AN ANISOTROPIC DYNAMIC ONE EQUATION SUBGRID SCALE MODEL FOR LARGE EDDY SIMULATION

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ABSTRACT

The main aim of the present work is to develop a anisotropic one equation model based on the subgrid scale (SGS) kinetic energy for the large eddy simulation (LES) and test on a fully developed channel flow with constant spanwise rotation. The present model considers an anisotropic eddy viscosity formulation. The production term is then computed using the local coefficient by using dynamic procedure without any averaging restrictions or clipping. For the prediction of the correct asymptotic behavior near wall an additional modification to the dissipation term is introduced following the RANS low Reynolds number modeling. The proposed model is first tested on fully developed channel flow and compared with some existing models results. Second, efficiency of this model has been demonstrated by the prediction of re-laminarization of grid scale (GS) turbulence on the suction side of the spanwise rotating channel flow.

INTRODUCTION

In Large eddy simulation (LES) the largest scales are resolved numerically, while the unresolved scales must be modeled with a subgrid scale model (SGS). The success of LES depends on how accurately the SGS stresses are modeled. Most commonly used SGS model is Smagorinsky (1963) model. It is recognized that the performance of Smagorinsky based models are improved by using the dynamic procedure relayed on variational methods, (e.g. with a least squares minimization method (1992) or Lagrangian method (1996)). However, when simulating high Reynold number confined flows, the results of the dynamic procedure are of doubtful reliability in the region close to the wall including both the viscous sublayer and the buffer layer as pointed out by Piomelli et. al (1999). The class of similarity models following Bardina et. al. (1983), even though it eliminates the main fundamental inconsistencies of the Smagorinsky dynamic models, has the drawback of being insufficiently dissipative. The mixed models do not sufficiently overcome the common drawbacks. All these models are not capable of predicting backscatter due to various numerical and physical reasons. Nevertheless this problem was overcome by the usage of one equation SGS models. Some approaches have already been proposed in the literature by Menon (1996) and Davidson (1997). These models have notable merits. The eddy viscosity does not become negative anywhere. SGS kinetic energy disappears automatically in non-turbulent region and it becomes zero on the solid wall due to the boundary condition. Moreover, wide variety of factors, such as non-equilibrium properties and additional energy sources or sinks such as particles or bubbles can be included. But these models are not able to predict the anisotropic effects well present in the near-wall regions.

The main aim of the present work is to develop a robust and efficient anisotropic one equation model based on the subgrid scale (SGS) kinetic energy for the large eddy simulation (LES). It has been shown recently by Speziale (1997) that the anisotropy exists at inertial as well as dissipation scales. Although the existing models well account for backscatter, but they are not suitable for the representation of anisotropy of the subgrid-scales due to the local isotropy assumption made in the eddy viscosity formulation. Due to this assumption, these models may lead to wrong prediction of backscatter, and thus of the production of SGS kinetic energy. The latter strongly influences the evaluation of the turbulent viscosity, which plays a dominant role in flow predictions. The present model considers an anisotropic eddy viscosity formulation. The production term is then computed using the local coefficient by using dynamic procedure without any averaging restrictions or clipping. For the prediction of the correct asymptotic behavior near wall an additional modification to the dissipation term is introduced following the RANS low Reynolds number modelling.

First in the present paper, we apply our anisotropic one-equation SGS model to a fully developed channel flow. Results by our model and existing models are compared with DNS database from AGARD test case PCH10 (1998). Second, the realizability of effect of Coriolis force in the rotating channel is examined. Especially in the Suction side of rotating channel, where Smagorinsky model gives SGS turbulence due to the mean velocity gradient even if GS flow is almost re-laminarized.

