ON THE INVESTIGATION OF A STRAIN AND ROTATION DEPENDENT SUBGRID-SCALE MODEL USING THE DYNAMIC PROCEDURE

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ABSTRACT

An anisotropic subgrid-scale model with five terms depending on strain and rotation rate is investigated. Single terms and the combinations of two of the five terms are tested in a turbulent channel flow at a turbulent Reynolds number of $Re_\tau = 395$. The model constants, one for each term, are determined dynamically. Some models with a combination of two terms achieved slightly better results than the single term models with about 25% additional computational costs compared to the Germano model.

INTRODUCTION

The most widely used model in large-eddy simulation (LES) is the Smagorinsky model. It is known that this model gives a rather inaccurate estimation of the subgrid-scale (SGS) stresses. Therefore Lund and Novikov (1992) proposed an anisotropic SGS model, which is illustrated below. Parts of this anisotropic SGS model have been employed for the calculation of a turbulent channel flow. Up to two of the five terms are taken into account at a time, because the computation of more than two coupled dynamic constants is not feasible, due to very high computational costs.

Presently, the wall model by Werner and Wengle (1991) is used to allow a coarse grid resolution in the near wall region.

SUBGRID-SCALE MODEL

Lund and Novikov (1992) proposed an anisotropic SGS model based on the assumption that the SGS stress is expressible as a tensor function of the strain and rotation rates and the unit isotropic tensor (δ):

$$\tau_{ij} = f_{ij}(\mathbf{S}, \mathbf{R}, \boldsymbol{\delta}) \tag{1}$$

where the strain and rotation rate tensors are respectively defined as

$$S_{ij} = \frac{1}{2} \quad \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \quad , \tag{2}$$

$$R_{ij} = \frac{1}{2} \quad \frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial \overline{u}_j}{\partial x_i} \quad , \tag{3}$$

with $\overline{u}_i$ as resolved velocity.

Expressing τ_{ij} as an integrity basis (Spencer 1971) of S_{ij} and R_{ij} and neglecting tensor products of order four and higher the model consists of five terms with a distinct model constant for each term:

$$\begin{aligned} \tau^*_{ij} &= C_1\Delta^2|S|S_{ij} + C_2\Delta^2(S_{ik}S_{kj})^* \\ &+ C_3\Delta^2(R_{ik}R_{kj})^* + C_4\Delta^2(S_{ik}R_{kj} - R_{ik}S_{kj}) \\ &+ C_5\Delta^2\frac{1}{|S|}(S_{ik}S_{kl}R_{lj} - R_{ik}R_{kl}S_{lj}). \end{aligned} \tag{4}$$

Here Δ is the grid filter width, $|S| = \sqrt{tr(S_{ik}S_{kj})}$ and $()^*$ denotes the trace-free part of a tensor. Here only the deviatoric part of the stresses are modeled, because it is common practice for incompressible flows to combine the isotropic part with the pressure. In the following the five terms are called term 1, term 2, etc..

Lund and Novikov (1992) did some *a priori* testing on filtered DNS data of homogeneous, isotropic turbulence. The Smagorinsky model (term 1) was found to be the dominant term. Including the remaining terms did not significantly improve the results.

DYNAMIC PROCEDURE

The determination of the model constants $C_1 - C_5$ is done dynamically with the procedure proposed by Germano et al. (1991) with the modification introduced by Lilly (1992). For a reduced model of the form (4) with two terms and two constants

$$\tau_{ij} = C_1\alpha_{ij}(\overline{u}) + C_2\beta_{ij}(\overline{u}) \tag{5}$$

the SGS error is given by:

$$E_{ij}(C_1, C_2) = L_{ij} + C_1M_{ij} + C_2N_{ij}, \tag{6}$$

where

$$\begin{aligned} L_{ij} &= \overline{\overline{u}_i\overline{u}_j} - \overline{u}_i\overline{u}_j, & (7)\\ M_{ij} &= \alpha_{ij}(\widehat{\overline{u}}) - \widehat{\alpha_{ij}(\overline{u})}, & (8)\\ N_{ij} &= \beta_{ij}(\widehat{\overline{u}}) - \widehat{\beta_{ij}(\overline{u})}. & (9) \end{aligned}$$

