

# AEROACOUSTICS OF HIGH-SPEED JETS: A PERSPECTIVE FROM NUMERICAL SIMULATIONS

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## ABSTRACT

The computational challenges faced in conducting numerical simulations which capture the noise radiated by turbulent high-speed jets are summarized. Recent work of Bodony & Lele using LES for jet aeroacoustics is illustrated with focus on comparing near-field turbulence and far-field noise with available data for heated and unheated jets. The opportunity provided by high-fidelity simulations for studying and modeling aeroacoustic phenomena is stressed via model problems of sound generation, including a discussion of open problems. Recent progress in simulating and modeling broadband shock associated noise is highlighted.

## PHYSICAL AND COMPUTATIONAL ISSUES

The prediction and reduction of jet noise has been a major theme of aeroacoustics research for over 50 years. More stringent community noise regulations require further reductions in jet noise along with other engine-noise components. Technical reviews of jet noise are available (see Lilley, 1991, Tam, 1995, and Huff et al., 2001).

Consider a high-speed jet at a moderate to high Reynolds number. The jet exit diameter is  $D_j$ , jet exit velocity  $U_j$ , jet density  $\rho_j$  (speed of sound  $C_j$ ). The ambient density and speed of sound are  $\rho_\infty$ , and  $C_\infty$ , respectively. It is assumed that the nozzle boundary layers are thin, i.e. their momentum thickness  $\theta/D_j \ll 1$ . The overall jet flow is quite insensitive to the jet Reynolds number  $Re_j = \rho_j U_j D_j / \mu_j$  for high  $Re_j$ ; the mean velocity profiles, Reynolds stress distributions, potential-core length etc. are not significantly affected by  $Re_j$ . The jet speed  $U_j$ , on the other hand does affect the jet flow and its noise: as the jet Mach number  $M_j = U_j / C_j$  increases the potential core gets extended due to a reduced shear-layer spreading. The spectra of the radiated noise also change with  $M_j$ . For the latter the acoustic Mach number of the jet  $M = U_j / C_\infty$ , and the temperature ratio  $T_j / T_\infty$  are more convenient. Tam (1995) has provided a recent review of jet noise, specially for supersonic jets.

As a specific example consider a laboratory-scale cold air jet with  $D_j = 2.5$  cm. at  $M_j = 0.9$ . We would estimate  $U_j = 290$  m/s and  $Re_D \approx 600,000$ . At  $x/D_j = 2$  using the mixing-layer estimates for the turbulence integral scale  $L$  gives  $Re_L \equiv u' L / \nu \approx 22,000!$  Experiments show that at  $90^\circ$  to the jet axis the noise spectra peak at a Strouhal number  $St_D \equiv f D / U_j = 0.3$  which for this jet corresponds to  $f_{p90} = 3.5$  KHz or  $\lambda_{p90} / D_j = 4.0$ . Since the peak level of jet noise, which occurs at smaller angles to the jet, does not show

Strouhal-scaling (Lush, 1971, Ahuja, 1974) but shows something like a Helmholtz-scaling, it is important to estimate this peak frequency as well. Experiments show  $f_p D_j / C_\infty = 0.16$  at  $30^\circ$  to the jet which yields  $f_{p30} = 2$  KHz or  $\lambda_{p30} / D_j = 6.25$ . These dominant acoustic wave lengths can be compared with the dominant scales of turbulence at various streamwise stations. For example, if the comparison is made at  $x/D_j = 2$ , we get  $\lambda_{p90} / L_{(x/D_j=2)} = 14$ , and  $\lambda_{p30} / L_{(x/D_j=2)} = 24$ . Such ratios are useful in selecting the size of the computational domain and the grid-spacing. Closer to the nozzle these scale ratios are much more extreme. It is also evident from these estimates that in a near-sonic jet the spatial scales of energy-containing turbulence never match the acoustic wavelength of the dominant radiation (at any angle). This mismatch in spatial scales (or scale disparity) is more severe for lower speed jets and is responsible for the radiation inefficiency of these flows. However, these estimates should **not** be taken to imply that the noise-sources in the jet, i.e. the turbulent eddies, radiate as localized compact acoustic sources. Spatio-temporal evolution of jet turbulence needs to be considered.

At any fixed station  $x$ , the turbulent fluctuations can be characterized using the space-time correlation  $\langle u_i'(x + \xi, t + \tau) u_j'(x, t) \rangle$ . For a fixed spatial separation  $\xi$ , a specific value of the time delay  $\tau = \tau_m(\xi)$  gives the maximum correlation. This allows a convection velocity  $U_c$  to be defined,  $U_c d\tau_m(\xi) / d\xi = 1$ , and the decay of the peak correlation gives a 'Lagrangian' decorrelation time  $\tau_L$ . Measurements (Davies et al., 1963) show that in the mixing layer of the jet  $U_c / U_j \approx 0.65$ ,  $\tau_L \approx 4.5 / \|dU/dr\|_{max}$  or equivalently that  $u' \tau_L / L = 0.9$  and  $U_c \tau_L / L = 4.1$ . Since (sub-sonic) convection of a frozen eddy-pattern does not radiate sound, it is natural to associate the radiated sound with the evolution of the turbulence in the convected frame (see Ffowcs Williams, 1963, Goldstein, 1976). Associating the local turbulence of Lagrangian time scale  $\tau_L(x)$  with radiation at a wavelength  $\lambda(x) = C_\infty \tau_L(x)$  implies that in the mixing layer  $\lambda(x) / L(x) \approx 7$  and  $\lambda(x) / (U_c \tau_L(x)) = C_\infty / U_c \approx 1.5$ . Evidently, the local turbulence contributes to sound at a spatial scale significantly larger than its integral scale. A similar conclusion is reached from the scaling of the noise peak at  $f_p D_j / C_\infty = 0.16$  which is equivalent to  $\lambda_p / D_j = 6.25$ . There is, however, a caveat. Lighthill (1952, 1954) stressed the acoustic inefficiency of noise radiation from bulk turbulence in the jet. This is consistent with the length scale ratios just discussed. However, if the flow processes in a jet 'generate'

very-large-scale motions<sup>1</sup>, *i. e.* with spatial scales much larger than the scales of jet turbulence, the radiation efficiency of these ‘global’ jet motions is likely to be rather high. Very-large-scale motions can dominate the acoustic radiation even when these motions are energetically weak.

