the hypotheses that filtering and differentiation in space commute. This process gives rise to terms which represents the effect of the sub-grid scale (SGS) motion on the resolved motion. The latter term is known as the SGS stress, and this must be modelled before the filtered equations can be solved.

The model for the SGS stress used in the present work was the dynamic model of Germano et al. (1991), implemented using the approximate localization procedure of Piomelli and Liu (1995) together with the modification proposed by di Mare and Jones (2003). This model represents the SGS stresses as the product of a SGS viscosity and the resolved part of the strain tensor, and is based on the possibility of allowing different values of the Smagorinsky constant at different filter levels. In this formulation the model parameter used is numerically well behaved, and the method is well conditioned and avoids the spiky and irregular behaviour exhibited by some implementations of the dynamic model. In the present work, test-filtering was performed in all space directions, with no averaging of the computed model parameter field.

Computations were performed using the computer program BOFFIN (Jones, 1991). The code implements an implicit finite-volume incompressible flow solver using a co-located variable storage arrangement. Because of this arrangement, fourth-order pressure smoothing, based on the method proposed by Rhie and Chow (1983), is applied to prevent spurious oscillations in the pressure field. Time advancement is performed via an implicit Gear method for all transport terms, and the overall procedure is second-order accurate in both space and time. The time step is chosen by requiring that the maximum Courant number lies between 0.1 and 0.3, with this requirement enforced for reasons of accuracy (Choi and Moin, 1994). The code is parallel and uses the message passing interface MPI-1.2.

The experimental arrangement was again represented for the computations by a jet issuing vertically downwards from a slot, with the jet impinging orthogonally on a horizontal flat surface. The previous finding that the flow of Yoshida et al. (1990) is effectively two-dimensional was used to reduce the size of the computational domain. In line with the findings of Beaubert and Viazzo (2003), therefore, the cross-stream extent of the domain was reduced from a width of 8 to 2 slot diameters, with periodic boundary conditions employed with the reduced domain in the span-wise direction. The fixed pressure boundaries were represented as convective outflow boundaries at ±10 diameters from the symmetry axis, again in line with findings of the RANS computations, whilst at solid surfaces the no-slip condition was applied. Inlet conditions were generated using a separate inflow turbulence generator based on digital filters (Klein et al., 2003). This technique generates turbulence structures, correlated in time and space, with specified turbulence length and time scales, and was applied together with the time-averaged inlet profiles used as the basis of the RANS calculations.

The numerical grid employed had dimensions of 270×140×88 nodes in the horizontal, vertical and cross-stream directions respectively, which compares with the 160×150 and 80×75 meshes (in the horizontal and vertical directions) used in RANS calculations which assumed symmetry along the jet centre-line, with the latter grid having been found to be free of numerical error. The mesh expansion ratio was less than 1.05 in the horizontal and vertical directions, with a uniform spacing being used in the cross-stream direction.

Time-averaged flow field variables were computed from running averages during the computations, and were accumulated once the initial flow field had been removed from the computational domain and the flow was fully established. Statistics were then collected every time step until time-averaged values converged.

RESULTS AND DISCUSSION

The flow field of an impinging planar jet can be divided into three zones: the free zone prior to impingement; the impingement region; and the wall jet region. Depending on the nozzle-to-plate spacing, the free region can also consist of one or more of the following zones: a potential core; a transition zone; and a fully developed zone, where the flow attains self-similar behaviour. In the impingement region a rapid decrease in the axial velocity and a corresponding increase in the static pressure occurs, with this region containing high levels of turbulence. The motion in the vicinity of the stagnation line comprises a nearly irrotational normal straining, while that nearer the edge of the impinging flow combines strong rotationality and streamline curvature. In the absence of a stabilizing co-flow, jet flapping can occur (Thomas and Goldschmidt, 1986) which alters the position of the stagnation line over time. The wall jet is a thin shear flow, with the maximum shear stress occurring outside the wall region, and with turbulence length scales near the wall strongly affected by those of the main flow turbulence.