The hat designates a test filtered quantity. Minimizing E^2_{ij} leads to two coupled equations for C_1 and C_2:

$$\begin{aligned} L_{ij}M_{ij} + C_1M_{ij}M_{ij} + C_2N_{ij}M_{ij} &= 0, & (10)\\ L_{ij}N_{ij} + C_1M_{ij}N_{ij} + C_2N_{ij}N_{ij} &= 0. & (11) \end{aligned}$$

With these two equations the contstants C_1 and C_2 can be determined.
The calculated dynamic constants are highly fluctuating. Due to an assumption in the derivation and numerical stability the constant has to be a smooth function of space and time. Therefore an averaging over homogeneous directions is applied to the parameter. Usually negative values for the model coefficient are set to zero for eddy viscosity type of models (term 1) to avoid negative turbulent viscosity. To allow negative values for the remaining terms, no clipping is applied in this study.

WALL MODEL BY WERNER AND WENGLE

The wall model proposed by Werner and Wengle (1991) is used to allow a rather coarse grid resolution close to the wall. The Basis of this model is the consistency between the wall shear stress and the tangential velocity at the wall nearest grid point. The difference to the wall model of Schumann (1975) is the use of instantaneous quantities instead of time averaged values, which makes it applicable to complex geometries.
The near wall region is divided into two regions. The wall nearest area is the viscous sublayer, for which a linear correlation between $\overline{u}^+$ and y^+ is known. For the ensuing universal region a power law is employed. Designating y as the wall normal direction the wall function is defined as:

$$\overline{u}^+ = y^+ \quad \text{for} \quad 0 \leq y^+ \leq 11.81, \tag{12}$$

$$\overline{u}^+ = a(y^+)^b \quad \text{for} \quad 11.81 < y^+ < 1000, \tag{13}$$

with the parameters $a = 8.3$ and $b = 1/7$.

NUMERICAL ASPECTS

The simulations are accomplished with the finite volume code FASTEST 3D. We use an implicit discretization in time (Crank-Nicholson method) and the spatial discretization is done using a second order scheme. The present test case is a turbulent channel flow at a turbulent Reynolds number of $Re_\tau = 395$. In $x-$ and $z-$directions we use periodic boundary conditions, while at the bottom and top walls no slip boundary conditions are employed. The size of the computational domain is 2π x 2 x π and the grid contains 64 x 32 x 32 points in $x-, y-$ and $z-$directions. In $x-$ and $z-$direction the grid is uniform. In wall normal direction the grid cells are refined using a geometric law. The grid point next to the wall is positioned at $y^+ = 12$. The flow is driven by a constant mass flow.

RESULTS

The results of the simulation are compared to results of the DNS of a channel flow by Moser et al. (1999).

Single term models

Additional computational costs of the single term models compared to the Germano model are negligible.
Figure 1 shows the distribution of the dynamic parameter for all five models. Interesting are the negative values at the second grid point next to the wall. Here the constant of the model term2, which is negative at the other grid points, turns to positive values. This phenomenon is due to the use of a wall model. This behaviour is usually not observed, because

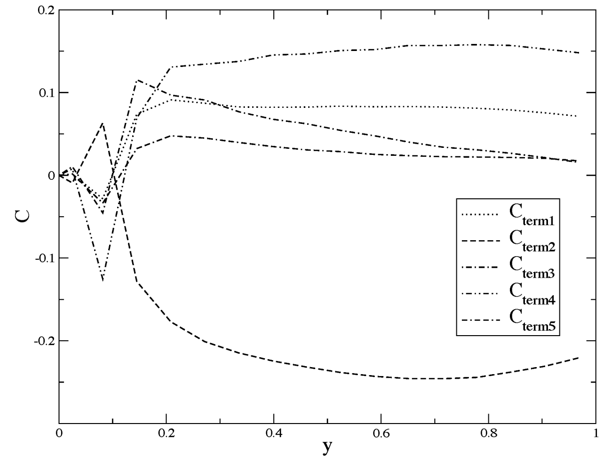

Figure 1: Distribution of the dynamic constant for one term models.