The acoustic compactness of energetically dominant eddies is valid only in the Lagrangian frame. If viewed in a frame of reference fixed to the nozzle the sound-source-region is acoustically non-compact due to the convection of the turbulent eddy during its life-time. The specific numerical factors in these estimates would change if jets at other operating conditions are considered. Jets at low acoustic Mach number  $U_j/C_\infty$  have a greater scale disparity  $\lambda(x)/L(x)$  and a greater amplitude disparity between the near-field turbulence fluctuations and the radiated sound. On the contrary when  $U_c > C_\infty$  (supersonic eddy-convection) strong radiation results (Ffowcs Williams, 1963, Tam & Burton, 1984). In the near-field this radiation has the structure of random Mach waves associated with the turbulent eddies. Similar but organized near-fields also arise for radiation from organized (supersonically-convected) instability waves in high-speed jets (Tam & Burton, 1984).

The physical aspects of high-speed jets were emphasized thus far. From a computational perspective they can be summarized as follows:

- High-speed turbulent jets represent unsteady compressible flows with a broad range of spatial and temporal scales. This broadband character is also reflected in the acoustic wave motions ‘generated’ by the jet. The spatial scales of the radiated sound are significantly larger than those of the jet turbulence. To capture them the computational domain must be large enough to contain the near-acoustic field.
- The magnitude of the acoustic waves generated by the jet is several orders of magnitude smaller than the turbulent fluctuations. The flow/turbulent fluctuations decay rapidly away from the jet leaving a wave-field which decays slowly and follows the inverse square law.
- The magnitude disparity between the turbulence and sound requires that special attention is given to minimize the dispersive and dissipative errors in the numerical approximations used to solve the governing equations.
- Computational cost considerations limit the size of the physical domain to be simulated. It is thus essential to use boundary conditions which allow *silent* passage of prescribed inflow disturbances into the domain, silent outflow of vortical disturbances at the outlet without an appreciable creation of spurious (upstream traveling) waves, and effective non-reflecting boundary conditions which allow acoustic waves to propagate out of all computational boundaries.

<sup>1</sup>The notion of *very large scales* is similar to Townsend’s (1956) notion of large-scales in turbulence. He categorized turbulent motions into energy containing scales (main turbulent motions), large-scales and small-scales. In his terminology the main turbulent motions are envisioned on scales smaller than the scale of mean flow inhomogeneity  $l_T$ . It is now well-known that energy-containing scales are *not* smaller than  $l_T$  but of similar scale. The adjective *very* is added to distinguish them from the energy-containing (large-scales) and the accepted usage in large-eddy simulation (LES).

## Representation of jet turbulence: organized motions

Jet flows contain both quasi-organized large-scale motions reminiscent of instability wave disturbances or wave-packets, and more irregular turbulent motions. Yet such a decomposition is not formally used in current jet noise theory. Methods such as the wavelet decomposition (Farge, 1992) and proper orthogonal decomposition (Freund and Colonius, 2002) are being applied to jet flows and their noise, but at present the available information is limited (see, Citriniti and George 2000, and Gordeyev and Thomas, 2000). This type of decomposition is yet to be used in a comprehensive method for noise prediction. One of the following two *extreme* positions<sup>2</sup> is commonly adopted for the jet flow:

- A) *all fluctuations are associated with turbulence*; no explicit representation of the organized wave-packets is used (e. g. Davies et al., 1963, Bradshaw et al., 1964, Zaman, 1986, Hussein et al., 1994, Panchapakesan & Lumley, 1993)
- B) *all large-scale motions or ‘large-scale turbulence structures’ correspond to instability waves* (e. g. Plaschko, 1981, 1983, Tam & Chen, 1979, Tam, 1987).

Viewpoint-A is purely statistical and leads to a representation of turbulence-associated noise sources in terms of space-time correlations. Lighthill (1952, 1962, 1963) adopted this view. Statistical representations of acoustic sources have been sought in many studies (Ffowcs Williams, 1963, Lilley, 1974, including recent work of Bailly et al., 1997, and Khavaran, 1999). Many studies take into account the effect the jet mean-flow has on the radiated noise, a feature shared by Tam and Auriault (1999) fine-scale noise model, also see Morris and Fararat (2002). Although a statistical representation does not rule out the presence of orderly structure in jet turbulence, this information is not explicitly reflected in current models.

Viewpoint-B treats the dynamics of *large-scales* as instability wave-packets<sup>3</sup>. This requires the mean-flow to be specified, or predicted. RANS equations, sometimes with adjusted model coefficients, are often used. Integral methods are used to represent the non-linear interactions between the wave-packet and other ‘background’ disturbances, including finer-scale turbulence (Mankbadi & Liu, 1981, Morris et al., 1992, Tam & Morris, 1985). It is arguable if an accurate prediction of instability wave-packets can be managed efficiently within the framework of a small set of interacting modes, such as the non-linear disturbance equations or nonlinear parabolized stability equations, NPSE. Recently Bertolotti and Colonius (2003) identified the potential importance of supersonically-convected entropic non-uniformities in the core of a heated jet (called ‘core-modes’) to the radiated noise. NPSE has also shown remarkable accuracy in strongly nonlinear two-dimensional shear layers (Day et al., 2001) and has been

<sup>2</sup>With notable exceptions of Liu (1974) and Michalke and Fuchs (1975) and related work cited therein.

<sup>3</sup>The large-scale eddy-structure in jets has been linked to the linearized instability characteristics of the mean-flow (Morris et al., 1992, Gaster et al. 1985). A flow disturbance at a fixed frequency initially grows in amplitude and subsequently decays due to mean flow spreading and nonlinear interactions, giving rise to a *wave-packet* with a carrier wavenumber and modulation amplitude which change slowly along the jet (Crighton & Gaster, 1976). This modulation results in noise radiation from subsonically convected ‘instability wave’ disturbances (Liu, 1974, Tam & Morris, 1980, Tam & Burton, 1984), Crighton & Huerre, 1990).

successful for predicting noise radiation (Cheung and Lele, 2004). DNS data from a supersonic turbulent jet (Mohseni et al., 2002) also reveals the importance of nonlinearity for modal amplitude prediction.

