A HYBRID LES-RANS FOR NEAR-WALL MODELING

Thien X. Dinh
Department of Mechanical Engineering
Ritsumeikan University
1-1-1Nojihigashi, Kusatsu, Shiga,525-8577 Japan
thien@cfd.ritsumei.ac.jp

Yoshifumi Ogami
Department of Mechanical Engineering
Ritsumeikan University
1-1-1Nojihigashi, Kusatsu, Shiga,525-8577 Japan
y_ogami@cfd.ritsumei.ac.jp

John C. Wells
Department of Civil Engineering
Ritsumeikan University
1-1-1Nojihigashi, Kusatsu, Shiga, 525-8577 Japan
jwells@se.ritsumei.ac.jp

ABSTRACT

In this paper we propose a zonal hybrid between large-eddy simulation (LES) and Reynolds-averaged Navier-Stokes equations (RANS) techniques, the latter taking over in near-wall regions. Both the LES and RANS use a one-equation model; the main difference between the two lies in the specification of the length scale. To avoid a sudden jump in this length scale, a transition region between LES and RANS is specified, in which a smoothly-varying weighted average length scale between LES and RANS is used. This approach was applied to a backward-facing step flow with an expansion ratio of 1.5, at a Reynolds number of 5540 based on inlet bulk velocity and the height of the step. These conditions match the experimental conditions of Kasagi and Matsunaga (1995). The backward step flow is a standard test case of massively separated flow. Turbulent structures at the inlet are calculated by channel DNS. The reattachment length is 6.97 step heights, an error of 7.1% compared with Kasagi and Matsunaga's experiment data. The peaks of Reynolds stresses are somewhat underpredicted near the step, but farther downstream they agree well. In contrast, mean stream velocity is well predicted near the step, but far from the step, the mean flow seems to be flexion down. The mean wall-normal velocity component was also agreed well with experimental data.

1. INTRODUCTION

"Direct Numerical Simulation" (DNS) resolves essentially all scales of motion in a turbulent flow, including the small-scale dissipative motions. When feasible, this technique is considered to reliably reproduce natural turbulent flow, in a statistical sense. However, the computational effort required to resolve the dissipative scales limits the applicability of this technique to simple flows at low to moderate Reynolds number. In Large Eddy Simulation, spatially-smoothed equations of motion are solved, and "stresses" generated by smaller scale motions are modeled. This filtering makes LES significantly cheaper than direct numerical simulation (DNS), even as the energy-containing unsteady motions are resolved. The latter attribute makes LES more accurate and reliable than RANS equations for flows characterized by non-equilibrium, three-dimensionality, boundary layer re-laminarization and re-transition, and massive separation.

The main restriction of LES to date appears to be in the application to high Reynolds-number wall bounded flows. Away from wall boundaries of a boundary layer, the requirement that the energy-carrying scales of motion be resolved results in a grid size proportional to the integral scale of motion. Since this is usually a weak function of the Reynolds number, the cost of LES does not depend strongly on the Reynolds number. But for the near-wall region, resolution requirements are qualitatively different, because the important motions scale with the viscous length scales, then computational cost is strongly Reynolds-number dependent. Bagett *et al*. (1997) considered a fluid volume whose size $L^3_{\varepsilon 0}$ is determined by the geometric scales, such as the channel half width. In the neighborhood of the wall the integral length decreases linearly as $L_\varepsilon \propto y$ and eddies remain anisotropic above $\Delta x \propto y$. The number of anisotropic modes in a slab of thickness dy is then $dN \propto L^3_{\varepsilon 0} dy / \Delta x^3$ and their total number is given by the integral

$$N = \int_{y_0}^{\infty} L^3_{\varepsilon 0} dy / y^3 \propto L^3_{\varepsilon 0} / y^2_0 \qquad (1)$$