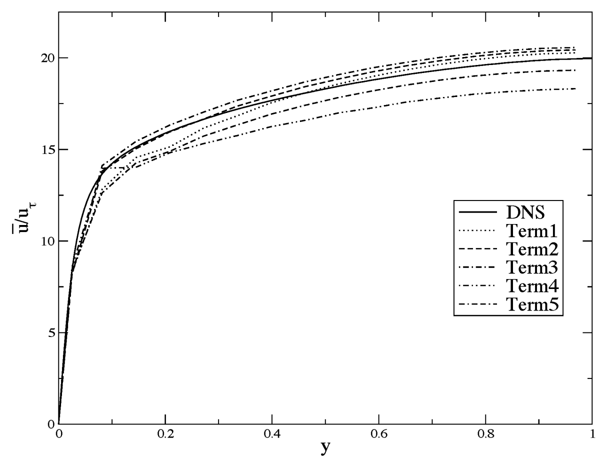

Figure 2: Mean velocity profile in streamwise direction for single term models in comparison to DNS at $Re_\tau = 395$ (Moser et al., 1991).

as mentioned above, negative values are set to zero for eddy viscosity models. The distribution of the dynamic constants and the coarse grid resolution in the near wall region leads to kinks in the distributions of turbulent statistics as one can see in the figures 3-5. Even an impact on the mean flow profile can be observed, shown in figure 2, where also weak kinks appear. For the sake of clarity only the best performing models are plotted. The models term4 and term5 underperformed and hence have been neglected. It can be stated that the models give rather accurate results for the Reynolds stresses in stream- and spanwise direction. The Reynold stresses in wall normal direction are too high near the wall for all models. The weak performance of model term1, which is similar to the Germano model, can be explained with the negative dynamic constants in the near wall region.

Two term models

The permutation of two model terms in equation (4) lead to ten different models. In the following the models are called according to the included terms: term1+2, term1+3, etc. The

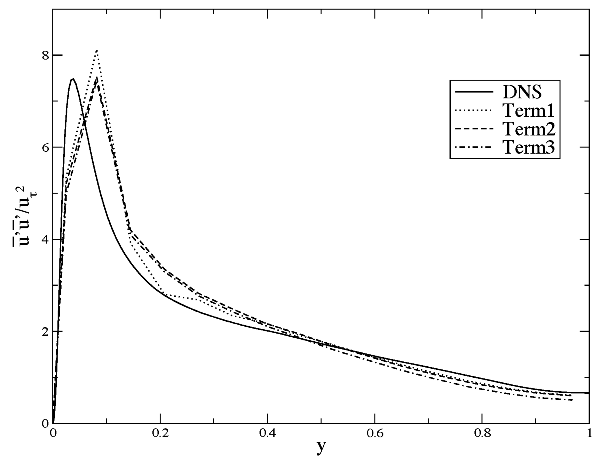

Figure 3: Reynolds stresses in streamwise direction for single term models in comparison to DNS at $Re_\tau = 395$ (Moser et al., 1991).