NUMERICAL PROCEDURE

For our computations, the governing equations are discretised on a block-structured boundary- fitted collocated grid following the finite-volume approach. Spatial discretisations are 2nd order with flux blending technique for the convective terms. The solution is updated in time using 2nd order accurate implicit Crank-Nicolson scheme. A SIMPLE type pressure correction is used for pressure-velocity coupling. The resulting set of linear equations is solved iteratively. Details of the method can be found in the paper by Mengler (2001)

For the incompressible and constant density flows considered here, the basic governing equations are the grid filtered continuity and Navier-Stokes equations.

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u}_i \overline{u}_j\right) = -\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \qquad (2)$$

where the overbar denotes a filtered variable. The effect of the unresolved subgrid scales is represented by the SGS stress

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j \qquad (3)$$

In the Smagorinsky model, the anisotropic part of the SGS-turbulent stress, τ_{ij}^a, is related to the resolved strain-rate tensor $\overline{S}_{ij}$ by

$$\tau_{ij}^a = -2(C_s \overline{\Delta})^2 \left(2\overline{S}_{mn}\overline{S}_{mn}\right)^{1/2} \overline{S}_{ij} \qquad (4)$$

where C_s represents the Smagorinsky constant.

Following Germano *et al* (1991), one introduces a test scale filter represented by a tilde. The purpose of doing this is to utilize the information between the grid- and test-scale filters to determine the characteristics of the SGS motion. The Smagorinsky constant can be then calculated dynamically using the following expression (5):

$$C_s^2 = \frac{L_{ij} M_{ij}}{M_{ij} M_{ij}} \qquad (5)$$

where $$M_{ij} = -2\tilde{\overline{\Delta}}^2 \left|\tilde{\overline{S}}\right| \tilde{\overline{S}}_{ij} + 2\overline{\Delta}^2 \left|\overline{S}\right| \overline{S}_{ij} \qquad (6)$$

$$L_{ij} = \widetilde{\overline{u}_i \overline{u}_j} - \tilde{\overline{u}}_i \tilde{\overline{u}}_j \quad or \quad = T_{ij} - \tilde{\tau}_{ij} \qquad (7)$$

$$T_{ij} = \widetilde{\overline{u_i u_j}} - \tilde{\overline{u}}_i \tilde{\overline{u}}_j \qquad (8)$$

$\tilde{\overline{\Delta}}$ is a test filter width and $\left|\tilde{\overline{S}}\right| = \left(2\tilde{\overline{S}}_{mn}\tilde{\overline{S}}_{mn}\right)^{1/2}$.

Different dynamic procedures can be applied to compute Eq. (5). Among these the Lagrangian dynamic model from Meneveau (1996) is geometry independent and hence can be used for the complex geometries.

In general one equation model is based on the transport equation of SGS kinetic energy given by

$$\frac{\partial k_{sgs}}{\partial t} + \left(u_j k_{sgs}\right)_{,j} = \left(\left(\upsilon_t + \upsilon_l\right)\left(k_{sgs}\right)_{,j}\right)_{,j} - \tau_{ij} S_{ij} - \varepsilon \qquad (9)$$

where

$$\nu_t = C\Delta\sqrt{k_{sgs}} \quad (10)$$

In the Eq. (9) the second and third terms represents production, dissipation.

In the present approach production term can be expressed as

$$P_{ksgs} = -\tau_{ij}^{a} S_{ij} \quad (11)$$

where

$$\tau_{ij}^{a} = -\left(\nu_{ik}\overline{S}_{kj} + \nu_{jk}\overline{S}_{ki}\right) \quad (12)$$

Here one can observe that we are using tensorial eddy viscosity instead of scalar eddy viscosity. The main reason behind this expression is to include the anisotropic effects in the prediction of the production of SGS kinetic energy. Tensorial eddy viscosity is expressed by (Gallerano 2000)

$$\nu_{ij} = C_p\overline{\Delta}\sqrt{k_{sgs}}\,\frac{L_{ij}^{m^a}}{L_{kk}^{m}} \quad (13)$$

where L_{kk}^{m} is the trace of the modified Leonard term, and given by

$$L_{ij}^{m^a} = L_{ij}^{m} - \frac{\delta_{ij}}{3}L_{kk}^{m} \quad (14)$$

which is based on the assumption that the anisotropy of the unresolved turbulence-velocity scales is equal to the anisotropy of the resolved part of the SGS turbulent stress tensor (that is, the modified Leonard term): this assumption is somewhat similar to the similarity hypothesis formulated by Bardina (1983), according to whom a strict analogy exists between the smallest resolved and the largest unresolved scales.