Tam et. al. (1996) show that far-field jet noise spectra are well described by two empirical spectra, one attributed to large-scales and the other to fine-scales. There is, however, no experimental evidence of a scale-gap between large and fine-scales in turbulent jets. Jet turbulence is intrinsically a multi-scale phenomena. Hence the predictions of the noise radiated by the large-scales need to be combined with the noise radiated by turbulence at other scales, intermediate and fine-scales. Goldstein (2003) suggests that the observed two components correspond to two different radiation ‘paths’ (one direct and the other mediated by the jet instability waves) but from the same sources.

Lighthill’s theory provides a scaling estimate for the jet mixing noise. At  $90^\circ$  from the jet axis this theory gives

$$\frac{p'}{p_\infty} \sim \left\{ \frac{\rho_j}{\rho_\infty} \left( \frac{u'}{U_j} \right)^2 \left( \frac{fD_j}{U_j} \right)^2 \left( \frac{L}{D_j} \right)^3 \right\} M_a^4 \frac{D_j}{r}, \quad (1)$$

where  $M_a = U_j/C_\infty$  is the ‘acoustic Mach number’ of the jet,  $f$ ,  $L$  and  $u'$  are *representative* scales for the peak frequency, correlation length, and turbulent velocity scale. Traditionally the prefactor in  $\{\}$  is taken to be a constant, yielding the famous  $U_j^8$  law for OASPL. Even though (1) does not account for many physical effects which are important in the jet flow, such as refraction of high-frequency sound by the flow and the coherent nature of low-frequency disturbances, it has served as a useful guide towards less noisy configurations; lowering the *mixed-flow* jet velocity gives dramatic noise reduction. The synergy with improved propulsion efficiency has compelled a trend towards higher bypass-ratio in modern turbo-fan engines. Other methods which alter jet eddy-structures by geometrical changes to the nozzle (such as with chevron, tabs, mixer lobes), or micro-jets and other actuators to reduce turbulence intensity near the end of the potential core have shown modest noise reduction benefits. But it is oversimplistic to characterize the entire jet flow field with a single length scale  $l$ , and time scale  $1/f$ , as implied in (1). The flow processes in different regions of the jet scale differently as the engine operating conditions are varied. Empirical models which use two or three distinct scales for different regions of the jet have been proposed, but their empirical origin limits their use to interpolation over a well parameterised design space. Manipulation/control of jet mixing to achieve less noisy flow at full-scale (in EPNL) remains an art.

It is well known that scaling such as (1), along with the attendant ‘Doppler factors’, is inadequate for the overall noise level in the jet-noise peak direction, at  $30$  to  $50^\circ$  from the jet axis. The spectral shape is significantly different, and OASPL varies more rapidly than Lighthill’s  $V_j^8$  scaling. The frequency of the peak noise is *curiously* independent of the jet speed  $V_j$  and  $fD_j/C_\infty \sim \text{const}$  is observed (Ahuja, 1974, Tanna, 1977, Tam et al., 1996)<sup>4</sup>. At present there is no comprehensive theory<sup>5</sup> which successfully predicts all major features of noise radiation from a jet.

<sup>4</sup>In recent work Goldstein & Leib (2005) have shown that a multiple-scales treatment of sound radiation from a jet, which keeps the instability wave response, is consistent with these trends.

<sup>5</sup>For supersonically convecting eddy-structures, *Mach wave* radiation is dominant along a preferential direction. This mecha-

nism permits a linearized theory which captures the directivity of this intense noise (Tam and Burton, 1984), and has stimulated work on low noise configurations for dual-stream jets (Papasochou and Debiasi, 2001).

A computational alternative to the methods based on specific decompositions is to lump all large-scale disturbances together and model only the unresolved-scales. This sidesteps the issue of wave-packet/irregular turbulence decomposition and is the general approach of large-eddy simulation (LES). Significant progress in the use of LES for jet noise predictions has been made recently (Bogey & Bailly, 2003), Zhao et al., 2000, Unzun et al., 2003, Bodony & Lele, 2004) and new insights on noise generation in turbulent jets are expected from such studies in the next few years. Due to computer resource limitations (see Freund & Lele, 2005) current jet LES studies use inlet conditions of an artificially thickened shear-layer. This can adversely impact the natural development of the jet and the radiated noise levels unless special care is taken. Efforts to include the nozzle geometry in the calculations are also being pursued and require careful development and validation.

## DNS OF TURBULENT JETS

DNS studies of a turbulent jet and its noise radiation have recently been reported (Freund, 2001, 2003, 2000). These are limited to a near-sonic unheated jet, and a supersonic jet at low Reynolds number. The overall features of the flow, such as the mean flow profiles, the rate of spreading of the jet, turbulence profiles, overall SPL and its directivity are in remarkable agreement with the available experimental data (Stromberg et al., 1980) at low Reynolds numbers. Also reported was analysis of the noise-sources based on Lighthill’s formulation, such as the spatial distribution of sound-source-strength, its decomposition into various source-components (self-noise, shear-noise, entropic component) and approximation, and its frequency-wavenumber dependence. It is intriguing that the source-distribution responsible for the dominant noise radiation from this turbulent jet has a very simple wave-packet structure. Whether this simple source structure would persist if the early shear-layers of the jet were also turbulent, although not certain at present, remains a distinct possibility. There is experimental evidence for wave-packet-like source distributions. Laufer and Yen (1983) and Crow (1972) interpreted their jet noise measurements in terms of a line-antenna model of the jet. Crighton (1975) and Crighton & Huerre (1990) analyzed the noise radiation from the jet’s orderly structure in terms of a wave-packet model and characterized its radiation as ‘superdirective’. Colonius et al. (1997) also found that the noise radiation from organized vortex-pairings could be represented as a ‘superdirective’ radiation from a wave-packet. Further analysis of the data gathered from such DNS/LES studies of jets can provide a fertile ground for developing new source approximations, and for developing hybrid methods for computations and noise predictions. An interesting use of jet DNS data is reported by Freund et al. (2002). The time dependent data was filtered to remove the scales containing most of the turbulent kinetic energy via spatial filtering and the dynamics of the very-large-scales were studied. This severe filtering did not affect the dominant low-frequency noise radiation from the jet confirming the importance of very-large scales to noise radiation.