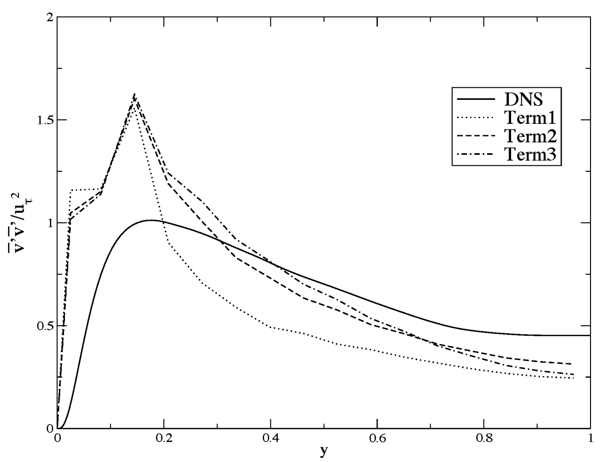

Figure 4: Reynolds stresses in spanwise direction for single term models in comparison to DNS at $Re_\tau = 395$ (Moser et al., 1991).

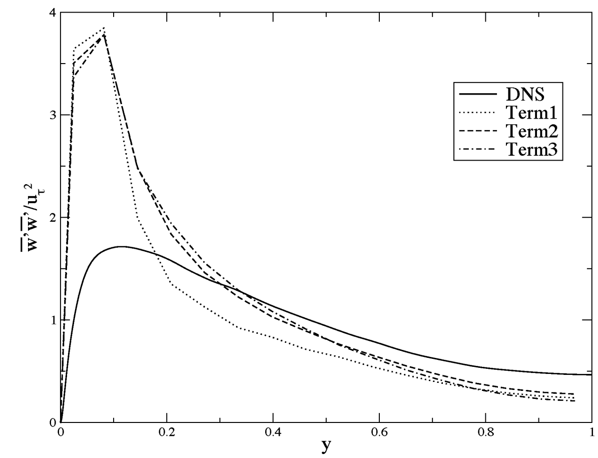

Figure 5: Reynolds stresses in wall normal direction for single term models in comparison to DNS at $Re_\tau = 395$ (Moser et al., 1991).

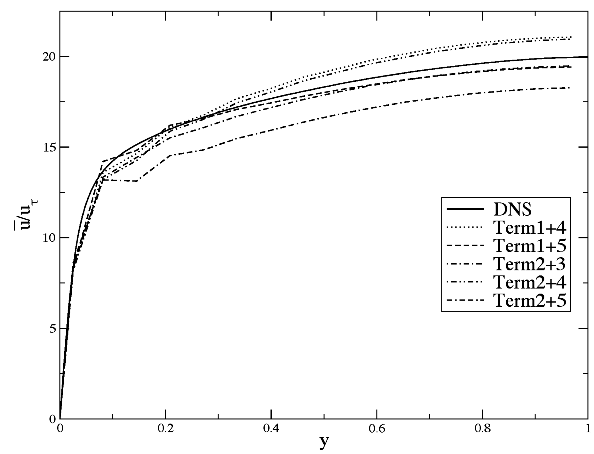

Figure 6: Mean velocity profile in streamwise direction two single term models in comparison to DNS at $Re_\tau = 395$ (Moser et al., 1991).

models with two of the five terms needed additional computation time of about 25% for one iteration compared to the single term model. The reason of these additional costs is the determination of two coupled dynamic parameters.
Simulations of three models (term1+2,term1+3 and term3+4) were numerically instable and diverged. The distribution of the dynamic constants for the two term models also show the phenomenon, which was observed for the single term models. The models including term 5, except the combination of term 1 and term 5, underperformed and were not plotted in the figures 6-9. As one can see in figure 6 the mean velocity profile in streamwise direction of model term2+3 is very close to the DNS data. The total mass flux is slightly too low, but the shape is exactly the same. The other turbulent statistics show good results for this model. Interesting enough, Lund and Novikov found this combination to be worst for a combination of two terms. They tested this model on filtered DNS data of homogeneous isotropic turbulence. They determined the correlation coefficient between the exact and the modeled SGS stresses. Herewith they defined the best and the worst groupings. The differences between the correlation coefficient of the combinations are small for variable model parameter. Compared to the results of the single term models the performance of the model term2+3 was slightly better. The other models performed comparable or worse.