Figures 1 to 4 compare time-averaged data and predictions of the RANS and LES approaches in the free region of the flow approaching the plate, from one diameter (d) downstream of the nozzle to the same distance from the impingement surface, where x is the axial distance and y the lateral distance. The figures show, respectively, the normal-to-wall mean (U) and fluctuating velocities (u'), the parallel-to-wall fluctuating velocity (v'), and the shear stress. These, and subsequent figures, also show cross-referenced data points, i.e. Yoshida et al. (1990) show the same data in different plots, and in Figs. 1-3 open circles represent data obtained from the equivalent figure in Yoshida et al. (1990)], whilst solid circles are used for data obtained from other plots of the same parameter.

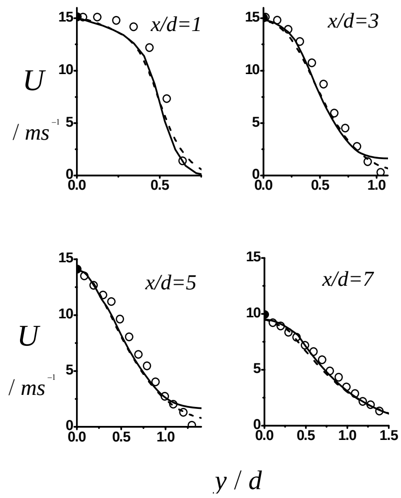

Fig. 1: Normal-to-wall mean velocities at various distances from the surface (○ data, ● cross-referenced data, —— RANS, ------- LES).

As noted, initial condition data are not available for this flow, and in its absence RANS computations of a fully developed slot flow were employed as the basis of both the RANS computations and LES. This approach cannot be guaranteed to reproduce the actual conditions employed in the experiments, and certainly in terms of the results of Figs. 1 to 4 differences between predicted and observed values can be attributed to this disparity. At one diameter downstream of the nozzle, x/d = 1, therefore, although the predicted peak mean velocities are in agreement with data, and the spreading rate of the flow has been predicted with reasonable accuracy, predictions under-estimate data over the majority of the width of the flow. Similarly, peak fluctuating velocities in the axial and transverse directions are predicted accurately, although there is an over-estimation of values on the centre-line and inner regions of the flow. In contrast, shear stresses are predicted with reasonable accuracy across the width of the flow, although the peak value is under-estimated by both predictive approaches. Overall, RANS and LES results are in close agreement with each other at this measurement location.

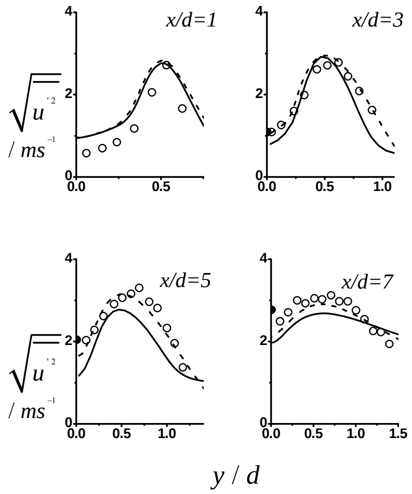

Fig. 2: Normal-to-wall fluctuating velocities at various distances from the surface (○ data, ● cross-referenced data, —— RANS, ------- LES).

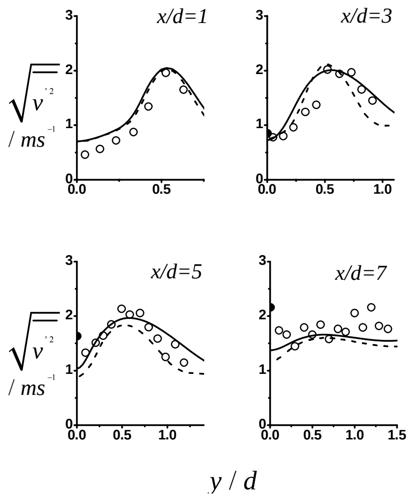

Fig. 3: Parallel-to-wall fluctuating velocities at various distances from the surface (○ data, ● cross-referenced data, —— RANS, ------- LES).

