ON STREAMWISE VORTICITY GENERATION IN COMPLEX TURBULENT JETS

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ABSTRACT

Budgets of the time-averaged stream-wise vorticity equation are examined in high Reynolds number 3D jets with complex cross-sections. Detailed results are presented from LES data of a rectangular jet. Experimental data are presented for elliptic and triangular jets, which issued from sharp orifices. The results show that the anisotropy of the turbulent stresses play significant roles in the evolution of the jets, in particular, in the axis-switching phenomena.

INTRODUCTION

Experiments (Tsuchiya and Horikoshi 1986, Quinn 1989) have shown that three-dimensional (3-D) jets can be used to enhance mixing and entrainment rates in comparison to axi-symmetric jets. A fundamental understanding of the dynamics of complex, turbulent jets is required for their prediction and control. Zaman (1996) used streamwise and azimuthal vorticity dynamics to explain the presence or absence of axis-switching in experiments on rectangular jets with different initial conditions. This study showed that the presence of stream-wise vorticity pairs with outflow rotation (pumping fluid from the core to the ambient perpendicular to the major axis plane) produced axis switching while pairs with the opposite sense of rotation did not. However, in jets with no streamwise vorticity at discharge some other mechanism must originate it. Hussain and Husain (1989) have explained that vortex self-induction which is an inviscid mechanism could be responsible. Wilson and Demuren (1998) used direct and large eddy simulation data to show the separate effects of self-induction and turbulence field anisotropy in evolution of rectangular jets.

In the present study, experimental data and numerical simulations of complex jets, issuing from orifices with rectangular, elliptical, triangular and round cross-sections, respectively, are analyzed. The generation and evolution of secondary motion and mechanisms, which produce axis-switching in 3D jets, are determined through direct computations of budgets of terms in the streamwise vorticity equation.

STREAMWISE VORTICITY GENERATION

The mechanism for streamwise vorticity generation can be examined by considering the time-averaged streamwise vorticity equation. Following Demuren and Rodi (1984), the stream-wise vorticity equation can be written, in Cartesian tensor notation, as:

$$\underbrace{U_i \frac{\partial \omega_1}{\partial x_i}}_{A_1} - \underbrace{\omega_i \frac{\partial U_1}{\partial x_i}}_{A_2} = \underbrace{\frac{\partial}{\partial x_1}\left(\frac{\partial \overline{u_1 u_2}}{\partial x_3} - \frac{\partial \overline{u_1 u_3}}{\partial x_2}\right)}_{A_3} + \underbrace{\frac{\partial^2}{\partial x_2 \partial x_3}\left(\overline{u_3^2} - \overline{u_2^2}\right)}_{A_4} - \underbrace{\left(\frac{\partial^2}{\partial x_3^2} - \frac{\partial^2}{\partial x_2^2}\right)\overline{u_2 u_3}}_{A_5} + \underbrace{\frac{1}{Re_D}\left(\frac{\partial^2 \omega_1}{\partial x_j \partial x_j}\right)}_{A_6}$$

The A_1 terms represent the convection of streamwise vorticity by the mean velocity while the A_2 terms represent the tilting and stretching of the vorticity vector by gradients of the mean velocity. Terms A_3, A_4, and A_5 contain the turbulent stresses and act to produce or destroy streamwise vorticity. In particular, terms A_4 and A_5 contain the effect of the turbulent secondary normal and shear stresses, respectively. In the study of turbulence-generated secondary flow in square ducts, Demuren and Rodi (1984) found these two terms to be dominant. The A_3 term, which involves gradients of the primary shear stresses did not play any significant role. The diffusion of streamwise vorticity is given by the terms, A_6. The generation of streamwise vorticity through vortex stretching and tilting (terms A_2) is known as secondary motion of Prandtl's first kind. This is an inviscid mechanism, which may be present in laminar as well as turbulent flows. Generation of stream-wise vorticity by gradients of the turbulent stresses (terms A_3, A_4, and A_5) is known as secondary motion of Prandtl's second kind. In turbulent jet flows, both mechanisms may be present.