where y_0 is some inner wall distance that determines the number of modes. In the absence of a good model for anisotropic turbulence, the near-wall motions scale with viscous length scales, we must choose this limit as a fixed number of wall units $y_0 = \nu y^+_0 / u_\tau$. Then the number of anisotropic modes becomes

$$N = \alpha \left(u_\tau L_{\varepsilon 0} / \nu \right)^2 = \alpha \, \mathrm{Re}^2_\tau \qquad (2)$$

which is only slightly lower than the estimate for DNS. Hence, the cost of LES with near-wall motions resolved increases with the square of Reynolds number; that is, the cost increases by a factor of 100 for each decade increase in Reynolds number.

To circumvent the high cost incurred to represent accurately the near-wall eddies, one can bypass the wall layer altogether, and model the effects of these eddies presenting in this region in a statistical sense. Modeling the wall layer saves a huge number of grid points. However, this process introduces empirical parameters, such as the Karman constant. Moreover, the empiricism is stable only for well known quantitatively regions, at least in equilibrium flows, and that fine grid is required in the region in which the empirical parameters is in control.

In recent years, an increasing number of hybrid models are being developed. A detailed overview of hybrid models making LES applicable can be found in the second edition of Sagaut's book (2002, Second Edition). In general, wall-stress models for LES can be divided into two types: equilibrium laws and zonal models. Equilibrium laws are based on the assumption that the dynamics of the wall-layer are universal and can be represented by a general law, such as the law-of-the-wall. The wall stress computed from this general law is applied as the walls boundary condition, instead of the usual no-slip condition. Schumann (1975) applied this kind of wall model in a simulation of a turbulent channel flow. The mean velocity was determined using the logarithmic law of the wall while the mean wall shear stress was determined from the driving pressure gradient. Later, Grotzbach (1987) used the same framework except that the mean velocity was calculated over the plane at a distance from the wall and parallel to the wall and the mean wall shear stress was determined from the logarithmic law of the wall. This is now referred to as the Schumann & Grozbach ("SG") model. Piomelli *et al* (1989) modified the SG model based on the observation of inclined coherent structures along the wall, in which the instantaneous filtered velocity signature is used to predict the instantaneous shear stress at the wall somewhat upstream. The equilibrium laws relax the constraint on the grid size, and have been used with considerable success in simple, attached flows, but rest on a very weak physical foundation. For complex geometries, or in flows without a-priori knowledge of the mean velocity profile, they cannot be easily applied; events fail to predict flows satisfactorily. For instance, For instance, in simulation of a rotating channel flow, the quasi-relaminarization observed one side of the channel (Balaras et al, 1996) could not be predicted by the equilibrium laws. Thus, their value is limited in engineering applications.

Zonal approaches are hybrid RANS/LES methods that use unsteady RANS (URANS) in the near wall region and LES elsewhere, in the so called "outer" flow. The simulation is extended to the wall, where the no-slip condition is still used. In zonal approaches, the explicit solution of a different set of equations in the near-wall layer supplies boundary conditions for the LES, then it is more dynamic than using a single friction law between stress and velocity at the wall-layer edge. A first technique of this type, known as the two layer model (TLM) solves two separate set of equations on two separated grids, while in others a single grids is used and only the turbulence model changes from one region to the other. Priori technique was proposed by Balaras and Benocci (1994) and Balaras *et al.* (1996), and has been practiced in both attached flows, and separated flows with or without knowing priori of separation point, such as high Reynolds number channel flows (Balaras *et al.* 1996), backward step (Cabot, 1996), and trailing edge of an airfoil (Wang and Moin, 2002), with favorably accurate results. The wall layer is solved by the thin boundary layer equations in a fine grid embedded under the coarser LES mesh and so no Poisson equation inversion is required. Hence, despite of the fact that needs to solve the additional equations in a very fine grid in the normal wall direction, this approach significantly reduces the computational cost compared with near-wall resolved LES (LES-NWR). Perhaps the best-known single-grid approach is Detached Eddy Simulation (DES) proposed by Spalart *et al.* (1997) for massively separated flows. DES combines URANS and LES solutions in a single grid, in which URANS is used to simulate the attached boundary layer and LES computes the remains. The most common URANS model employed in DES applications is the Spalart-Allmaras one-equation model (Spalart and Allmaras 1994, or "S-A model"). To switch between the turbulent eddy model and the subgrid scale model, the length scale of the S-A destruction term is taken to be the minimum of the distance to the closest wall and a length scale proportional to the local grid spacing; this ensures that URANS treatment is retained within the boundary layer. In DES, the transition between URANS and LES is seamless because a common equation is used without specification the interface between RANS and LES zones. However, using a simple min cut-off function leads to a discontinuity in the gradient of the length scale that enters the destruction term of the turbulence model.