As the flow develops downstream, both predictive techniques come more in line with data. In Fig. 1, therefore, both RANS and LES predictions of the mean velocity are in good agreement with data and each other up until the final measurement station at *x/d* = 7. Some small differences are noted at the edge of the flow, with LES results being closer to observations at these locations. In terms of the normal-to-wall fluctuating velocities, shown in Fig. 2, by *x/d* = 3 the LES results are in good agreement with data, and this agreement persists as the solid surface is approached. Some under-prediction of data is evident at the last two measurement locations, with a slight shift in the peak stress location observed at *x/d* = 5, although at all distances from the nozzle the predictions are in good qualitative agreement with the measurements. A degree of scatter in the data is also evident at *x/d* = 5 and 7, with the cross-referenced data points indicating some uncertainties in the data set on the centre-line. In contrast, RANS predictions, although capturing the peak value at *x/d* = 3, subsequently under-estimate maximum normal stresses at all locations, with some over-prediction of data also evident towards the edge of the flow as the surface is approached. A shift in the peak value again occurs at *x/d* = 5, in line with LES results. Parallel-to-wall fluctuating velocity data (Fig. 3) show more scatter than equivalent normal-to-wall values, with cross-referenced data again bringing the reliability of the data set in to question. Overall, predictions of the *y*-directed normal stress are less satisfactory than those for the equivalent *x*-directed stress, with the LES failing to capture peak values, although providing reasonable estimates of the width of the shear layer. RANS predictions fair reasonably well in the estimation of peak values, but over-predict data towards the edge of the shear layer at *x/d* = 5. Fig. 4 gives values of the shear stress as the surface is approached. Overall, both the RANS and LES results are in reasonable qualitative agreement with data at all locations, although the LES is clearly superior. However, both approaches fail to capture peak shear stress values, and a shift in the location of the maximum shear stress, relative to the data, is apparent at *x/d* = 7. Again, significant scatter is evident in the data, although cross-referenced data are not available for the shear stress.

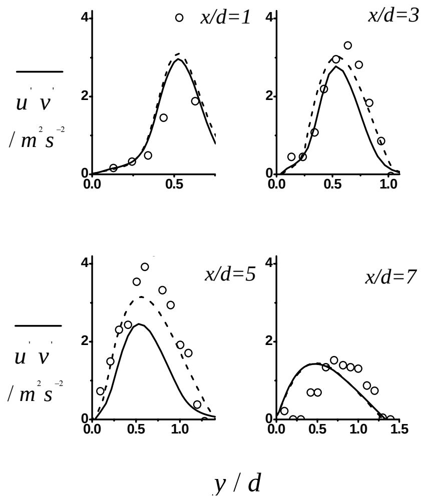

Fig. 4: Shear stress at various distances from the surface (○ data, —— RANS, ------- LES).

The under-prediction by both approaches of peak shear stresses warrants further comment, particularly given the reasonable agreement found between measured and predicted fluctuating velocities. The shear stress data given in Yoshida et al. (1990) exhibit a significant increase around *x/d* = 5, with the peak value dropping rapidly at the final measurement station. This increase does not seem plausible since the influence of the solid surface is not felt at such distances from the plate. Given that the data obtained at the first three measurement locations, between *x/d* = 1 and 5, correspond to the free regions of the flow, the data of Yoshida et al. (1990) were compared with the free slot jet data of Quinn and Millitzer (1988). This reveals that the data of Yoshida et al. (1990) show reasonable qualitative and quantitative agreement, in non-dimensional terms, with that of Quinn and Millitzer (1988) for both the *x*- and *y*-directed normal stresses, although normal-to-wall fluctuating velocities obtained by Yoshida et al. (1990) are slightly higher than those of Quinn and Millitzer (1988). In contrast, the shear stress data of the former authors are significantly higher than those obtained by Quinn and Millitzer (1988), and demonstrate an irregularity that is not reflected in the data of the latter authors.

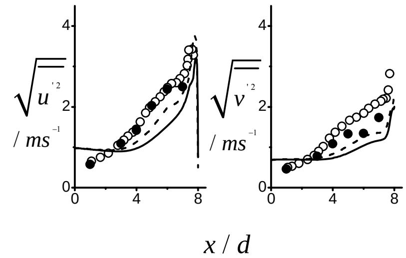

Fig. 5: Fluctuating velocities along the stagnation line (○ data, ● cross-referenced data, —— RANS, ------- LES).

Although some influence of the solid surface is evident in the results at *x/d* = 7 in Figs. 1 to 4, Fig. 5 more clearly shows the effect of the impingement plate on the fluctuating velocities in the main body of the flow by plotting results along the stagnation line approaching the plate. Again, cross-referenced centre-line data, extrapolated from that of Figs. 2 and 3, are included in this figure, together with the raw data from the equivalent plots in Yoshida et al. (1990).

