ATTEMPTS TO MODEL DENSITY EFFECTS ON MIXING LAYER GROWTH RATE

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ABSTRACT

This paper investigates several new attempts to model the influence of the density ratio of the two streams on the mixing layer growth rate. The model recently proposed by Kreuzinger et al. is not sensitive to the relative orientations of the velocity and density gradient. A revised version has been proposed but is far from being satisfactory. It was concluded that baroclinic effect should be modelled, but not at the dissipation level. Other strategies were thus investigated. For that, the length scale equation, deduced from two-point correlations, was extended to compressible flows. Extra terms linked to density gradient were evidenced and modelled. Models available in the literature were also analyzed. If the high speed flow is lighter, which is the case of propulsion jets, density effects are small and can be captured by some models. But no model is able to correctly capture the density effect on the mixing layer growth rate whatever the density and velocity ratios.

INTRODUCTION

Mixing layers occur in a wide variety of flows. More often, the best mixing capabilities are looked for as e.g. in combustion chambers, to increase the efficiency and reduce the pollution, or in nozzle jets, to reduce the infrared signature. However, in some cases as in wall cooling by tangential blowing, the lowest possible mixing is wanted.

Mixing layers rapidly reach a self similar state in which the velocity, temperature, density, species mass fraction... profiles can be recast in a simple form

$$\frac{u-u_2}{u_1-u_2} = W\left(\frac{y}{\delta(x)}\right) \quad \frac{T-T_2}{T_1-T_2} = \Theta\left(\frac{y}{\delta(x)}\right) \quad \ldots \tag{1}$$

where subscripts $_1$ and $_2$ respectively refer to high- and low-speed streams, y is the distance along the mixing layer thickness and δ a characteristic mixing layer thickness.

Various mixing layer thicknesses δ can be considered, based upon Schlieren (optical thickness), the stagnation pressure profile or the velocity profile. The most classical ones are $\delta_{1\%}, \delta_{10\%}$ and the Stanford thickness δ_S respectively defined as the distance between the points where the non-dimensional velocity $W = \frac{u-u_2}{u_1-u_2}$ is equal to 1% (resp. 10% and $\sqrt{0.1} \approx 0.32$) and 99% (resp. 90% and $\sqrt{0.9} \approx 0.95$) and the vorticity thickness based upon the maximum velocity gradient as

$$\delta_\omega = (u_1-u_2)/\left(\frac{\partial u}{\partial y}\right)_{max} \tag{2}$$

As the velocity profile is hardly affected by density changes or by compressibility, these thicknesses nearly remain proportional. Therefore, the analysis presented below will be restricted to the vorticity thickness δ_ω.

The standard analysis (Brown and Roshko, 1974; Bogdanoff, 1983; Dimotakis, 1991) thus shows that the mixing layer growth rate depends upon three parameters:
- the velocity ratio of the two streams $r = \frac{u_2}{u_1}$,
- the density ratio of the two streams $s = \frac{\rho_2}{\rho_1}$,
- the convective Mach number M_c which characterizes the compressible character of the turbulent motion.

The standard analysis yields the following expression for the mixing layer spreading rate

$$\delta' = \frac{d\delta}{dx} = C_\delta\, f(M_c) \frac{(1-r)(1+\sqrt{s})}{2(1+\sqrt{s}r)} \tag{3}$$

which is well supported by experiments.

Most turbulence models correctly reproduce the variation of the mixing layer growth rate with the velocity ratio r. They can be educated to reproduce the effect of the convective Mach number M_c (see, e.g. Aupoix, 2004).

The effect of density ratio s is more difficult to capture. Guézengar (1996) pointed out that the $k-\varepsilon$ model gives the wrong sensitivity. A systematic study performed by Aupoix (2004) showed that simple models in which the length scale is prescribed (mixing length model, model with one equation for the turbulent kinetic energy) are able to reproduce the density ratio effect but that models in which the turbulence length scale is computed (Spalart and Allmaras, $k-\varepsilon$, $k-\varphi\ldots$) fail.

These conclusions hold for density gradients due as well to temperature variations as to differences in chemical composition. They also hold whatever the model constants (in an acceptable range), whatever the turbulent Prandtl or Lewis number and whatever the constitutive relation.

TEST PROCEDURE

Attempts to predict density ratio effects will be analyzed in the present paper. All model tests were performed with a code solving self-similarity equations developed by Bézard (2000). The self-similarity hypothesis leads to a simple one-dimensional problem for which grid converged results are easily achieved. Moreover, it allows to perform systematic tests, covering a wide range of velocity and density ratios.