RESULTS AND DISCUSSION

The dominant mechanism for secondary motion production and the consequent jet evolution can be obtained from budgets of terms in the mean streamwise vorticity equation, which were computed by post-processing the instantaneous results from experimental data and LES of the different jet flows. Figure 1 shows results for the LES of a rectangular jet at 3 cross-sections. The jet issued from a contoured nozzle with an exit Reynolds number of 75,000. There was negligible streamwise vorticity and hence no secondary velocity at the exit plane. Cross-sectional contours of the A_2, A_4, A_5, and the $A_4 + A_5$ terms, at $x/D_e = 3$, along with the mean streamwise vorticity and velocity show that eight pairs of streamwise vortices are present and their locations and signs are consistent with that of the $A_4 + A_5$ term. Hence, it is shown that gradients of the difference of the turbulent normal stresses are responsible for the initial generation of streamwise vorticity. Contour patterns of streamwise vorticity and of the A_4 term show little bias towards the major or minor axis planes, while contours of the $A_4 + A_5$ term begin to show an anisotropic pattern. Further downstream, the sense of rotation of the vortex pairs about the minor axis plane is such that core velocity fluid is pumped from the jet centerline to the ambient leading to a distortion of the initially rectangular mixing layer. The four pairs of streamwise vortices oriented along the minor axis strengthen until the end of the potential core where the initially flat major axis sides develop a bulge and the corners are flattened leading to the observed diamond shape at $x/D_e = 5.25$ and the eventual switching of the major and minor axes. That process is shown to be complete by $x/D_e = 8.25$. Comparison of the structures of the A_4, A_5 terms and the streamwise vorticity suggests that the A_4 term acts largely as a source and the A_5 term as a sink, and the difference between them produces the streamwise vorticity. The A_2 and A_3 terms are the most insignificant. The inviscid mechanism, based on the Biot-Savart law or A_2 plays no significant role in this case.

Figure 2 shows experimental data for an elliptic jet, which issues from sharp-edged orifices at an exit Reynolds number of 100,000. In this case, there is significant streamwise vorticity, and hence secondary flow, at the exit plane. Streamwise vorticity contours are presented at 3 cross-sections, along with the corresponding gradients of the secondary normal stresses, the A_4 term. The secondary shear stress is extremely difficult to measure accurately, so that data is not available and the A_5 term cannot be calculated. At $x/D_e = .25$, there is a well-organized streamwise vorticity field around the jet and a correspondingly strong A_4 distribution. The evolution of the jet occurs much more rapidly, in comparison to that in Fig. 1, such that by $x/D_e = 1.25$, the jet has become almost circular, and by $x/D_e = 2$, the jet major and minor axes have almost completely switched. Throughout the process, the sense of the A_4 term is consistent with that of the dominant mechanism for the observed evolution as discussed for the rectangular jet. The lack of complete data in the experimental study prevents a definitive conclusion.

Figure 3 presents experimental data for an equilateral triangular jet, which issues from a sharp-edged orifice at a Reynolds number of 100,000. As in the elliptical jet case, there is strong organized streamwise vorticity at the jet exit, and the evolution of the axis-switching phenomenon occurs fairly rapidly. By $x/D_e = 1$, the jet is almost circular in shape, and by $x/D_e = 2$, the vertices are inverted. Again in this case, the A_4 term is consistent with the dominant mechanism responsible for this evolutionm

CONCLUDING REMARKS

Origin of secondary flow or streamwise vorticity for different jet cross-sections have been investigated through budgets of the mean streamwise vorticity derived from the LES data and experimental data. In turbulent jets, streamwise vorticity is generated by terms involving derivatives of the secondary Reynolds normal and shear stresses. Detailed LES data have been used to illustrate this process and how it leads to axis switching in complex turbulent jets. These conclusions are supported with experimental data, though these are not as complete or detailed as the LES data.