$$\begin{aligned} &C = -\frac{L_{ij}M_{ij}}{2M_{ij}M_{ij}}; \; L_{ij} = \widetilde{\tilde{u}_i\tilde{u}_j} - \tilde{\tilde{u}}_i\tilde{\tilde{u}}_j; \\ &K = \tilde{k}_{sgs} + \frac{1}{2}L_{ii} \\ &M_{ij} = \tilde{\Delta}K^{1/2}\tilde{S}_{ij} - \widetilde{\Delta k_{sgs}^{1/2}S_{ij}} \end{aligned} \quad (15)$$

Dissipation term can be expressed as

$$\varepsilon = C_\varepsilon\frac{k_{sgs}^{1/2}}{\overline{\Delta}} \quad (16)$$

Model constant in dissipation term is evaluated by

$$C_\varepsilon = \frac{F}{G} \quad (17)$$

where

$$\begin{aligned} &F = \upsilon\left(\left\langle\frac{\partial\overline{u}_i}{\partial x_j}\frac{\partial\overline{u}_i}{\partial x_j}\right\rangle - \frac{\partial\overline{\overline{u}}_i}{\partial x_j}\frac{\partial\overline{\overline{u}}_i}{\partial x_j}\right) \\ &G = \left(\frac{K^{1/2}}{\tilde{\Delta}} - \left\langle\frac{k_{sgs}^{1/2}}{\overline{\Delta}}\right\rangle\right) \end{aligned} \quad (18)$$

An additional modification to the dissipation term is introduced following the RANS low Reynolds number modelling

$$\varepsilon^{m} = \varepsilon + \varepsilon_w \quad (19)$$

$$\varepsilon_w = 2\nu\frac{\partial\sqrt{k_{sgs}}}{\partial x_j}\frac{\partial\sqrt{k_{sgs}}}{\partial x_j} \quad (20)$$

Model constant in the production term can be evaluated by using Germano identity from Eq. (7).

$$\tau_{ij}^{a} = -\left(\nu_{ik}\overline{S}_{kj} + \nu_{jk}\overline{S}_{ki}\right) \quad (21)$$

$$\begin{aligned} &-C_p\frac{\tilde{\Delta}\sqrt{K}}{L_{kk}^{T}}\left(L_{ik}^{Ta}\tilde{S}_{kj} + L_{jk}^{Ta}\tilde{S}_{ki}\right) + \\ &\overline{\left(C_p\frac{\overline{\Delta}\sqrt{k_{sgs}}}{L_{kk}^{m}}\left(L_{ik}\overline{S}_{kj} + L_{jk}\overline{S}_{ki}\right)\right)} = \Theta_{ij}^{a} \end{aligned} \quad (22)$$

Θ_{ij}^{a} is resolved turbulent stress tensor and L_{ij}^{T} represents test filter level term.

RESULTS AND DISCUSSION

Numerical simulations were performed first on a fully developed turbulent channel flow at Reynolds number 395, which is based on the friction velocity and half-width of the channel. The two walls of the channel are treated as no-slip boundaries. In the streamwise and spanwise directions the domain is truncated to a finite size and periodic boundary conditions are imposed. For the present case the domain size of $2\pi\delta \times \pi\delta \times 2\delta$ in the streamwise, spanwise and wall-normal has been considered. Simulations are carried out on a coarse grid with cells 64×32×32. An evaluation of the proposed model is performed using DNS data of AGARD test case PCH10 (1998). Results are also compared to that obtained with the Smagorinsky model and its Lagrangian dynamic version. Comparison between obtained normalized mean velocity and normalized Reynolds stress profiles (normal to the wall) are shown in the Fig. 1 and Fig. 2. Both one equation and Lagrangian models predict near wall flow phenomena very well compared to Smagorinsky model.