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## LES FOR JET AEROACOUSTICS

Development of LES for aeroacoustic predictions is a very active area of on-going work. Appropriate treatment of turbulent inflow, SGS models, and noise models for the scales not captured in LES are being pursued. Comments on the current status of LES for jet noise were made earlier. Bogey & Bailly (2003) reported the first successful jet noise results from jet LES at near sonic conditions. They used the DRP scheme with artificial-selective damping (Tam & Webb, 1993) on a cartesian mesh with fine spacing in the jet. Constantinescu & Lele (2001) also conducted LES of a near-sonic jet using high-order compact finite difference schemes. They devised a special treatment of the governing equations near the cylindrical coordinate singularity (Constantinescu & Lele, 2002). They also note that aliasing errors from non-linear terms cause energy pile up at near-grid scales; the sub-grid model alone was insufficient. A spatial filtering of the solution, with an eighth-order compact filter (Gaitonde & Visbal, 1998) was applied every 200 time steps to remove the spurious grid-to-grid oscillations. Bodony & Lele (2004) refined the numerical implementation and used an inlet forcing based on a large number of linear stability eigen-modes of the jet mean flow near the inlet. The phase of these modes had a random component. This method generates a turbulent flow after a modest development length and does not create spurious noise. The importance of inflow conditions is also noted by Bogey & Bailly (2005a).

Bodony & Lele (2004, 2005a, 2005b) have reported results from a comprehensive study of jet aeroacoustics using LES. Jet speed and jet temperature were varied covering a range of flows from low speed-, to near sonic-, and supersonic-jets with and without heating. A subset of the flows studied by them is shown in Table 1. Some highlights from these simulations are reproduced here. The axial development of the jet half-width is shown in figure 1 and the decay of the mean velocity along the jet centerline is plotted in figure 2 with the axial distance scaled according to the empirical correlation due to Witze (1974). Also shown is comparison with experimental data and Freund's DNS data. On the whole the mean flow decay follows the trends seen in the experiments, but the jet spreading rate is slightly over-estimated. This is believed to be associated with the lack of full developed turbulence in the shear layers in early regions of the jet (Bodony & Lele, 2005a). The spreading rate of the jet is reduced as the jet Mach number increases. Also notable is the more rapid spreading of the heated jet when jet speed is kept constant. Both trends are correctly reproduced in the LES. Turbulence properties in the jet also follow the expected behavior. Figure 3 shows the development of rms streamwise velocity fluctuations. The peak levels are approx. 10% larger than laboratory measurements, again largely due to the differences in the early shear layers.

Figures 4 - 5 show composite visualizations of the two highest speed jet flows. Both flows have the same jet speed or acoustic Mach number. Figure 4 shows the heated case and figure 5 the unheated case. In the latter case the jet Mach number is also supersonic. Careful examination of the near acoustic field in these cases shows two dominant radiation patterns: Mach wave radiation at small angles to the jet which is more prominent for the unheated case and a less directive finer-scale radiation originating from the region where the potential core ends. In animations the interference between these two types of radiation can be observed. Figure 6 shows the OASPL directivity predicted by the LES for the two

ID	TID	$U_j/a_\infty$	$U_j/a_j$	$T_j/T_\infty$	$M_c$	$Re_D^\ddagger$
M05TR176	sp23	0.50	0.38	1.76	0.21	27,000
M05TR095	sp3	0.50	0.51	0.95	0.25	79,000
M09TR270	sp46	0.90	0.55	2.70	0.34	13,000
M09TR086	sp7 <sup>†</sup>	0.83	0.90	0.86	0.43	88,000
M15TR230	sp39	1.47	0.97	2.30	0.58	84,000
M15TR056	sp62	1.47	1.95	0.56	0.83	336,000

Table 1: Conditions of the simulations presented. The nomenclature sp $N$ , where  $N$  is an integer, and listed in the 'TID' column, refers to conditions tabulated in Tanna (1977). <sup>†</sup>The conditions for run M09TR086 are approximately the same as those used by Tanna (1977). <sup>‡</sup>The Reynolds numbers, with  $Re_D = \rho_j U_j D_j / \mu_j$ , are those used in the present LES and are not the same as in the experiments.

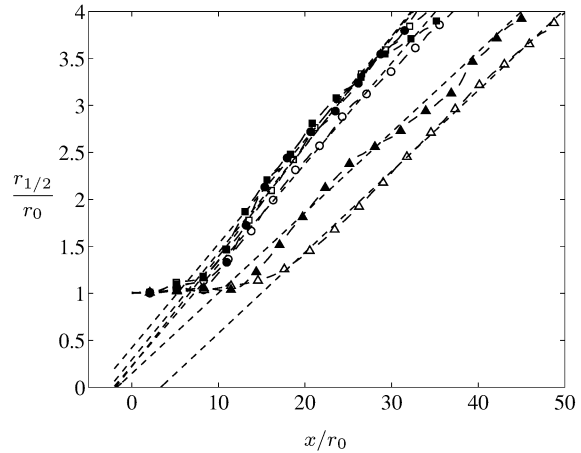


Figure 1: Evolution of jet half width for different jet flows. From the LES study by Bodony & Lele (2004). The abscissa is given in the computational coordinate (unshifted & unscaled)  $x/r_0$ . Legend: M09TR086,  $\circ$ —; M09TR270,  $\bullet$ —; M05TR095,  $\square$ —; M05TR176,  $\blacksquare$ —; M15TR056,  $\triangle$ —; M15TR230,  $\blacktriangle$ —.

highest speed jets along with the experimental data. The radiated noise levels agree with data and clearly show the trend towards lower noise with heating at all observer angles. Figure 7 shows a similar comparison for the near sonic unheated and heated jets. Agreement with the data is still quite good, and larger scatter amongst the experimental data should be noted. Figure 8 shows a comparison of narrowband sound spectra with the measurements taken by Viswanathan (2004). The predicted noise spectra agree with data up to a Strouhal number of approx 1.5 above which the LES spectra show a rapid fall-off. This is a signal of the bandwidth limitation of the turbulent fluctuations in the jet (computed here with  $2 \times 10^6$  grid points) and not associated with numerical dissipation (Bodony, 2004).