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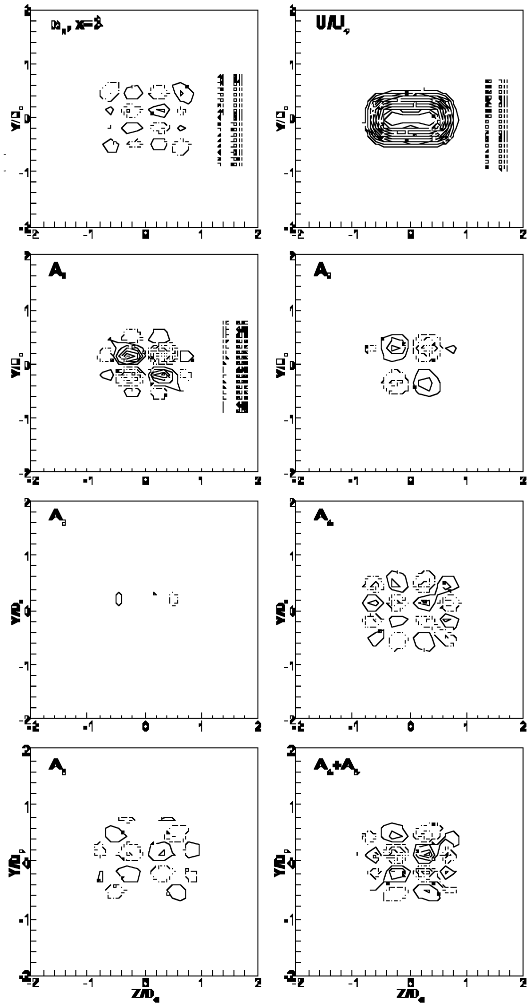

Figure 1: LES results for rectangular turbulent jet at Reynolds number of 75,000, (a) x/D=3.

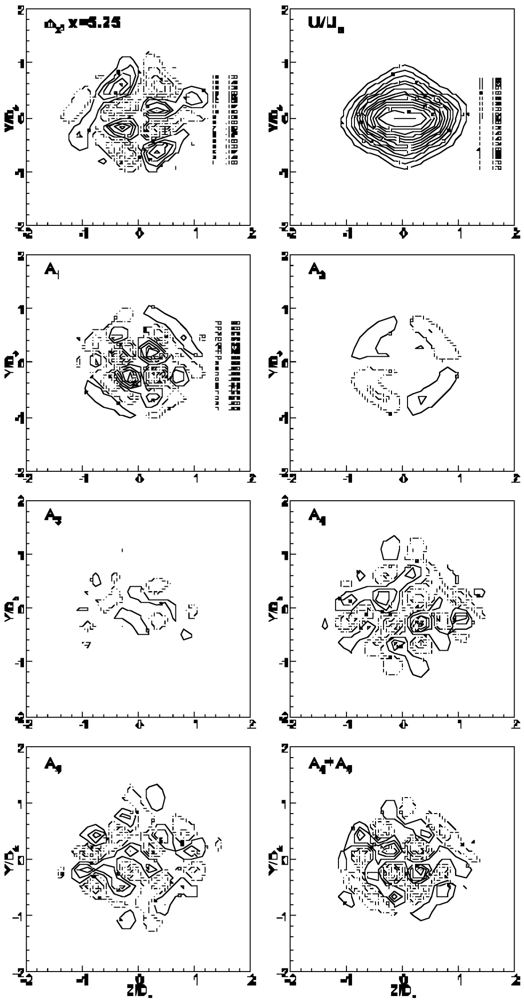

Fig1 (b) x/D=5.25.