All computations presented below were performed with Bézard (2000) $k-\varepsilon$ model, together with Catris' compressibility correction, to be discussed later. A drawback of Bézard's model is to be tuned to give a value of the mixing layer expansion rate coefficient $C_{\delta_\omega} \approx 0.135$ while experimental data suggest higher values in the range 0.16-0.18

(Brown and Roshko, 1974; Aupoix and Bézard, 2006). However, it has been checked that the conclusions hold when other models, giving higher expansion rates, are used. Consequently, on the figures, computations will be compared to the theoretical solution, given by equation (3), with $C_{\delta_\omega} = 0.135$.

DENSITY CORRECTIONS BASED UPON BOUNDARY LAYER SCALING

Huang et al. (1994) pointed out the inconsistency of turbulence models in the logarithmic region of compressible boundary layers in presence of strong density gradients. From boundary layer scalings, Catris and Aupoix (2000) proposed a general strategy to correct the length scale equation in order to retrieve this consistency. The basic idea is that extension of turbulence models to compressible flows should consider the turbulent kinetic energy per mass unit ρk and the turbulent length scale L as variables having the same properties in compressible flows as in incompressible flows. This led to a simple strategy to extend any transport equation model to compressible flows.

When this correction is applied to mixing layer flows, the wrong sensitivity to density gradients of e.g. $k-\varepsilon$ models disappears but the effect of the correction is far too small (see Aupoix (2004) or curve $C=0$ in figure 1). New density corrections are thus required, either alone or coupled with Catris' correction, to preserve the logarithmic region and give correct predictions of the mixing layer growth rate.

BAROCLINIC CORRECTIONS

Introduction

Turbulent motion is first of all a vortical motion. Therefore, a natural way to take into account the effect of density gradient upon a turbulent, vortical, motion is to advocate the baroclinic effect, i.e. the production of vorticity due to the combined action of density and pressure gradients as

$$\frac{D\underline{\omega}}{Dt} = \ldots + \frac{1}{\rho^2}\,\underline{\mathrm{grad}}\,\rho \wedge \underline{\mathrm{grad}}\,p + \ldots \tag{4}$$

In turbulence models, vorticity naturally appears if the turbulent kinetic dissipation rate is approximated as

$$\varepsilon = \nu\overline{\underline{\omega}' \cdot \underline{\omega}'} \tag{5}$$

Krishnamurty and Shyy (1997) decomposed the baroclinic term in the transport equation for the dissipation term into three contributions as

$$\epsilon_{pqi}\frac{\nu}{\overline{\rho}^2}\left(\frac{\partial\overline{\rho}}{\partial x_q}\overline{\omega''_p\frac{\partial p'}{\partial x_i}} + \frac{\partial\overline{p}}{\partial x_i}\overline{\omega''_p\frac{\partial\rho'}{\partial x_q}} + \overline{\omega''_p\frac{\partial\rho'}{\partial x_q}\frac{\partial p'}{\partial x_i}}\right) \tag{6}$$

For base flow computations, they deduced from an order of magnitude analysis that the leading term is the one associated with the mean pressure gradient.

Aupoix' model

This conclusion, and the associated model, does not hold in mixing layers which are isobaric. This led Aupoix (2004) to prefer the term involving the mean density gradient. He related the fluctuating pressure gradient to the velocity field through the momentum equation as

$$\frac{\partial p'}{\partial x_i} \approx \rho u''_l\frac{\partial\tilde{u}_i}{\partial x_l} \tag{7}$$

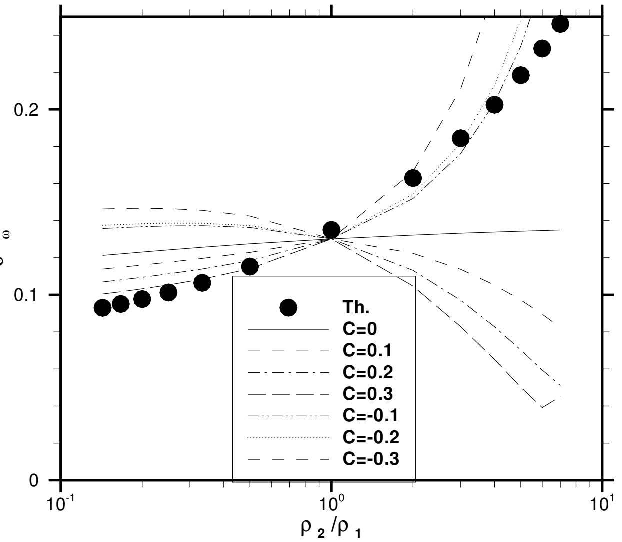

Figure 1: Mixing layer growth rate versus density ratio for $u_2 = 0$ – Kreuzinger et al. model