This comparison also highlights the need for developing models for predicting the noise associated with the missing scales in LES. Bodony & Lele (2003) have developed such a framework which is currently being applied to jet flow computations. They also discuss the numerical implementation and noise source modeling issues in this context. Another recent study of jet LES by Bogey & Bailly (2005b) relies entirely on high wave number filtering as a surrogate SGS model

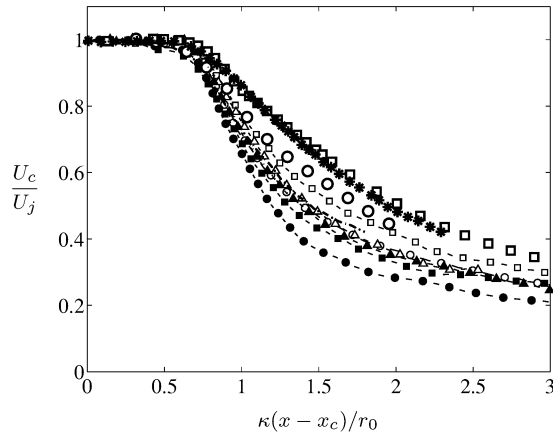


Figure 2: Evolution of mean velocity along the jet centerline for different jet flows. The x-coordinate has been scaled based on Witze's correlation. From the LES study by Bodony & Lele (2004). See Fig. 1 for legend and  $-\cdot-\cdot-$ , Freund (2001);  $---$ , Bogey & Bailly (2003); Experimental data:  $\circ$ , Tanna (M09TR086);  $\square$ , Bridges & Wernet (2003) (M05TR095);  $\star$ , Crow & Champagne (1971) (incompressible).

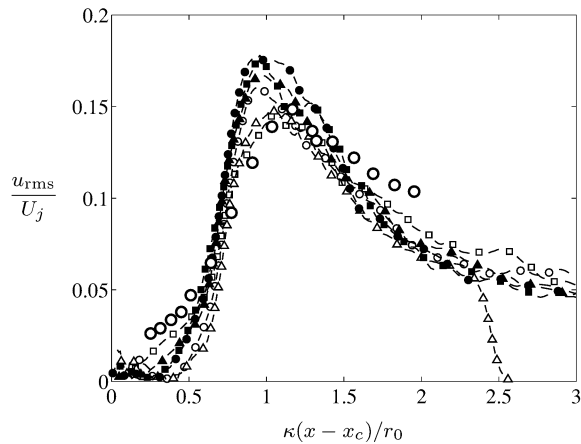


Figure 3: Evolution of rms streamwise velocity fluctuation along the jet centerline for different jet flows. The x-coordinate has been scaled based on Witze's correlation. From the LES study by Bodony & Lele (2004). See Fig. 2 for legend.

with striking results on jet turbulence and noise. They stress the importance of limiting the influence of the SGS model to near-grid scales. Development of LES as a predictive tool for broadband noise requires careful examination of the role of SGS model, missing-scales noise model, as well as numerical issues associated with turbulent inflow prescription, and high-fidelity treatment of unsteady multi-scale flows in domains with embedded complex geometrical boundaries, e. g. realistic nozzle geometry.

#### JET SCREECH AND SHOCK-ASSOCIATED NOISE

Imperfectly-expanded jets produce additional noise due to the interaction of the jet turbulence with the shock-cell structure existing within the jet. Its tonal components called jet

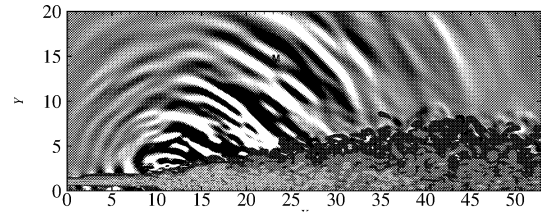


Figure 4: Contours of dilatation (outside the jet) overlaid on contours of  $\|\omega\|$  (in the jet) from LES of a  $Ma = 1.5$  heated jet by Bodony (2004)

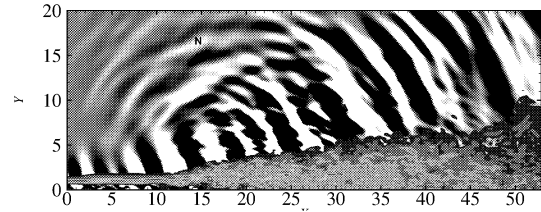


Figure 5: Contours of dilatation (outside the jet) overlaid on contours of  $\|\omega\|$  (in the jet) from LES of a  $Ma = 1.5$  unheated jet by Bodony (2004)

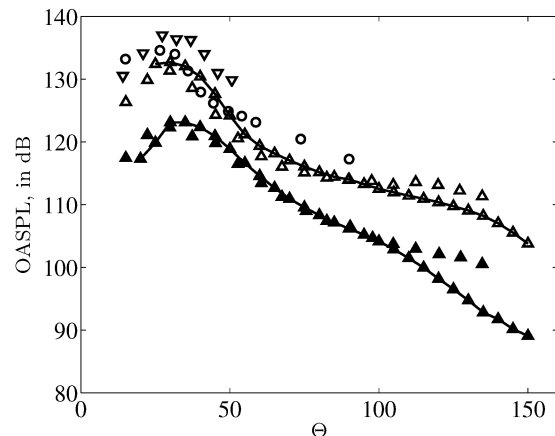


Figure 6: OASPL directivity of the radiated noise for jets with  $Ma = 1.5$ . The far-field data has been extrapolated to a distance of  $100D_j$ . From the LES study by Bodony & Lele (2004). Legend:  $-\triangle-$ , M15TR056,  $-\blacktriangle-$ , M15TR230. Experimental data:  $\triangle$ , Tanna (M15TR056);  $\blacktriangle$ , Tanna (M15TR230);  $\circ$ , Troutt & McLaughlin (1982) (cold,  $M_j = 2.0, Re_D = 5.2 \times 10^6$ );  $\nabla$ , Troutt & McLaughlin (1982) (cold,  $M_j = 2.1, Re_D = 7.0 \times 10^5$ ).