Mean axial velocity along the stagnation line (not shown) is predicted well by both approaches, with their results being indistinguishable. Normal- and parallel-to-wall fluctuating velocities conform less well with data. Close to the nozzle, both the axial and transverse normal stresses are over-predicted by both methods, with this again being most likely associated with differences between the actual and simulated inlet profiles. In terms of the axial normal stress, there is a considerable under-estimation of data from *x/d* = 3 onwards, with both the RANS and LES approaches failing to predict the rapid rise in this stress. However, the data are seen to increase almost linearly from *x/d* = 1 which is again physically unlikely given the early break-down of the potential core that this implies. Closer to the impingement surface good agreement is found with data, with the LES results clearly superior. Peak values close to the plate are slightly over-predicted by the LES, and this may to some extent be due to a lack of numerical grid resolution in this region, and the wall function prescription employed. Resuts for the *y*-directed normal stress largely lead to the same conclusions, although a clear under-estimation of peak values close to the surface is evident in this case. The under-estimations of the normal stress data shown in Fig. 5 were not, however, as apparent in the comparisons of Figs. 2 and 3, and accordingly cross-referenced data on the centre-line of the flow, extrapolated from the latter figures, are included in Fig. 5. These data again demonstrate uncertainties in the measurements and, in terms of the parallel-to-wall fluctuating velocities at least, indicate closer agreement with the predictions. Overall, however, and despite any questions relating to the reliability of the data, the LES results are clearly superior to equivalent RANS predictions.

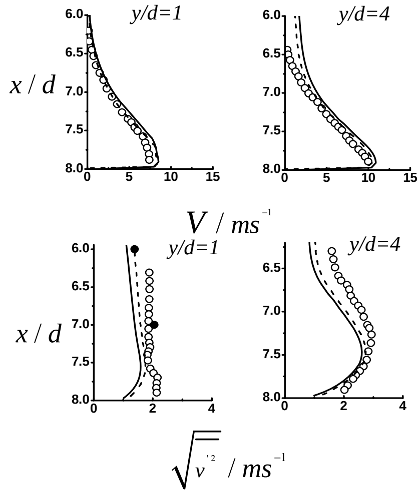

Fig. 6: Parallel-to-wall mean and fluctuating velocities in the wall jet region (○ data, ● cross-referenced data, —— RANS, ------- LES).

Turning to the wall jet region, Fig. 6 shows parallel-to-wall mean (*V*) and fluctuating velocities at two measurement stations moving away from the flow centre-line. Mean velocity predictions show good agreement with data, although some small differences are apparent close to the wall, most likely due to the wall functions used within both predictive techniques, although these are minor. Additionally, in the outer regions of the wall jet at *y/d* = 4, an over-prediction by both methods of the mean velocity is apparent. This may, to some extent, be due to a lack of grid resolution in the LES in this region, although RANS predictions were free of numerical error. More likely is an under-prediction of turbulence levels, as exemplified by the parallel-to-wall fluctuating velocity results. Although predictions of both the RANS and LES methods are largely qualitatively correct, therefore, they do underestimate data, particularly in the outer regions of the flow where the mean velocity is under-predicted. Deviations with observations are again apparent close to the wall, where the wall functions used may again be a contributing factor, although measurement of fluctuating velocities is difficult at such locations. A significant under-estimation of data also occurs at *y/d* = 1, although the cross-referenced data indicate closer agreement between predictions and observations at one location, and clearly the mean velocity profile at this position is well predicted, again calling the reliability of the data in to question. In line with previous findings, the LES produces results in closest accord with data.

CONCLUSIONS

The application of RANS and LES to the prediction of an impinging, plane air jet has been reported, and comparisons with the data of Yoshida et al. (1990) demonstrate good agreement with observations. Despite uncertainties in the experimental inlet conditions, predictions of both methods in the free and wall regions, and along the stagnation line, are in reasonable accord with data. Some under-prediction by RANS of turbulence quantities in both regions is apparent, and overall LES is in closer agreement with data. The differences between LES and RANS for the flow considered are not large, however, and this is likely related to the effort already expended in the development of second-moment turbulence closures for impinging flows. Examination of the data of Yoshida et al. (1990), and comparison with that of other authors, does, however, cast doubts on the reliability of their data.

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