For two-dimensional flows, under the thin layer hypothesis, the correction in the dissipation equation reads

$$Ck^{3/2}\frac{\partial\rho}{\partial y}\frac{\partial u}{\partial y} \tag{8}$$

This correction improves the prediction of the mixing layer growth rate versus velocity and density ratios. Nevertheless, it is far from being satisfactory. Moreover, no general tensorial form could be found and the above expression alters the prediction of boundary layer flows.

Kreuzinger et al. model

Kreuzinger et al. (2006) analyzed DNS of temporally growing mixing layers and evidenced that the leading term is not as expected a term involving a mean quantity gradient, but the correlation of fluctuating density and pressure gradients. They related these gradients to the variance of the quantity and the Taylor microscale, and obtained a correction term in the dissipation equation of the form

$$C\|\underline{\mathrm{grad}}\,\rho\|\sqrt{k}\varepsilon \tag{9}$$

They recommended a constant $C = 2 \text{ x } 0.18 \text{ x } \sqrt{2} \approx 0.5$.

Figure 1 shows the growth rate versus density ratio when the low speed stream is at rest ($r=0$), i.e. when the mixing layer growth rate is maximum. On this figure, a semi-logarithmic scale is used for the density ratio, in order to give the same influence to the density ratios $s = \frac{\rho_2}{\rho_1}$ and $\frac{1}{s}$. Symbols correspond to the theoretical values, given by equation (3) and lines to computations.

Various values of the Kreuzinger et al. model constant C were tested. As expected from equation (9), the model is unable to distinguish whether the low speed stream or the high speed stream flow is the heaviest as it is only sensitive to the modulus of the mean velocity gradient. Positive values of the constant lead to an increase of the dissipation and hence a reduction of the turbulence and of the growth rate, whatever the density ratio. Conversely, a negative value of the constant leads to increased spreading rate. Plotted in linear coordinates, the curves are symmetric with respect to

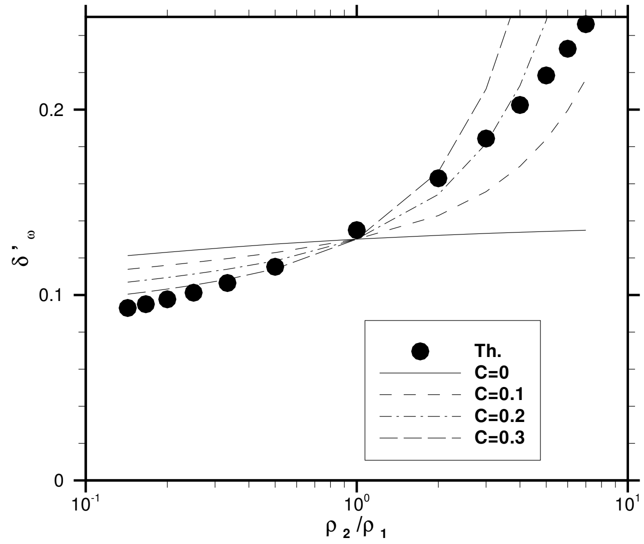

Figure 2: Mixing layer growth rate versus density ratio for $u_2 = 0$ – Revised version of Kreuzinger et al. model

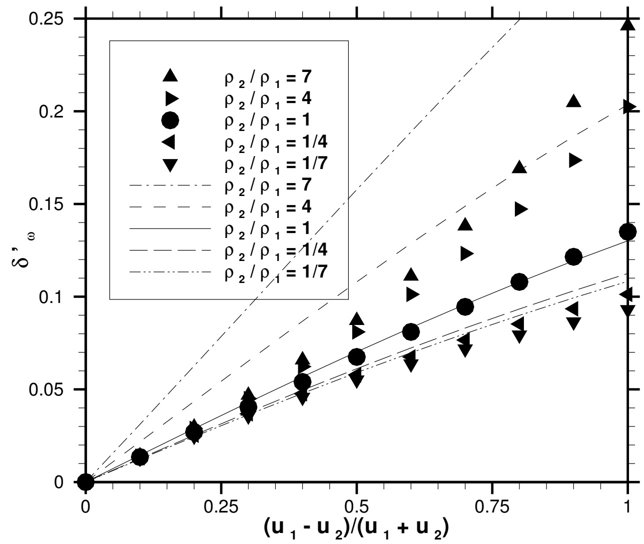

Figure 3: Mixing layer growth rate versus velocity ratio for several density ratios – Revised version of Kreuzinger et al. model

$s = 1$. This model drawback is consistent with the fact that the notion of high speed and low speed side is not relevant for temporally growing mixing layers.