Based on Dahlstrom and Davidson (2003) work and an idea to overcome the discontinuity in the gradient of the length scales, we propose a hybrid LES/RANS method, which applied a one-equation model to both LES and RANS. Subgrid scales model is based on one equation of SGS by Yoshizawa and Horiuti (1985) and URANS turbulence model is Chien & Patel (1988) one-equation model. This approach is tested by the backward-facing step flow of Kasagi and Matsunaga (1995) for which the expansion ratio of the flow was 1.5, and the flow had Reynolds number of 5540 based on inlet bulk velocity and the height of step. Next the numerical results on the turbulent statistics will be shown. Finally, some conclusions and recommendations for future work will be presented.

2. IMPLEMENTATION

The problem used to test our approach is backward-facing step flow, which is a widely used benchmark problem to evaluate the performance of turbulence models in the prediction of separated flows. The governing equations are filtered equations of conservation and momentum for incompressible Newtonian fluid resulted in the forms

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0 \tag{3}$$

$$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \overline{u_i u_j}}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{\mathrm{Re}}\nabla^2 \overline{u_i} - \frac{\partial \tau_{ij}}{\partial x_j} \tag{4}$$

where the overbar denotes filtered variables, and the extra term τ_{ij} represents the effect of SGS to resolved scales. The additional stress are parameterized by using an eddy-viscosity model

$$\tau_{ij} - \frac{2}{3}k\delta_{ij} = -2\nu_T \overline{S}_{ij} \tag{5}$$

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{6}$$

The eddy-viscosity is determined by a one-equation model which results in following form:

$$\nu_T = C_\nu l_\nu \sqrt{k}$$

$$\frac{\partial k}{\partial t} + \overline{u}_j \frac{\partial k}{\partial x_j} = 2\nu_T \overline{S}_{ij}\overline{S}_{ij} - C_\varepsilon \frac{k^{3/2}}{l_\varepsilon} + \frac{\partial}{\partial x_j}\left((\nu + \frac{\nu_T}{\sigma_k})\frac{\partial k}{\partial x_j}\right) - \varepsilon_\omega \tag{7}$$

in which the LHS contains the unsteady and convective terms, and the RHS contains respectively the production, dissipation, diffusion, and low Reynolds number correction term which is based on the version of $k-\varepsilon$ model (Jones and Launder, 1972, published in Wilcox D. C. 'Turbulence modeling for CFD', second edition)

$$\varepsilon_\omega = 2\nu \frac{\partial \sqrt{k}}{\partial x_j}\frac{\partial \sqrt{k}}{\partial x_j} \tag{8}$$

Equations 7 represent the motion of subgrid scale kinetic energy in LES, and of turbulent kinetic energy in RANS. The other closure constants are referred to one-equation SGS model by Yoshizawa and Horiuti (1985) and the one-equation model turbulence model by Chen and Patel (1988). They are summarized in Table 1; in which y is distance to the nearest wall, and Δ is filter width.

