# STUDYING STREAMWISE ROTATION VARIATIONS OF A TURBULENT CHANNEL FLOW USING DIRECT NUMERICAL SIMULATION AND COHERENT VORTEX EXTRACTION 

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## ABSTRACT

In this contribution a study of a turbulent channel flow rotating about the streamwise direction is presented by using direct numerical simulation (DNS). The theory giving a theoretical basis for the mean flow is based on the investigations of Oberlack et al. (2006) employing symmetry group theory. It was found both in DNS and in the theory that a cross flow in the spanwise direction is induced. A series of DNS has been conducted at Reynolds number $R e_{\tau}=180$ for both the non-rotating case and different rotation numbers up to $R o=100$ to examine the effects of rotation. A further purpose of this paper is to investigate the effects of the structures of the vorticity fluctuations. Hence, the coherent vortex extraction (CVE) algorithm based on the orthogonal wavelet decomposition of vorticity is applied to six selected vorticity fluctuation fields.

## INTRODUCTION

Rotating turbulent flows play an increasing role in many engineering applications such as in gas turbine blade passages, pumps and rotating heat exchangers. In these cases secondary flows are induced caused by centrifugal or Coriolis forces.

In the past intensive research has been dedicated experimentally as well as numerically to turbulent channel flows rotating about the spanwise direction (e.g. Johnston et al., 1972).

However, rotation about the streamwise direction has been studied only recently. First investigations were done by Oberlack et al. (2006) employing symmetry group theory. The theoretical results show that the flow presents several common features with the classical rotating channel flow rotating about the spanwise direction (Johnston et al., 1972), but also exhibits different characteristics. The induction of


Figure 1: Sketch of the flow geometry of a turbulent channel flow rotating about the mean flow direction
a mean velocity in $x_{3}$-direction (Oberlack, 2001) is the most striking difference compared to the classical case. This cross flow can be deduced by investigating the mean momentum equation and the Reynolds stress transport equation.

The aim of the paper is to investigate the effect of rotation on both the mean velocity as well as the structure of the vorticity fluctuations. In the first part the results of a DNS at Reynolds numbers $R e_{\tau}=180$ first for the nonrotating case and second for different rotation rates up to Ro $=100$ are presented and the influence due to the different rotation rates are analyzed. Further a wavelet analysis by using the coherent vortex extraction (CVE) method has been performed for same selected computations to identify and extract coherent vorticities.

DIRECT NUMERICAL SIMULATION (DNS)

## Numerical Method and Computations

The numerical technique which was chosen is a standard spectral method with Fourier decomposition in streamwise

Table 1: Computations at Reynolds number $R e_{\tau}=180$

| Ro | Box | Grid | Grid points |
| :---: | :---: | :---: | :---: |
| 0 | $4 \pi \times 2 \times 2 \pi$ | $128 \times 129 \times 128$ | 2.1 Mio |
| 1 | $4 \pi \times 2 \times 2 \pi$ | $128 \times 129 \times 128$ | 2.1 Mio |
| 3.2 | $4 \pi \times 2 \times 2 \pi$ | $128 \times 129 \times 128$ | 2.1 Mio |
| 4 | $4 \pi \times 2 \times 2 \pi$ | $128 \times 129 \times 128$ | 2.1 Mio |
| 5.2 | $4 \pi \times 2 \times 2 \pi$ | $128 \times 129 \times 128$ | 2.1 Mio |
| 7 | $8 \pi \times 2 \times 4 \pi$ | $256 \times 129 \times 128$ | 4.2 Mio |
| 10 | $8 \pi \times 2 \times 4 \pi$ | $256 \times 129 \times 128$ | 4.2 Mio |
| 14 | $8 \pi \times 2 \times 4 \pi$ | $256 \times 129 \times 128$ | 4.2 Mio |
| 20 | $8 \pi \times 2 \times 4 \pi$ | $256 \times 129 \times 128$ | 4.2 Mio |
| 60 | $24 \pi \times 2 \times 4 \pi$ | $640 \times 385 \times 256$ | 63.1 Mio |
| 100 | $32 \pi \times 2 \times 4 \pi$ | $640 \times 385 \times 256$ | 63.1 Mio |

and spanwise direction as well as Chebyshev decomposition in wall-normal direction. The numerical code for channel flow was developed at KTH/Stockholm (see Lundbladh et al., 1992). Additional features such as the streamwise rotation and statistics were added during the project.