screech, requires a feedback loop (Powell, 1953). Tam (1995) discusses the present physical understanding of these noise components. At the nozzle lip *embryonic* shear-layer disturbances are generated, which convect and amplify in the developing shear-layers. Their interaction with the second/third shock-cell generates acoustic waves which travel upstream to the nozzle lip and close the loop. The phase criterion for constructive reinforcement over the feedback loop provides a

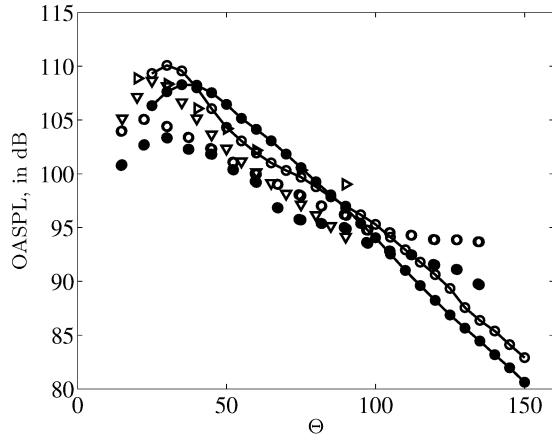


Figure 7: OASPL directivity of the radiated noise for jets with  $Ma = 0.9$ . The far-field data has been extrapolated to a distance of  $100D_j$ . From the LES study by Bodony & Lele (2004). Legend:  $\circ$ —, M09TR086,  $\bullet$ —, M09TR270. Experimental data:  $\circ$ , Tanna (M09TR086);  $\bullet$ , Tanna (M09TR270);  $\diamond$ , Mollo-Christensen et al. (1964);  $\nabla$ , Stromberg et al. (1980).

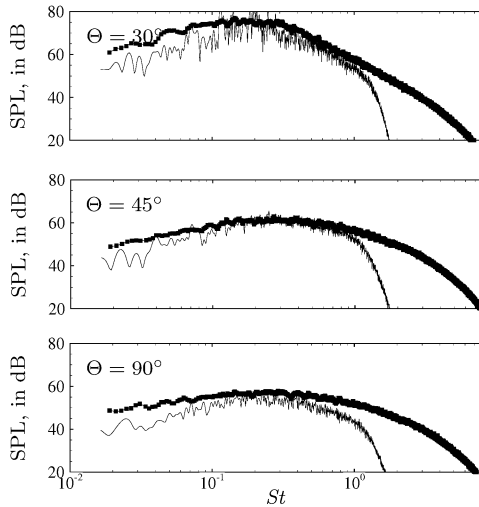


Figure 8: Narrow band spectra the radiated noise for jets with  $Ma = 0.9$ . The far-field data has been extrapolated to a distance of  $160D_j$ . From the LES study by Bodony & Lele (2004). Legend: —, present LES;  $\text{---}\blacksquare\text{---}$ , Viswanathan (2004). The data are scaled to a common distance of  $160D_j$ . In the middle figure, experimental data at  $\Theta = 50^\circ$  has been used.

formula for the frequency of screech tones with good agreement with data (see Raman, 1998). However, predictions of screech amplitude, its directivity and the nonlinear stalling phenomena are not available (Tam, 1995). Developing methods for noise predictions in flows involving a resonance, including jet screech and cavity flows, is an area of active research.

Shock-associated noise is typically most intense in the direction upstream to the jet and exhibits spectral bumps with peak frequency increasing with the inlet angle  $\chi$ . Harper Bourne &

Fisher (1974) explained these features using a phased-array of simple noise sources located at the end of each shock-cell each phased according to the convective time delay for a turbulent eddy to pass over each shock-cell. The Lagrangian correlation time limits the spatial coherence of the noise sources. Tam & Tanna (1982) noted that a phased-array of simple sources predicts noise radiation at harmonically related tones<sup>6</sup> which are not observed, and proposed a distributed-source model for the shock-cell noise generation. In their model the turbulent motions are regarded as stochastic instability waves (view-point B). Its non-linear interaction with the wave-guide modes (representing the shock-cell structure) radiates sound. This interaction implies flow disturbances with upstream-directed supersonic phase speed and results in upstream-directed Mach waves. Specific frequencies radiate preferentially at particular angles from the inlet axis. This basic model was given a firm mathematical foundation in later work (Tam, 1987), who also gave a semi-empirical formula for predicting the shock-cell noise, and showed its effectiveness with detailed comparisons to the narrow band shock-cell noise measurements by Norum & Seiner (1982) and others. This semi-empirical model has been refined further (Tam, 1990, 1992) and represents the present state-of-the-art in shock-cell noise prediction.

Despite its success some significant limitations exist in using this model. The noise prediction is based on an *assumed* spatio-temporal distribution of near-field pressure disturbances which are supposed to result from a nonlinear interaction between the large-scale instability waves of the jet and its shock-cell structure. The former is modeled with a Gaussian wavepacket shape and the latter is an empirical modification of the Prandtl-Pack solution. In this sense Tam's prediction is based on a *model* of the near-field pressure fluctuation; it does not model the sound sources. As a result, further extensions of this semi-empirical formula<sup>7</sup>, to account for co-annular or other nozzle configurations are difficult, and the semi-empirical near-field pressure model leaves unsettled questions about the scaling of shock-cell noise.

In principle LES can be used to study the noise-generation mechanisms in an imperfectly-expanded jet, but this is computationally demanding. Besides representing the multi-scale jet turbulence, the shock-cell structure which involves steep gradients in the early jet, would also need to be accurately captured. Such a calculation is yet to be attempted, but detailed study of related model problems (Lui and Lele 2002, 2003) has provided insights into improving shock-cell noise prediction methods.

## SOUND GENERATION IN JET SCREECH

Shen & Tam (1998) used a hybrid method to study jet-screech. They solve the jet 'mean' flow using the unsteady RANS approach, with the  $k-\epsilon$  equations, and the Euler equations are solved in the exterior region. The nozzle geometry is retained with a multiple block mesh with the highest resolution in the near-nozzle region. The calculations which use DRP scheme aim to predict the amplitude and directivity of screech tones. This type of URANS approach can be justified when the spatial and temporal scales of the dominant acous-

<sup>6</sup>As noted by Harper Bourne & Fisher (1974) the spectral coherence of the source-array controls the relative level of these tones.