SUMMARY AND OUTLOOK

Parts of the anisotropic SGS model proposed by Lund and Novikov were investigated. Simulations with single term and two term models were done. It was found that the use of the wall model by Werner and Wengle has an influence on the flow. Therefore the dynamically determined model parameters become negative in the near wall region when no clipping is applied. The improvement which can be achieved with two terms as model is rather small. The additional computational cost for this improvement is about 25% compared to the Germano model. The simulations should be done with a wall resolved LES to eliminate the impact of the wall model for a further grounded statement on the model performance.

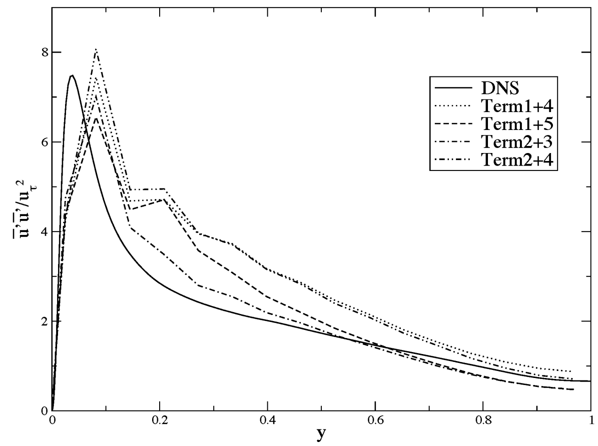

Figure 7: Reynolds stresses in streamwise direction for two term models in comparison to DNS at $Re_\tau = 395$ (Moser et al., 1991).

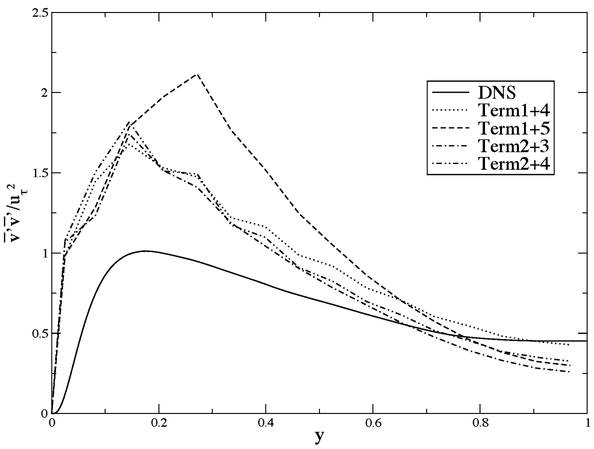

Figure 8: Reynolds stresses in spanwise direction for two term models in comparison to DNS at $Re_\tau = 395$ (Moser et al., 1991).

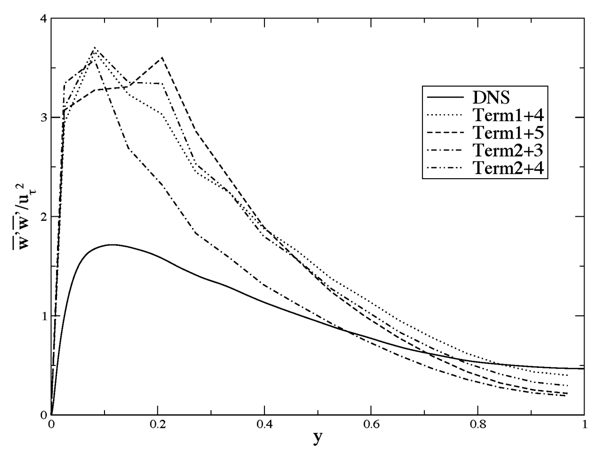

Figure 9: Reynolds stresses in wall normal direction for two term models in comparison to DNS at $Re_\tau = 395$ (Moser et al., 1991).

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