All flow quantities are non-dimensionalized by $h / 2$ and $u_{C L}$, where $h$ is the channel half-height and $u_{C L}$ is the centerline (CL) velocity of the flow field. The boundary conditions are no-slip at $x_{2}= \pm 1$ and periodic in $x_{1}$ - and $x_{3}$-direction. For all computations the pressure-gradient is kept constant $\frac{\partial p}{\partial x_{1}}=$ const. The calculations start from a laminar velocity channel flow profil.

The time integration is performed using a third order Runge-Kutta scheme. The transformation between physical and spectral space is done by Fast Fourier Transform (FFT). Further details on the numerical scheme may be obtained from Lundbladh et al. (1992).

After the simulations all flow quantities were normalized on the friction velocity $u_{\tau}$

$$
\begin{equation*}
u_{\tau}=\sqrt{\left.\nu \frac{\partial \bar{u}_{1}}{\partial x_{2}}\right|_{W a l l}} \tag{1}
\end{equation*}
$$

Hence, the Reynolds number is defined as follows:

$$
\begin{equation*}
R e_{\tau}=\frac{h u_{\tau}}{2 \nu} \tag{2}
\end{equation*}
$$

and the rotation number as

$$
\begin{equation*}
R o=\frac{\Omega h}{u_{\tau}} \tag{3}
\end{equation*}
$$

Table 1 gives an overview on the different computations, the flow domain and corresponding grid points. All presented results for the non-rotating case $R o=0$ are in good agreement with the data from Kim et al. (1987).

## Main Profiles

Figure 2 shows the streamwise mean velocity profiles at different rotation rates. In general, the profiles decrease monotonically with increasing rotation number.

In particular, a significant decay of the maximum velocity i.e. $\bar{u}_{1}^{+}=16.94$ down to $\bar{u}_{1}^{+}=13.98$ between $R o=5.2$ and 10 is observed, which at the same time means a significant decrease in mass flux or increase in drag. A more detailed view of this effect can be taken from fig. 3, where the development of the mass flux is plotted over the rotation rate. The significant decay is also visible between $R o=5.2$ and 10 .

For the highest rotation rates $R o=60$ and $R o=100$ the decrease of the mean velocity profiles $\bar{u}_{1}$ is only marginal, it seems that a minium is approached. The mass flux (fig. 3) consequently shows the same tendency, thus it seems the flow is still turbulent. In the case that Ro tends to infinity, we expect a relaminarization. Fig. 4 shows the mean velocity profiles $\bar{u}_{1}$ of two rough computation at $R o=500$ and1000 compared to $R o=100$, which demonstrates the relaminarization of the flow.


Figure 2: Streamwise mean velocity profiles $\left(\bar{u}_{1}\right)$ at different rotation rates.


Figure 3: Development of $\bar{u}_{\text {bulk }}$ for increasing rotation rates.


Figure 4: Streamwise mean velocity profile $\left(\bar{u}_{1}\right)$ at $R o=$ 100, 500 and 1000 at $R e_{\tau}=180$.

As predicted from the Lie Group theory by Oberlack et al. (2006) the cross flow can be observed in all DNS. A parallel experimental study (Recktenwald et al., 2006) verifies also the existence of the cross flow. Fig. 5 displays the spanwise mean velocity profiles at different rotation rates of the numerical computations.


Figure 5: Spanwise mean velocity profiles $\left(\bar{u}_{3}\right)$ at different rotation rates.

The profiles show an $S$-shaped skew-symmetric cross flow with a flow reversal in the core region of the channel with
three zero-crossings. The larger velocities near the wall can be explained by the Coriolis forces (Alkishriwi et al., 2006). The flow reversal, if it occurs at all, is due to another mechanism, which is probably driven by larger turbulent structures, which are encountered at higher rotation rates.

Further, it is important to notice, that with increasing $R o$ the maximum value of the cross flow reaches a maximum at about $R o=10$ and then decreases for higher Ro. In figure 6 the development of $\bar{u}_{3_{\max }}$ as a function of the rotation rate is shown.


Figure 6: Development of $\bar{u}_{3_{\max }}$ as a function of the rotation rate.


Figure 7: Distance from the wall $y^{+}$of $\bar{u}_{3_{\max }}$ as a function of the rotation rate.