Attempt to improve Kreuzinger et al. model

This model was then sensitized to the relative orientation of the density and velocity gradient by linking the sign of the model constant to the sign of $\overline{\underline{\mathrm{grad}}\,\rho \cdot \underline{\mathrm{grad}}\,\|\underline{u}\|}$. This modification is not fully satisfactory as it is not Galilean invariant. Nevertheless, figure 2 shows that model predictions are thus improved but no model constant really fits the theoretical curve. A value of the constant about 0.2, lower than Kreuzinger et al. recommendation, gives a good compromise.

Growth rate evolutions versus velocity ratio, predicted with this revised model, are plotted in figure 3 for several density ratios. The abscissa is

$$\frac{u_1 - u_2}{u_1 + u_2} = \frac{1-r}{1+r} \tag{10}$$

as the mixing layer growth rate is proportional to this quantity for constant density flows. Again, symbols correspond to relation (3) and curves to computations. The value of the model constant used here is $C = 0.18$. The overall agreement is poor as the modified model is unable to reproduce the concave or convex curvature of the growth rate curve according to the density ratio. Similar results are obtained for other values of the constant C.

A higher value of the constant, about 0.3, allows at least a fair agreement for density ratios s less than unity but degrades the predictions for density ratios higher than unity.

Other attempt

Baroclinic effects are a good candidate to explain density effects on turbulence but density effects should first alter the large scale dynamics so that a direct modelling of terms in the dissipation equation, which is linked to the viscous scales, may not be the best approach.

A quantity which is linked to the large scales, or not linked to the viscosity, but still linked to the vorticity, is the helicity

$$H = \overline{\underline{u}' \cdot \underline{\omega}'} \propto \frac{k}{L} \tag{11}$$

Nevertheless, the use of this quantity, which is a pseudo-scalar, as a turbulence scale, is problematic. It is surely not a good candidate to evaluate a turbulence length scale in all kinds of flows. Using the modelling strategies previously proposed by Aupoix (2004) or Kreuzinger et al. (2006) to represent the baroclinic influence on the vorticity fluctuation did not lead to any convincing model.

LENGTH SCALE EQUATION

Length scale equation for compressible flows

In order to capture the right dynamics, it seems interesting to deal with turbulent scales determining variables directly linked to the energy containing range. A turbulence time scale or length scale is thus a good candidate. As models with an imposed turbulent length scale give satisfactory results and as the turbulent length scale also naturally appeared in the modelling of density effects in compressible boundary layers (Catris and Aupoix, 2000), the length scale equation was favoured.

For incompressible flows, Wolfshtein (1970) proposed to define the turbulent length scale from the integral of two-point correlations, as

$$2kL = \int_V \overline{\underline{u}'_A \cdot \underline{u}'_B} \frac{d^3 \underline{r}_{AB}}{\|\underline{r}_{AB}\|^2} \tag{12}$$

where $\underline{r}_{AB}$ is the vector linking points A and B. Should this definition be extended to compressible flows considering the integral of $\overline{\rho_A \underline{u}''_A \cdot \underline{u}''_B}$ or of $\overline{\rho_A \underline{u}''_A \cdot \rho_B \underline{u}''_B}$? The second option seems more natural as $\rho \underline{u}''$ is a centered quantity. Moreover, it leads to simpler calculations as both points play a similar role.

The first option was nevertheless favoured as the previous study of modelling of density effects in compressible boundary layers (Catris and Aupoix, 2000) led to deal with variables which are combinations of ρk and L, which is automatically done with the first option. Moreover, not introducing the density at point B, which is the point over which the integration is performed, allows to better evidence the contribution of the mean density gradient in the final equation form.