Table 1: Summary of parameters

	Chen & Patel	Yoshizawa
C_ε	1	1.05
C_ν	0.09	0.07
l_ε	$2.495y\left(1-e^{-0.2y\sqrt{k}/\nu}\right)$	Δ
l_ν	$2.495y\left(1-e^{-0.0143y\sqrt{k}/\nu}\right)$	Δ

The length scale in Equation 7 is taken to be a weight averaged between those of LES and RANS $l = \alpha l_{LES} + (1-\alpha)l_{RANS}$ $\quad 0 \le \alpha \le 1$. The value of α is asymptotic to 1 as far from the walls and 0 as near to the wall. This ensures that RANS will be used at the near wall region. As a consequence there is transition region instead of sharp interface, and the eddy-viscosity has smaller values than for RANS. At a distance from the wall which depends on the weight function, the length scale switches from the RANS value to that for LES. In the present investigation, the weight function α is similar to the Van Driest function, $\alpha = 1 - e^{-y+/20}$ $\quad y^+ = \frac{yu_\tau}{\nu}$, where y is distance to the nearest walls and u_τ is friction velocity at the inlet. The LES length scale Δ is set to the cube root of the computational cell volume, $\Delta = (\Delta_x\Delta_y\Delta_z)^{1/3}$.

This paper reports results for the backward-facing step condidered experimentally by Kasagi and Matsunaga (1985). *x*, y and *z* represent streamwise, normal and spanwise directions respectively. Denoting step height by *h*, the computational domain size in each direction is Li=2.5*h* Lx=22.5*h*, Ly=3*h*, Lz=3*h* as shown in Figure 1. The expansion ratio is thus 1.5 and the Reynolds number, based on the step height and mean bulk inlet velocity U_c, is taken to be 5540. The flow is assumed to be spanwise periodic, and turbulent velocities and pressure at the inlet are calculated by DNS of a fully developed periodic turbulent channel. Out flow is standard condition.

The set of governing Equations 3 and 4 are numerically solved by using a second-order accurate finite-difference method on staggered grids. The spatial derivatives are approximated by central finite difference of 2^{nd} order of accuracy. The discretized equations are

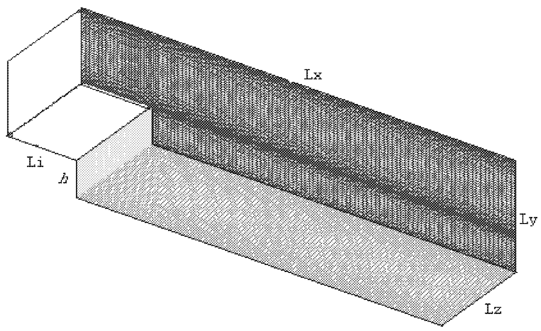

Figure 1 Geometry of the backward-facing step

advanced in time by the SMAC scheme, using the 2^{nd} order Adams-Bashforth method for prediction step. Poisson equation is solved by FFT in the periodic direction and preconditioned Bi-CGSTAB method with tri-diagonal factorization pre-conditioner in the other directions. Then velocity is corrected by pressure gradient to make the field solenoidal. The eddy-viscosity is determined at the pressure nodes. Spatial discretizations and time marching of Equations 7 are the same as those of the governing equations. Non-uniform grid distributions are used in both the streamwise and wall-normal directions, and uniform grid distribution in the spanwise direction. Grids are clustered near the step edges and the walls using tangent hyperbolic functions. The number of grid points used is 164×64×32 in the streamwise, wall-normal and spanwise directions, respectively. In the wall-normal direction, 44 grids are used in the range $y > h$.

3. RESULTS

Figure 2 shows a part of the mean streamlines near the step.