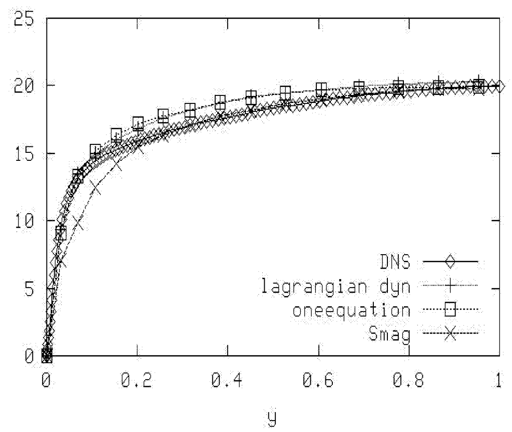

Fig. (1) Comparison between normalized mean velocity profile <u>

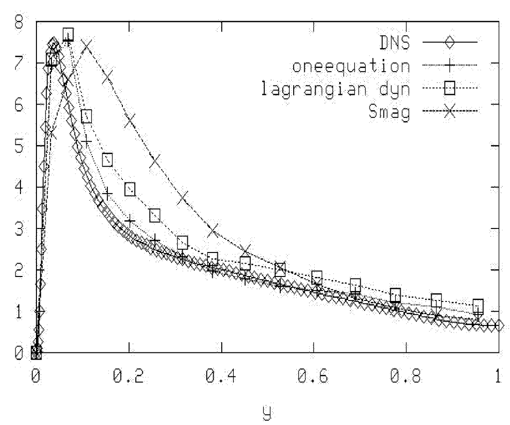

Fig. (2) Comparison between normalized Reynolds stress profile <u′u′>

Fig. 3 shows the comparison between forward and backscatter energy, which are evaluated by using following expression

$$P^{+} = \frac{1}{2}\left(P_{ksgs} + \left|P_{ksgs}\right|\right) \text{ , } P^{-} = \frac{1}{2}\left(P_{ksgs} - \left|P_{ksgs}\right|\right) \quad (21)$$

where P^+ and P^- represents forward and backward scatter. One can observe the significant contribution of backscatter. In Fig. (4) one can observe the variation of the model coefficients normal to the wall. The variation of the model constants in the diffusion and production terms of the SGS kinetic energy equation are similar.