The evolution equation for the correlation $\overline{\rho_A \underline{u}''_A \cdot \underline{u}''_B}$ can be deduced from the Navier–Stokes equation, after tedious calculations, as

$$
\begin{aligned}
&\frac{\partial}{\partial t}\overline{\rho_A u''_{iA} u''_{iB}} + \frac{\partial}{\partial y_l}\left(\tilde{u}_{lA}\,\overline{\rho_A u''_{iA} u''_{iB}}\right) = \\
&+\frac{\partial}{\partial r_l}\left(\tilde{u}_{lA}\,\overline{\rho_A u''_{iA} u''_{iB}}\right) - \tilde{u}_{lB}\,\frac{\partial}{\partial r_l}\left(\overline{\rho_A u''_{iA} u''_{iB}}\right) \\
&\quad -\overline{\rho_A u''_{lA} u''_{iB}}\left(\frac{\partial \tilde{u}_{iA}}{\partial y_l} + \frac{\partial \tilde{u}_{lB}}{\partial r_i}\right) \\
&\quad -\frac{\partial}{\partial y_l}\left(\overline{\rho_A u''_{iA} u''_{iB} u''_{lA}}\right) \\
&\quad +\frac{\partial}{\partial r_l}\left(\overline{\rho_A u''_{iA} u''_{iB} u''_{lA}} - \overline{\rho_A u''_{iA} u''_{iB} u''_{lB}}\right) \\
&+\frac{\partial}{\partial y_l}\left(\overline{\rho_A u''_{iA} u''_{lA}}\,\overline{u''_{iB}}\right) - \frac{\partial}{\partial r_l}\left(\overline{\rho_A u''_{iA} u''_{lA}}\,\overline{u''_{iB}}\right) \\
&\quad +\overline{\frac{1}{\rho_B}\frac{\partial}{\partial r_l}\left(\rho_A u''_{iA}\overline{\rho_B u''_{iB} u''_{lB}}\right)} \qquad (13) \\
&-\frac{\partial}{\partial y_l}\left(\overline{p'_A u''_{iB}}\right) + \frac{\partial}{\partial r_l}\left(\overline{p'_A u''_{iB}}\right) - \overline{\frac{1}{\rho_B}\frac{\partial}{\partial r_i}\left(p'_B \rho_A u''_{iA}\right)} \\
&\quad +\frac{\partial}{\partial y_l}\left(\overline{\tau'_{ilA} u''_{iB}}\right) - \frac{\partial}{\partial r_l}\left(\overline{\tau'_{ilA} u''_{iB}}\right) \\
&\quad +\overline{\frac{1}{\rho_B}\frac{\partial}{\partial r_l}\left(\tau'_{ilB}\rho_A u''_{iA}\right)} \\
&\quad -\overline{\rho'_A u''_{iB}}\left(\frac{\partial \tilde{u}_{iA}}{\partial t} + \tilde{u}_{lA}\frac{\partial \tilde{u}_{iA}}{\partial y_l}\right) \\
&-\overline{\left(\frac{\rho_A \rho'_B u''_{iA}}{\rho_B}\right)}\left(\frac{\partial \tilde{u}_{iB}}{\partial t} + \tilde{u}_{lB}\frac{\partial \tilde{u}_{iB}}{\partial r_l}\right) + \overline{\rho_A u''_{iA} u''_{iB}\frac{\partial u''_{lB}}{\partial r_l}}
\end{aligned}
$$

where, following Wolfshtein, r_k denotes the separation vector between points A and B and y_k the coordinates of point A. Wolfshtein (1970) form is retrieved for incompressible flows.

The first two lines represent advection, the third one the influence of mean velocity gradients, the fourth to seventh lines turbulent diffusion, the eighth one pressure transport, the ninth and tenth ones viscous diffusion. The sixth and seventh lines are null in incompressible flows since $\overline{u'} = 0$. The last two lines also are extra terms introduced by the compressible character of the flow.

A priori, the last term, which is linked to the divergence of the fluctuating motion, is negligible compared to the previous one, turbulent motion dilatation being generally small.