Additionally, with an increasing rotation rate, the maxima get closer to the wall and at the same time the $\bar{u}_{3}$ boundary layer decreases. A more detailed perspective is given in figure 7 , where the distance from the wall $y+$ of $\bar{u}_{3_{\max }}$ is plotted as function of the rotation rate. The curve decreases rapidly up to $R o=20$ and goes on with a smaller gradient for higher rotation rates. That means that the boundary layer thickness approaches a minimum for $R o \rightarrow \infty$.

Since both profiles exhibit a significant change at about the same rotation number $R o=10$, a significant change in the flow dynamics at this value is conjectured. The above observations will be given the name rotation drag effect (RDE), due to increased drag leading to a reduced mass flux in $x_{1-}$ direction.

## Coherent Vortex Extraction (CVE) Algorithm

An orthogonal wavelet-based method to extract coherent vortices from 3D turbulent flow was proposed by Farge et al. (2001). Considered is the vorticity field $\boldsymbol{\omega}=\nabla \times \boldsymbol{u}$ at resolution $N=2^{3 J}$, where $N$ is the number of grid points and $J$ the corresponding number of octaves. Each component is developed into an orthogonal wavelet series, from the largest scale $l_{\max }=2^{0}$ to the smallest scale $l_{\min }=2^{1-J}$, using as 3D multi-resolution analysis (MRA) (Daubechies, 1992, Farge, 1992):

$$
\begin{align*}
& \boldsymbol{\omega}(\boldsymbol{x})=\tilde{\boldsymbol{\omega}}_{0,0,0,0} \phi_{0,0,0,0}(\boldsymbol{x})+ \\
& \sum_{j=0}^{J-1} \sum_{i_{x_{1}}=0}^{2^{j}-1} \sum_{i_{x_{2}}=0}^{2^{j}-1} \sum_{i_{x_{3}}=0}^{2^{j}-1} \sum_{\mu=1}^{7} \tilde{\boldsymbol{\omega}}_{j, i_{x_{1}}, i_{x_{2}}, i_{x_{3}}}^{\mu} \psi_{j, i_{x_{1}}, i_{x_{2}}, i_{x_{3}}}^{\mu}(\boldsymbol{x}) \tag{4}
\end{align*}
$$

with

$$
\psi_{j, i_{x_{1}}, i_{x_{2}}, i_{x_{3}}}^{\mu}(\boldsymbol{x})=\left\{\begin{array}{r}
\psi_{j, i_{x_{1}}}\left(x_{1}\right) \phi_{j, i_{x_{2}}}\left(x_{2}\right) \phi_{j, i_{x_{3}}}\left(x_{3}\right),  \tag{5}\\
\mu=1, \\
\phi_{j, i_{x_{1}}}\left(x_{1}\right) \psi_{j, i_{x_{2}}}\left(x_{2}\right) \phi_{j, i_{x_{3}}}\left(x_{3}\right), \\
\mu=2, \\
\phi_{j, i_{x_{1}}}\left(x_{1}\right) \phi_{j, i_{x_{2}}}\left(x_{2}\right) \psi_{j, i_{x_{3}}}\left(x_{3}\right), \\
\mu=3, \\
\psi_{j, i_{x_{1}}}\left(x_{1}\right) \phi_{j, i_{x_{2}}}\left(x_{2}\right) \psi_{j, i_{x_{3}}}\left(x_{3}\right), \\
\mu=4, \\
\psi_{j, i_{x_{1}}}\left(x_{1}\right) \psi_{j, i_{x_{2}}}\left(x_{2}\right) \phi_{j, i_{x_{3}}}\left(x_{3}\right), \\
\mu=5, \\
\phi_{j, i_{x_{1}}}\left(x_{1}\right) \psi_{j, i_{x_{2}}}\left(x_{2}\right) \psi_{j, i_{x_{3}}}\left(x_{3}\right), \\
\mu=6, \\
\psi_{j, i_{x_{1}}}\left(x_{1}\right) \psi_{j, i_{x_{2}}}\left(x_{2}\right) \psi_{j, i_{x_{3}}}\left(x_{3}\right), \\
\mu=7
\end{array}\right.
$$

and

$$
\begin{equation*}
\phi_{j, i_{x_{1}}, i_{x_{2}}, i_{x_{3}}}(\boldsymbol{x})=\phi_{j, i_{x_{1}}}\left(x_{1}\right) \phi_{j, i_{x_{2}}}\left(x_{2}\right) \phi_{j, i_{x_{3}}}\left(x_{3}\right) \tag{6}
\end{equation*}
$$