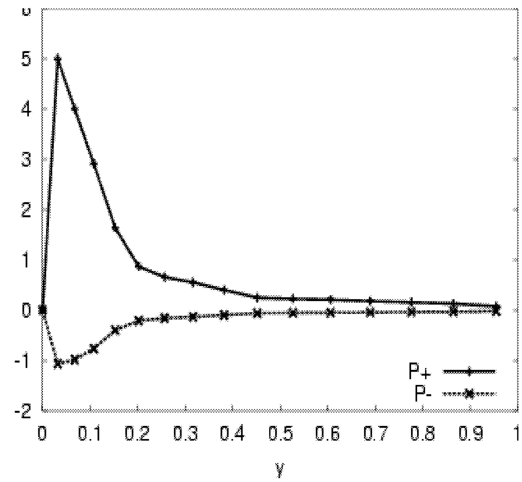

Fig.3 Comparison between forward and backward scatter

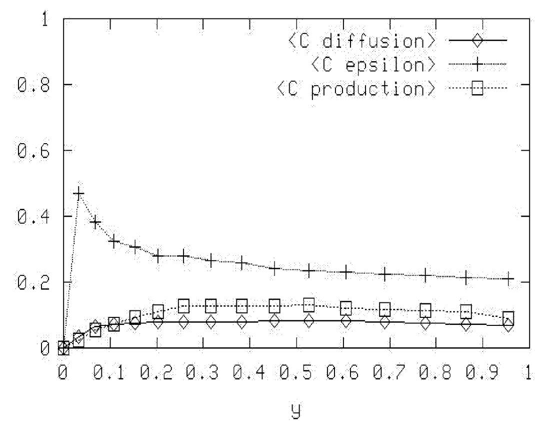

Fig. (4) Variation of the model constants

For the second test case fully developed spanwise rotating channel has been considered. For this case computational domain of $4\pi\delta \times 4\pi\delta/3 \times 2\delta$ in the streamwise, spanwise and wall-normal with grid size of 48×51×64 is considered. Reynolds number and rotation number are 177 and 0.144 respectively, which are based on the friction velocity and half-width of the channel. Results are compared to that obtained with the DNS results of AGARD test case

PCH21 (1998). Comparison between obtained mean velocity and Reynolds stress profiles are shown in the Fig. 5, 6 and 7.

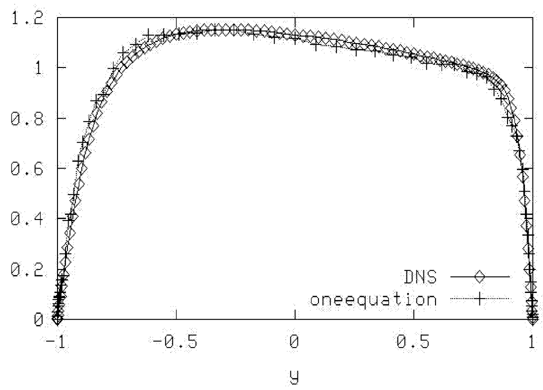

Fig. (5) Comparison between normalized mean velocity profile <u>

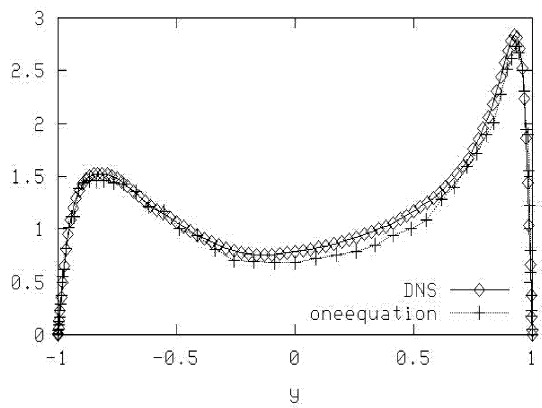

Fig. (6) Comparison between normalized Reynolds stress profile <u′u′>

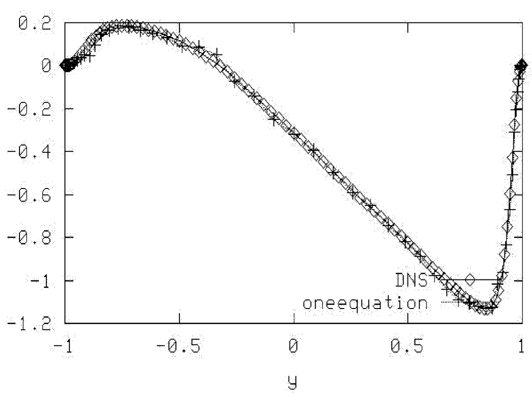

Fig. (7) Comparison between normalized Reynolds stress profile <u′v′>

This comparison clearly demonstrates the advantage of the proposed one equation model in the prediction of the body force.

CONCLUSIONS

New one equation model based on SGS kinetic energy is developed by introducing tensorial viscosity in the production term of the SGS kinetic energy. This model is capable of predicting the anisotropic effects and also predicts backscatter. Due to the RANS low Reynolds number model type correction to the dissipation term one can predict flow behaviour near the wall more accurately.

The test by fully developed channel flow proves the agreement between proposed model and the DNS database even for the coarse grid. The test by rotating channel flow proves the advantage of the proposed model in the prediction of the body force.

In the present model there is no clipping for the model constant in the production term.

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