Model for the density gradient effects

The leading terms for the compressibility effects therefore seem to be the last two lines, except the last term. In the transport equation for $2\rho k L$, they give the contribution

$$
\begin{aligned}
&\int_V -\overline{\rho'_A u''_{iB}}\left(\frac{\partial \tilde{u}_{iA}}{\partial t} + \tilde{u}_{lA}\frac{\partial \tilde{u}_{iA}}{\partial y_l}\right)\frac{d^3\underline{r}_{AB}}{\|\underline{r}_{AB}\|^2} \qquad (14) \\
&-\int_V \overline{\left(\frac{\rho_A \rho'_B u''_{iA}}{\rho_B}\right)}\left(\frac{\partial \tilde{u}_{iB}}{\partial t} + \tilde{u}_{lB}\frac{\partial \tilde{u}_{iB}}{\partial r_l}\right)\frac{d^3\underline{r}_{AB}}{\|\underline{r}_{AB}\|^2}
\end{aligned}
$$

The modelling is performed following Wolfshtein's strategy, i.e. all quantities are expanded around point A as

$$\overline{\rho'_A u''_{iB}} = \overline{\rho'_A u''_{iA}}\, f_1\left(\frac{\underline{r}_{AB}}{L}\right) \tag{15}$$

so that the final term is proportional to

$$-\overline{\rho'_A u''_{iA}}\,\frac{D\tilde{u}_{iA}}{Dt} L \tag{16}$$

Various models for the mass flux term $\overline{\rho'_A u''_{iA}}$ are available in the literature for compressible flows. The only relevant one, to have a model valid as well for low speed as high speed flows, is to relate the mass flux to the density gradient. The correction thus reads

$$C_1 \nu_t \frac{\partial \rho}{\partial x_i}\frac{D\tilde{u}_i}{Dt} L \tag{17}$$

This correction term has been obtained for the transport equation for $2\rho k L$. It can be applied to the transport equation for any length scale determining variable. For example, if the dissipation rate ε is used, the corresponding term reads

$$C_1 \nu_t \frac{\partial \rho}{\partial x_i}\frac{D\tilde{u}_i}{Dt} L \frac{\varepsilon}{kL} = Ck \frac{\partial \rho}{\partial x_i}\frac{D\tilde{u}_i}{Dt} \tag{18}$$

This model form satisfies our two requirements. On the one hand, it is null in isochoric flows as $\frac{\partial \rho}{\partial x_i} = 0$ and, on the other hand, it is negligible in the logarithmic region of the boundary layer where $\frac{D\tilde{u}_i}{Dt} \approx 0$.

When this model form is transformed using the self-similarity assumptions, because of the advection term, the final term is proportional to the square of the density gradient. This means that, again, this model cannot give a reduction or an increase of the mixing layer growth rate, according to the respective orientations of the density and velocity gradients. This is confirmed by figure 4 on which the predictions of the mixing layer growth rate when the slowest stream is at rest, with the above correction and for various values of the model constant C, are plotted. Moreover, only a small increase of the mixing layer growth rate can be observed and the model becomes unstable for large values of the constant C.

Second model for the density gradient effects

As pointed out previously, there are several contributions of density gradients to the two-point correlation equation (13). The sixth and seventh lines were already identified as purely compressible terms. Using Wolfshtein's strategy (15), the volume integration of the first term of the sixth line is straightforward.

$$
\begin{aligned}
&\int_V \frac{\partial}{\partial y_l}\left(\overline{\rho_A u''_{iA} u''_{lA}}\,\overline{u''_{iB}}\right)\frac{d^3\underline{r}_{AB}}{\|\underline{r}_{AB}\|^2} \\
&= \frac{\partial}{\partial y_l}\left(\overline{\rho_A u''_{iA} u''_{lA}}\int_V \overline{u''_{iB}}\,\frac{d^3\underline{r}_{AB}}{\|\underline{r}_{AB}\|^2}\right) \qquad (19) \\
&\propto \frac{\partial}{\partial y_l}\left(\overline{\rho_A u''_{iA} u''_{lA}}\,\overline{u''_{iA}} L\right)
\end{aligned}
$$

The two other terms of the sixth and seventh lines are expected to give similar contributions so that, using again a first gradient hypothesis to model the mass flux $\overline{u''_{iA}}$, the final model form reads

$$\frac{\partial}{\partial x_l}\left(\overline{\rho u''_i u''_l}\,\frac{\mu_t}{\rho^2}\frac{\partial \rho}{\partial x_i} L\right) \tag{20}$$

This term is of course null in an isochoric flow but not in the logarithmic region of a boundary layer. It cannot