where $\phi_{j, i}$ and $\psi_{j, i}$ are the 1D scaling function and the corresponding wavelet, respectively. Due to orthogonality, the scaling coefficients are given by $\bar{\omega}_{0,0,0,0}=\left\langle\boldsymbol{\omega}, \phi_{0,0,0,0}\right\rangle$ and the wavelet coefficients by $\tilde{\boldsymbol{\omega}}_{j, i_{x_{1}}, i_{x_{2}}, i_{x_{3}}}^{\mu}=\left\langle\boldsymbol{\omega}, \psi_{j, i_{x_{1}}, i_{x_{2}}, i_{x_{3}}}^{\mu}\right\rangle$, where $\langle\cdot, \cdot\rangle$ denotes the $L^{2}$-inner product and $\mu$ corresponds to seven discrete directions in three dimensions.

The extraction algorithm can be summarized as follows:

1. Given $\boldsymbol{\omega}$, sampled on a grid $\left(x_{1_{i}}, x_{2_{i}}, x_{3_{i}}\right)$ for $i, j, k=$ $0, N-1$ and the total enstrophy $Z=\frac{1}{2} \int|\boldsymbol{\omega}|^{2} d x$.
2. Perform the 3D wavelet decomposition by applying the Fast Wavelet Transform (FWT) to each component of $\boldsymbol{\omega}$ to obtain the three components of $\tilde{\boldsymbol{\omega}}_{j, i_{x_{1}}, i_{x_{2}}, i_{x_{3}}}^{\mu}$ for $j=0, J-1, i_{x_{1}}, i_{x_{2}}, i_{x_{3}}=0,2^{J-1}$, and $\mu=1, \ldots, 7$.
3. Thresholding, where Z is the enstrophy and N the number of grid points,

$$
\begin{equation*}
T=\sqrt{\frac{4 Z}{3} \ln N} \tag{7}
\end{equation*}
$$

and split the coefficients $\tilde{\boldsymbol{\omega}}$ into

$$
\tilde{\boldsymbol{\omega}}_{c o h}=\left\{\begin{array}{lll}
\tilde{\boldsymbol{\omega}} & \text { for } & |\tilde{\boldsymbol{\omega}}| \geq T \\
0 & \text { for } & |\tilde{\boldsymbol{\omega}}|<T
\end{array}\right.
$$

and

$$
\tilde{\boldsymbol{\omega}}_{i n c}=\left\{\begin{array}{ccc}
\tilde{\boldsymbol{\omega}} & \text { for } & |\tilde{\boldsymbol{\omega}}|<T  \tag{8}\\
0 & \text { for } & |\tilde{\boldsymbol{\omega}}| \geq T
\end{array}\right.
$$

4. Perform the 3D wavelet reconstruction by applying the inverse FWT to compute $\boldsymbol{\omega}_{c o h}$ and $\boldsymbol{\omega}_{i n c}$ from $\tilde{\boldsymbol{\omega}}_{c o h}$ and $\tilde{\boldsymbol{\omega}}_{\text {inc }}$, where coh and inc corresponds to coherent and incoherent. For the decomposition one FWT is performed for each component of the vorticity vector $\boldsymbol{\omega}$. To reconstruct the coherent vorticity $\boldsymbol{\omega}_{c o h}$ in physical space one inverse FWT is required for each component. The incoherent vorticity vector is obtained by taking the difference between the total and the coherent vorticity in physical space. This yields in total six FWTs.

In this study orthogonal Coiflet 12 wavelets are used which have a filter length of twelve and four vanishing moments (Daubechies, 1992, Farge, 1992). The choice of the threshold (eq. 7) is based on Donohos theorems (Donoho et al ., 1993, Farge et al., 2001).

## Application to the present turbulent rotating channel flow

The CVE software package was applied to six data sets at Reynolds number $R e_{\tau}=180$ (see table 1) to study coherent vortices and their statistics. At first for the non-rotating case and further for the rotating cases at five different rotation rates $R o=1,3.2,5.2,10$ and 20. In this frame the respective instantenous 3 D vorticity fields were split in two parts namely the coherent and the incoherent component. The coherent part represents the coherent structures, while the incoherent part corresponds to incoherent structures, the so-called background flow.

Fig. 9 displays the results for an example at $R o=10$. Although the CVE was done for the whole channel for the visualizations in figure 9 a subzone is selected to have a better zoom of the vorticities. For the subzone a quarter of the streamwise (64-128), half of the spanwise (1-64) and all of the wall-normal grid points (1-128) were selected. In figure 8 a sketch of the channel to illustrate the subzone which is visualized in fig. 9 is shown.