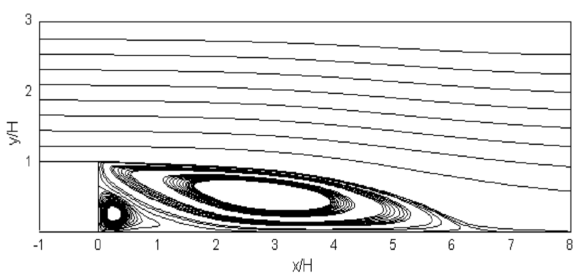

Figure 2 Mean streamlines

There are two recirculation regions as observed in Kasagi and Matsunaga's experiment. The position of the reattachment point determined by zero-average wall stress value is 6.97 step heights from the step. This value gives an error in prediction around 7% compared with Kasagi and Matsunaga's experiment results (x/h=6.51). The length of the secondary recirculation zone in the corner of the step is well predicted, too. Figure 3 shows the mean streamwise and vertical velocity components profiles and in Figure 4 the Reynolds stresses distributions at some different distance downstream. In the separated mixing layer, the Reynolds stresses are somewhat underpredicted compared with experimental data. This cause for this may be unsuitable small filter size for the LES that results from the dense vertical gridding, and corresponding small computational cell volume. In contrast, mean stream velocity is better predicted at near step region. Generally, the mean flow seems to curve downward.

From the results in Figure 5, the flow upstream of the step behaves like a channel flow. Lower predicted Reynolds stress indicated in Figure 6 seems to be cause by high interface location. This fact was concluded in U. Piomelli *et al*. (2003) simulation. In their calculation of a turbulent channel flow, lowering the interface did result in increased resolved stress.

4. CONCLUSION

We have proposed a hybrid LES/RANS calculation in which the near-wall region is simulated by RANS technique and the outer flow by LES. The hybrid approach blends length scales between RANS and LES regions by weight interpolation, which is slightly difference from DES method, and smoothly-varying weights yield a length scale that is continuous to high order of derivative. The weight function is based on Van-Driest function for which the location of the transient region is valuable. Moreover, we can freely choose the models for LES and RANS. It is simple and easy to apply to flow with complex geometry, and thus could be widely applicable. Applying this technique to backward-facing step has yielded encouraging results. The length of reattachment point differs by 7.1% from experimental data. However, this method has not circumvented an inherent problem of hybrid LES/RANS: a mismatch of scales between RANS and LES zones. In the RANS layer, the turbulence model supplies most of the Reynolds stress, while in the LES region, resolved eddies dominates. Beside the mismatch in length scales, the time scales resolved computationally in the RANS region is larger than those in LES region, in contrast to the physical turnover times. In the future, a model based on numerical considerations as well as physical arguments is needed to circumvent this problem.

Acknowledgements

This worked is supported by Ministry of Education of Japan, and also was supported in part by the "Environmental Flows Research Group" of Ritsumeikan University.

References

Baggett J. S., Jimenez J., Kravchenko A. G., 1997 Resolution requirement in Large Eddy Simulation of shear flows. Center of Turbulence Research, Annual Research Briefs 1997