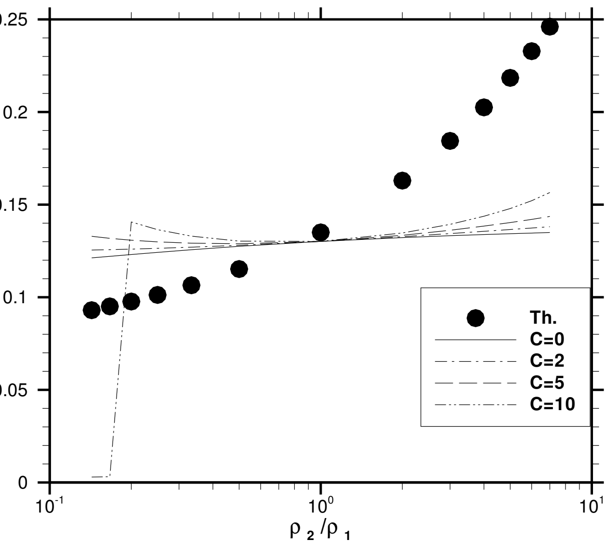

Figure 4: Mixing layer growth rate versus density ratio for $u_2 = 0$ - Model (18) for various model constant values

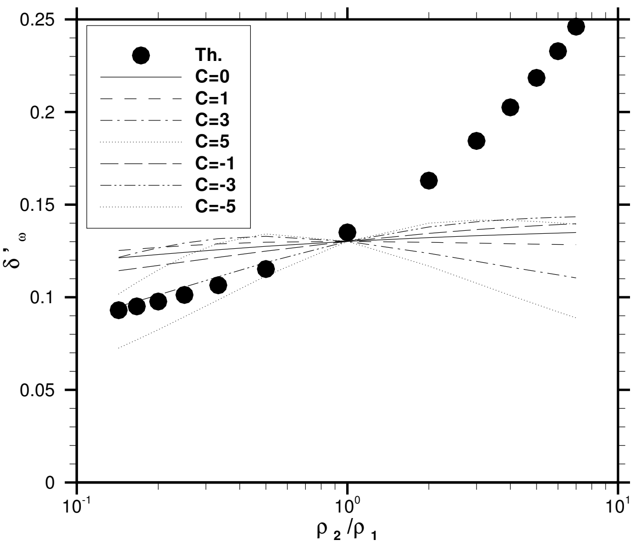

Figure 5: Mixing layer growth rate versus density ratio for $u_2 = 0$ - Model (25) for various model constant values

be used alone to correct the model in the boundary layer. Therefore, the only strategy is to consider that part of this model is already included in the Catris' correction and to extract the part which is not active in the logarithmic region of the boundary layer but may affect the mixing layer. With thin layer assumptions, the model (20) can be rearranged as

$$\frac{\partial}{\partial y}\left(\overline{\rho v''^2}\,\frac{\mu_t}{\rho^2}\,\frac{\partial \rho}{\partial y}L\right) \propto \frac{\partial}{\partial y}\left(\rho k \frac{\mu_t}{\rho^2}\,\frac{\partial \rho}{\partial y}L\right) \qquad (21)$$

while, in the logarithmic region of the boundary layer, these various quantities evolve as

$$\rho k \propto \rho_p u_\tau^2 \quad \mu_t = \sqrt{\rho \rho_p}\, u_\tau \kappa y \quad L \propto y \qquad (22)$$

Therefore, expanding model (20), the only contribution which is null in the logarithmic region is

$$\frac{\partial \overline{\rho u_i'' u_l''}}{\partial x_l}\,\frac{\mu_t}{\rho^2}\,\frac{\partial \rho}{\partial x_i}L \qquad (23)$$

For flows without pressure gradients, as the self-similar mixing layers presently investigated, the divergence of the Reynolds stress tensor can be expressed with the help of the momentum equation, so that the model reads

$$\frac{\mu_t}{\rho}\,\frac{\partial \rho}{\partial x_i}\,\frac{D\tilde{u}_i}{Dt}L \qquad (24)$$

i.e. model form (17) previously investigated.

Equation (23) also suggest to test the following model

$$\frac{\mu_t}{\rho^2}\,\frac{\partial \rho k}{\partial x_i}\,\frac{\partial \rho}{\partial x_i}L \qquad (25)$$

which cannot be strictly derived from that equation but at least reduces to the same form under thin layer assumptions. For $k-\varepsilon$ models, the correction term (25) becomes

$$\frac{k}{\rho}\,\frac{\partial \rho k}{\partial x_i}\,\frac{\partial \rho}{\partial x_i} \qquad (26)$$

It must be pointed out that this model form has contributions of opposite signs on both sides of the mixing layer since the sign of $\frac{\partial \rho}{\partial y}$ remains the same but that of $\frac{\partial \rho k}{\partial y}$ changes. Analysis of DNS data (Aupoix et al., 2004) have shown that, compared to isochoric flows, the dissipation increases on the high density side of the mixing layer. To get such a behaviour, the above term must be multiplied by a negative constant.