Fig. 9 shows the streamwise isosurfaces of a) the total subzone, b) the coherent part and c) the incoherent part of an instantinous 3D vorticity field at $R o=10$ and Reynolds number $R e_{\tau}=180$. The streamwise component was chosen because it is the dominant part of $|\omega|$. The isosurfaces of the total subzone and the coherent part were plotted for the same values $\omega_{x_{1}}= \pm 1.9$ and the incoherent part for $\omega_{x_{1}}= \pm 0.28$. For all figures the same colormap is used (shown in fig. 8). Interesting enough we found that few strong wavelet coefficients (3.61\%) represent the coherent vortices and retain most of the enstrophy of the flow (fig. 9b) while the remaining weak coefficients correspond to the incoherent background flow (Fig. 9c).

In table 2 the rates of the coherent and incoherent components of the compression rate $C R$ and the enstrophy $Z$ are summarized for all rotation rates. The compression rate yields the number of coherent and respectively incoherent wavelet coefficients in percent of the particular component. Consequentially the compression rate of the total field must corresponds $100 \%$. Compared to the non-rotating case the compression rate for the coherent structures is decreasing to a minimum at $R o=5.2$. At $R o=10$ the compression rate appears to revers it increases suddenly up to $3.46 \%$ and keeps on increasing monotonically for higher rotation rates.
$\omega \dot{\circ} \dot{\circ} \dot{\sim} \times \dot{\sim}$


Figure 8: Sketch of the channel to illustrate the subzone which is visualized in fig. 9

 $96.39 \%$ of the wavelet coefficients. For all figures the colormap shown in fig. 8 is used.

Figure 10 shows the percentage of wavelet coefficients corresponding to the coherent flow and the percentage of the coherent enstrophy which are plotted versus the rotation number.

Concerning enstrophy the values are inversely proportional to the compression rate (see fig. 11). For small rotation rates the coherent part increases compared to the non-rotating case and at $R o=10$ the enstrophy decreases down to $93.70 \%$. This means that fewer coefficients retain more enstrophy, which might be interpreted as an increased coherency of the flow.

Table 2: Coherent and incoherent components of the compression rate $C R$ and enstrophy $Z$ at different rotation rates at $R e_{\tau}=180$.

|  | Compression rate |  | Enstrophy |  |
| :---: | :---: | :---: | :---: | :---: |
| $R o$ | coh[\%] | inc[\%] | coh[\%] | inc[\%] |
| 0 | 3.41 | 96.59 | 94.85 | 5.15 |
| 1 | 3.24 | 96.76 | 96.54 | 3.46 |
| 3.2 | 3.14 | 96.86 | 97.07 | 2.93 |
| 5.2 | 3.12 | 96.88 | 97.22 | 2.78 |
| 10 | 3.46 | 96.54 | 93.70 | 6.30 |
| 14 | 3.53 | 96.47 | 93.55 | 6.45 |
| 20 | 3.57 | 96.43 | 93.66 | 6.34 |



Figure 10: Compression rate $C R$ as a function of the rotation rate.


Figure 11: Coherent enstrophy $Z$ as a function of the rotation rate.

Important to note that the RDE was found at about the same rotation rate where the curves of the compression rate and enstrophy reverse. This confirms a change in the flow dynamics for a specific $R o$.

## CONCLUSIONS

The influence of different rotation rates of a rotating turbulent channel at Reynolds number $R e_{\tau}=180$ is examined. The results of DNS agree well with the results of the symmetry theory. Not only the predicted cross flow could be verified in the DNS at different rotation rates but also some new properties were detected.

In addition $R e_{\tau}=180$ a significant decay of the streamwise maximum velocity between $R o=5.2$ and 10 was noticed. At the same time the cross flow reaches a maximum at about $R o=10$ and then decreases for higher rotation rates Ro. These observations are called rotation drag effect (RDE). To summarize we have clear indication from several above given parameters that for a specific $R o$ a significant change in flow dynamics occurs. From the present DNS at $R e=180$ we find $R o \approx 5.2$.

Further the CVE was conducted for six selected data sets. We found that for all data sets and statistics a few strong wavelet coefficients represent the coherent vortices of the flow while the remaining weak coefficients correspond to the incoherent background flow. A similar reverse of the curves compared to DNS was found in both the compression rate and the enstrophy. This seems to confirm a change in the flow dynamics for a specific Ro.

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