<sup>7</sup>As noted already by Tam (1987, 1990), the spectral peaks predicted by the formula for small inlet angles are too narrow.

tic waves *and* their ‘source processes’ are larger than those scales of the turbulence. The former condition is easily satisfied but the latter holds only marginally for the observed screech tones. URANS has also been used in other resonant acoustics phenomena such as supersonic cavity tones, Zhang et al. (1995). In recent work Shen & Tam (2002) have extended this approach to non-axisymmetric modes and shown that the mode-switching phenomenon observed in screech from circular jets is reproduced. This raises expectations that a simple dynamical systems model could predict jet screech behavior, also see Walker & Thomas (1997).

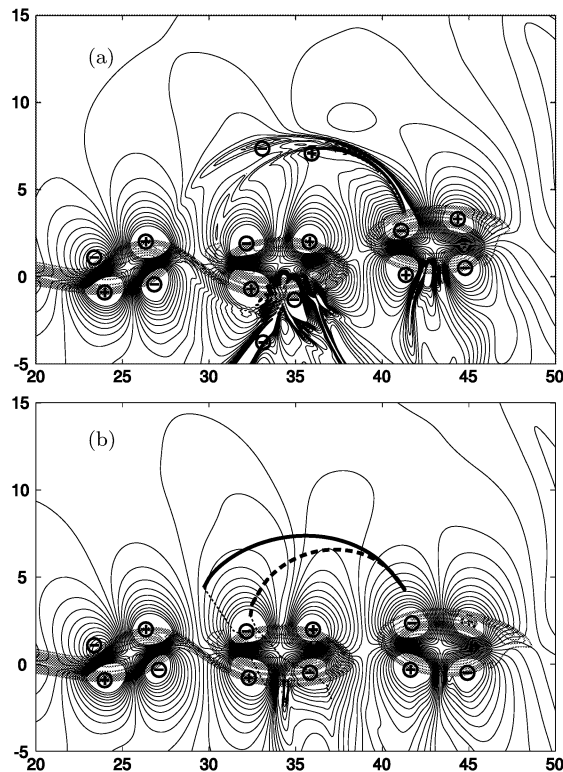


Figure 9: A snapshot of results from screech model problem. a) DNS result; b) Geometrical acoustics result. Solid lines show contours of dilatation whose sign is marked. Also overlaid are contours of vorticity at the same instant. From Suzuki & Lele (2003)

A similar suggestion that the sound generation process in jet screech can be modeled with simpler models has emerged from DNS studies. Manning & Lele (2000) studied an isolated screech-noise source region in a shear layer. Inflow boundary conditions were used to specify instability-wave eigenmodes which grow downstream and form large-scale vortices. Boundary conditions were used to independently specify a stationary compression wave in the supersonic stream. The interaction of the vortices with the compression-expansion wave generates sound. Upstream boundary conditions ‘absorb’ the upstream travelling sound and thus suppress the re-excitation of the shear-layer. An outflow zone allows large-scale vortices and sound to travel out of the domain without a significant reflection. The numerical boundary conditions are quite challenging and require a careful validation (Manning & Lele, 1998). The snapshots from DNS illustrate the sound generation process.

As shear-layer vortices convect past the interaction location, the tip of the compression wave oscillates significantly. During the time when the so-called braid region passes over the compression wave tip, a part of this wave is observed to leak across the shear-layer towards the ambient region initiating a sharp cylindrical compression wave. Refraction of this wave back into the supersonic flow can also be observed. This is an upstream travelling Mach wave within the supersonic flow.

The DNS data shows that the radiated sound pressure level is proportional to the ‘local’ pressure rise in the shock-cell structure at the noise-source location. This is in agreement with experimental data on broadband shock-associated noise which shows a proportional scaling. The linear scaling is further verified by a separate calculation using the linearized Euler equations (LEE) about the non-linear unsteady flow of the shear layer. Primary features of the radiated signal observed in DNS agree very well with the LEE calculation. The radiated sound level is, however, *not* necessarily proportional to the amplitude of the unsteady disturbances in the shear layer. An amplitude threshold is observed below which the radiated sound is proportionally weak, but above it strong radiation occurs. As the shear-layer disturbances grow, they form distinct vortices (clumps of vorticity) and braid regions between them. Using Stuart’s solution (Stuart, 1967, wherein an amplitude parameter  $A$  controls vorticity clumping) of the (incompressible) Euler equation as a baseflow, it was found that a vortex-laden mixing layer also shows the same thresholding behavior as the full DNS.

The model of geometrical acoustics to track ray trajectories and wavefronts through the unsteady baseflows of the vortex-laden mixing layer offers an explanation (Manning & Lele, 2000) of the DNS results. The initial ray direction corresponds to the stationary Mach waves in the supersonic stream. For small  $A$  all rays are reflected back towards the supersonic flow, but as  $A$  increases a small window of rays penetrate across the mixing layer; the onset of transmission occurring around  $A \approx 0.54$ . The size of the ray bundle which is transmitted above this threshold grows initially but saturates at high  $A$ ; this limits the maximum transmission across the mixing layer as observed in full DNS. In a recent study Suzuki & Lele (2003) have provided a mathematical justification of the ray-acoustics limit for the jet-screech problem. They show that the problem of propagation of a weak shock, with a steep gradient, in a vortex-laden mixing layer is analogous to the ray-tracing approach of high-frequency geometrical acoustics. Taking this approach to the next order, the amplitude of the radiated sound can be satisfactorily predicted. An example of this is shown in figure 9. The prediction of the radiated shock front agrees closely with the DNS observation. Recent measurements (Alkisar et al., 2003) in a screeching jet have provided an unprecedented detail of the unsteady jet flow during the screech cycle. These data support the notion that vorticity clumping in the jet shear layer plays a major role in determining the screech amplitude.