Figure 5 confirms that a negative value of the constant is required to have the correct evolution. For $C = -3$, this model gives fair predictions when $\rho_2 \leq \rho_1$ but it is not possible to have good agreement when $\rho_2 > \rho_1$, the model underestimating the density effect.

CORRECTIONS AVAILABLE IN THE LITERATURE

Other corrections to improve mixing layer and jet flow predictions are available in the literature. A review of some recent models for jet flows has been provided by Georgiadis et al. (2006).

PAB temperature correction

The PAB temperature correction has been proposed by Abdol-Hamid et al. (2004). The C_μ coefficient in the eddy viscosity definition is sensitized to the gradient of the stagnation or total enthalpy T_t as

$$C_\mu = 0.09\left[1 + \frac{T_g^3}{0.041}\right] \qquad T_g = \frac{k^{3/2}}{\varepsilon}\,\frac{\|\mathrm{grad}\, T_t\|}{T_t} \qquad (27)$$

This model suffers from two drawbacks. It is not Galilean invariant as the total temperature depends upon the reference frame. But mainly, it is only sensitized to the modulus of the total temperature gradient. It therefore cannot distinguish whether the low speed stream is the hotter or the colder. Therefore, it has the same kind of symmetric behaviour as the basic Kreuzinger et al. model.

Tam and Ganesam model

Tam and Ganesam (2004) performed a stability analysis to evidence couplings between the Kelvin–Helmholtz instability and the density gradient. They considered only density ratio close to unity so that the effect of density gradient is small and can be linearized. Therefore, they directly introduced a correction on the eddy viscosity which reads

$$\mu_t = \mu_{t\,\rho=cst} + \mu_\rho \tag{28}$$

and the correction μ_ρ is expressed as

$$\mu_\rho = \begin{cases} C_\rho \dfrac{k^{7/2}}{\varepsilon^2} \| \dfrac{\partial \rho}{\partial r} \| & \text{if } \dfrac{\partial \rho}{\partial r} \dfrac{\partial u}{\partial r} < 0 \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

The above formula is of course valid only for axisymmetric jets. They therefore attempted to extend it to general geometries as

$$\mu_\rho = \begin{cases} C_\rho \dfrac{k^{7/2}}{\varepsilon^2} \dfrac{\| \underline{\text{grad}}\, \rho \cdot \underline{\underline{\text{grad}}}\, \| \underline{u} \| \|}{\| \underline{\underline{\text{grad}}}\, \| \underline{u} \| \|} & \text{if } \underline{\text{grad}}\, \rho \cdot \underline{\underline{\text{grad}}}\, \| \underline{u} \| < 0 \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

This model has the same drawback as the modified Kreuzinger et al. model, using the gradient of the velocity modulus, which is not Galilean invariant. Moreover, this model only acts if the density and velocity gradients have opposite directions, i.e. if $s < 1$, which is not the most difficult case.

CONCLUSION

These model tests show that the prediction of density effects on the mixing layer growth rate remains a challenge.

Baroclinic effects may explain the influence of density gradient on the dynamics of the big rollers encountered in mixing layers. Nevertheless, baroclinic effects should be small when considering dissipative scales so that accounting for baroclinic effects in the dissipation rate equation may not be relevant. Nevertheless, the modified version of the Kreuzinger et al. model remains the best compromise whatever the density ratio, as it can better predict the mixing layer growth rate than models without modifications and preserves boundary layer flows. The big drawback of this model is that it is not Galilean invariant.

Another attempt to model density effect using the length scale transport equation deduced from the integral of two-point correlations led to a model which is efficient only when the low speed stream is the heavier.

For applications such as propulsive jets in which the low speed stream is the heaviest, this model (25, 26) as well as the basic Kreuzinger et al. model, with a constant value about 0.3, or other models already available in the literature can give fair predictions. The real, still unresolved problem, occurs when the low speed flow is the lighter ($s > 1$), a situation which can be encountered e.g. in film cooling.

No model preserving the logarithmic region in presence of density gradient and giving the right prediction of mixing layer growth rate whatever the density and velocity ratios is presently available.

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