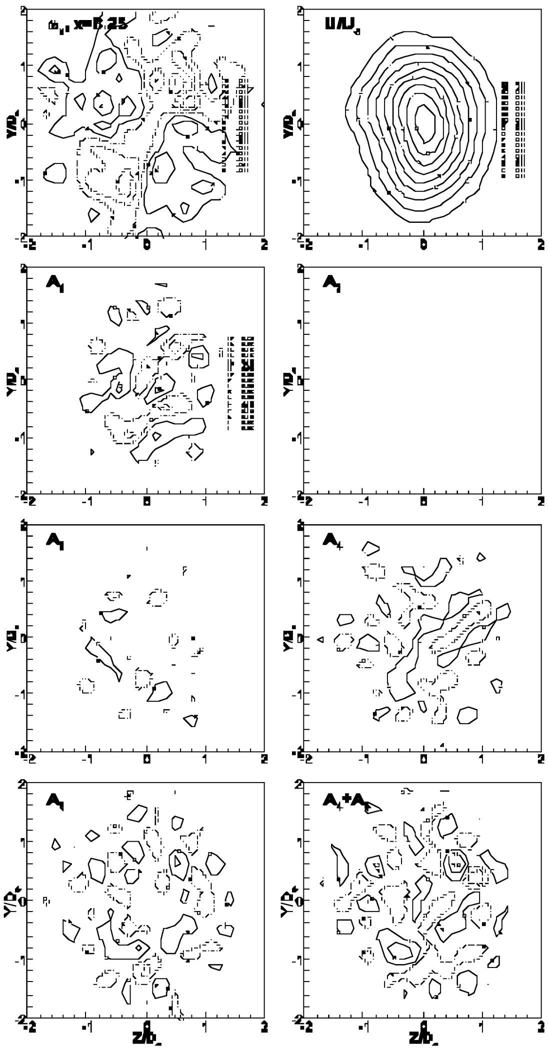

Fig. 1 (c) x/D = 8.25.

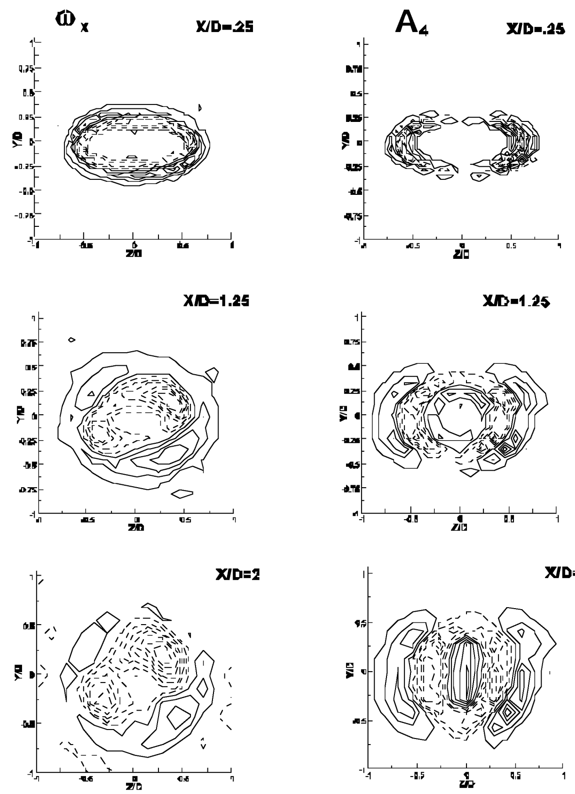

Figure 2: Experimental data of mean streamwise vorticity and gradients of secondary normal stresses for elliptic jet at Reynolds number of 100,000.

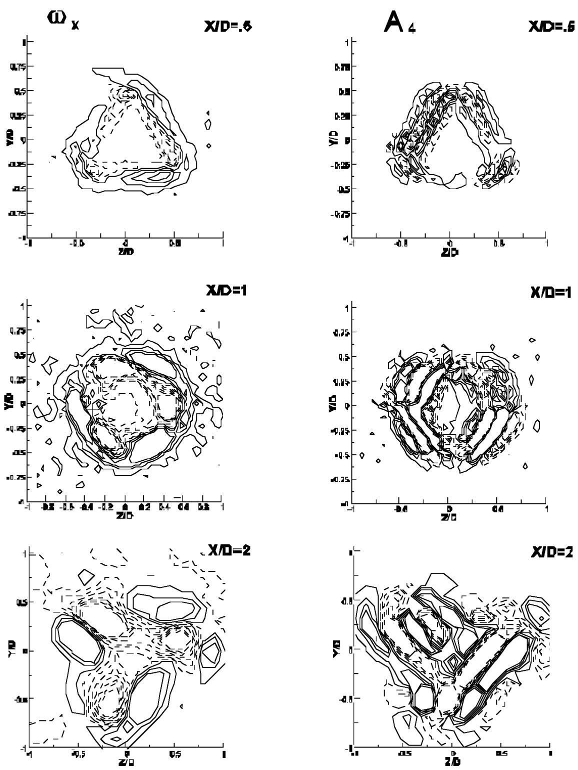

Figure 3: Experimental data of mean streamwise vorticity and gradients of secondary normal stresses for triangular jet at Reynolds number of 100,000.