#### BROADBAND SHOCK-CELL NOISE GENERATION

Recent numerical simulations have also given new insights into the broadband shock-cell noise generation. The turbulent eddies in the early jet have a short life time, thus each interaction with the shock-cell structure behaves as an isolated source region. Lui & Lele (2002, 2003) have conducted DNS studies of noise generation from such an ‘isolated noise

source'. Figure 10 shows a visualization of the noise radiation from the interaction of a turbulent shear layer with an isolated shock-cell. The cylindrical waves originating from the interaction region are attributed to this interaction. This radiation is approximately omni-directional with an apparent origin somewhat downstream of the interaction site. A detailed study of the noise-generation (Lui & Lele, 2003) shows that the observed sound is generated closer to the supersonic edge of the shear-layer consistent with the downstream shift in the apparent source. The magnitude of the radiated pressure was found to scale with the imposed pressure change in the incident compression wave, a scaling observed in experiments. Interestingly, the spectral peak of the radiated noise corresponds closely with the spectral peak of the turbulence in the shear-layer at the interaction location. This can be observed in figure 11 (a)-(b) which shows the frequency spectrum of TKE near the interaction location and the (noise) pressure spectrum at an upstream observer point, respectively. This coincidence is significant since the elevated spectral level of shock-cell noise typically extends to frequencies much higher than the mixing-noise peak at  $\chi = 90^\circ$ . First few shock-cell shear-layer interactions can potentially generate noise at these high frequencies. A theoretical model which captures many features of the DNS results has also been developed recently (Lele, 2003). This model provides analytical predictions for the noise radiation from the interaction of shear-layer disturbances with an isolated shock-cell. Its formulation draws from previous theoretical work by Kerschen & Cain (1995) and Tam (1987). The noise source terms associated with the shock shear-layer interaction are reformulated using generalized functions and the result is simplified by appeal to observations from DNS. The radiated noise depends on the local shock and turbulence properties; the model shows the streamwise (turbulent) velocity fluctuation,  $\frac{u'_j}{U_j}$ , and the local (shockcell) pressure amplitude  $\Delta p$  to be controlling variables.

It also explains why shock-cell noise is independent of the jet velocity (for a given pressure ratio). This remarkable property of heated jets appears to be without a rational explanation in the published literature! The explanation found is very simple (Lele, 2003). The shock-cell noise sources, for example the unsteady stress-fields due to the interaction, are proportional to the local (mean) jet density and are bilinear in the turbulence velocity  $u_i^{(t)}$  and shock-cell-associated velocity disturbance  $u_i^{(s)}$ . For small amplitude disturbances the latter (Mach waves) is proportional to  $\Delta p$  but is also inversely proportional to the local (mean) jet density. If the local turbulence intensity  $u_i^{(t)}/U_j$  is insensitive to the jet temperature, the radiated shock-cell noise is proportional to  $\Delta p$  and independent of  $U_j$  and  $T_j$ , etc. This simple scaling property is shared by a more complete shock-cell noise model which accounts for the spatially extensive noise-sources. This simple scaling can be contrasted with the empirically determined amplitude factor in Tam's formula (Tam, 1990) which shows good agreement with data but with no explanation offered for the specific empirical function used. The new model can provide useful extensions of the existing shock-cell noise prediction methods, and efforts along these lines are underway.

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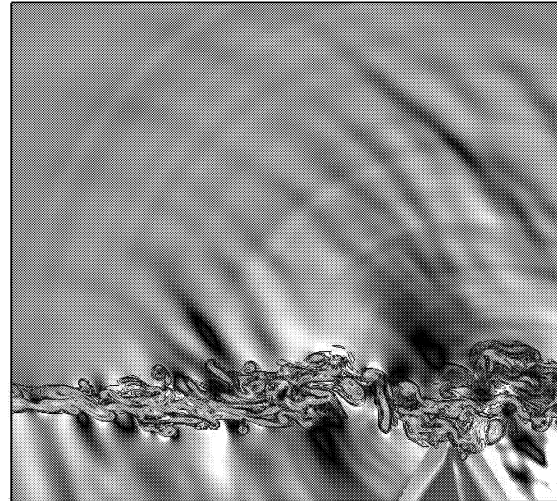


Figure 10: Visualization from DNS of the interaction of a compression wave with a turbulent shear layer. Contours of vorticity magnitude show the shear layer and the compression wave and its reflection are observed in the pressure contours (triangle shape). Contours of dilatation in gray-scale show the sound field. The noise generated by the interaction of turbulence with the compression wave appears as weak cylindrical waves.  $M_1 = 1.2$ ,  $M_2 = 0.0$ ,  $\Delta p/p_1 = 0.2$  Figure 3 from Lui & Lele, 2002.

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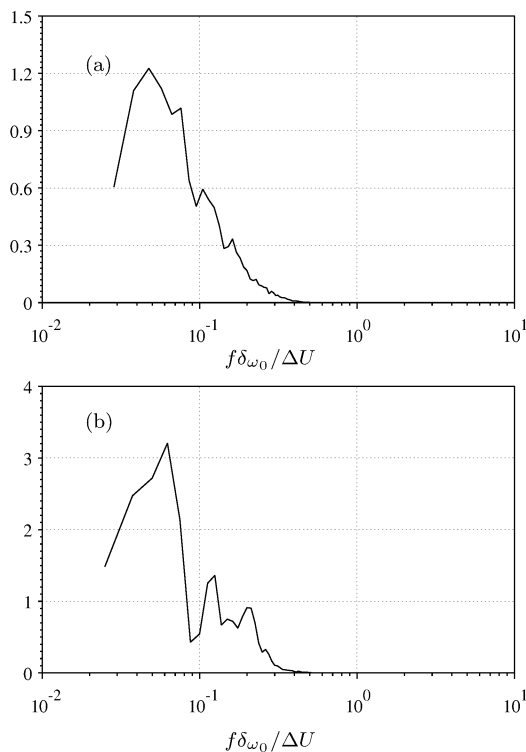


Figure 11: (a) Frequency spectrum of  $u_1'$  near the interaction location and (b) spectrum of the radiated noise (pressure) measured at approximately  $45^\circ$  from the upstream direction. Both spectra are pre-multiplied by the frequency  $f$ . From Lui & Lele (2003)

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