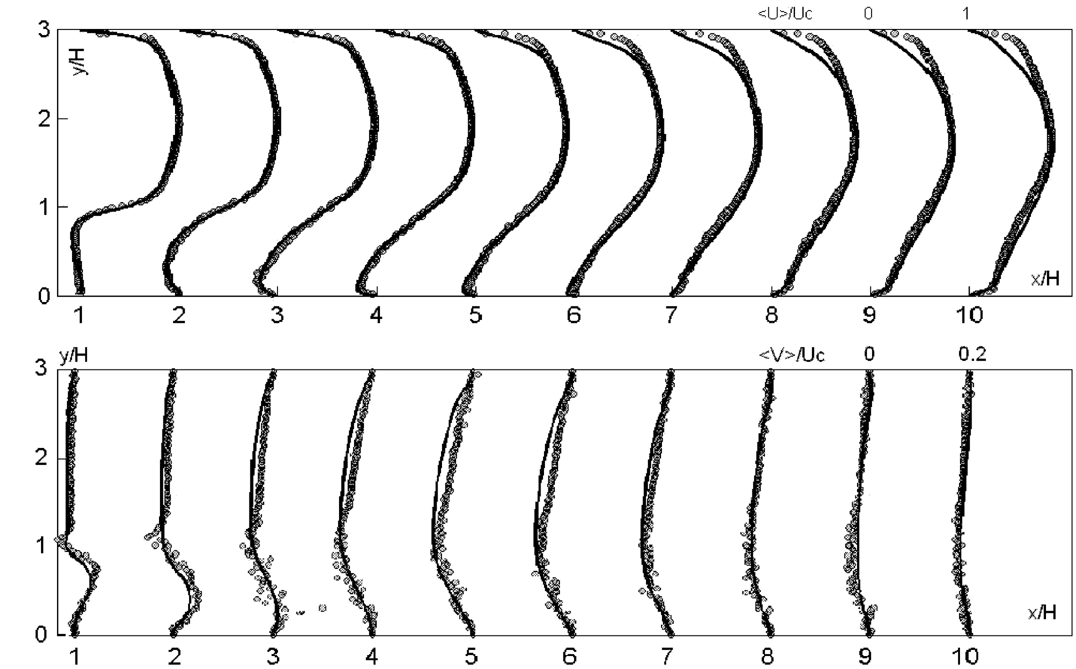

Figure 3 Mean velocity distributions (downstream of the step): streamwise component (above) and wall-normal component (below). Solid lines: present simulation, circles: Kasagi and Matsunaga's experiment

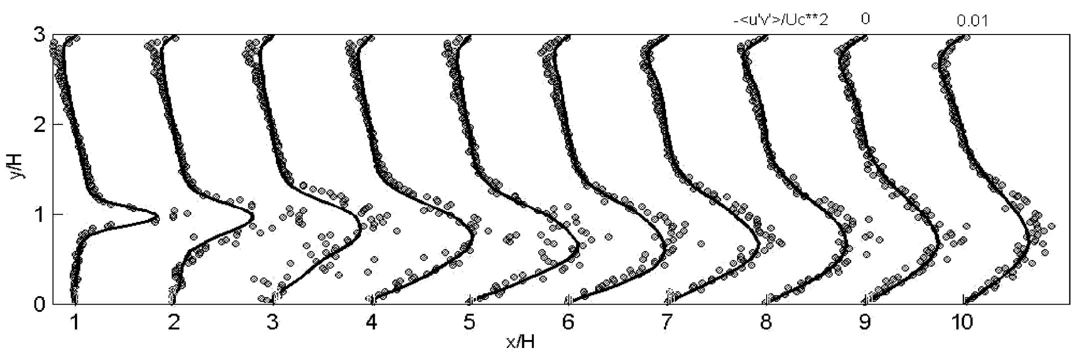

Figure 4 Reynolds stress distributions (downstream of the step): solid lines: present simulation, circles: Kasagi and Matsunaga's experiment

Balaras, E., Benocci, C., Piomelli, U., 1996. Two-layer approximate boundary conditions for large-eddy simulation, AIAA J. 24, 1111

Cabot, W. H., 1996. Near-wall models in large-eddy simulations of flow behind a backward-facing step. Center of Turbulence Research, Annual Research Briefs 1996

Chen, H. C., Patel V. C., 1988. Near-wall turbulence models for complex flows including separation. AIAA 14 Journal, 26:641-648

Dahlstrom, S., Davidson, L., 2003. Hybrid RANS/LES employing Interface Condition with Turbulent Structure. Turbulence, Heat and Mass Transfer 4, pp. 689-696.

Grotzbach, G. 1987. Direct numerical and large eddy simulations of turbulent channel flows. Encyclopedia of Fluid Mechanics (ed. N. P. Cheremisino), Vol. 6, pp. 1337-1391. Gulf.

Kasagi, N. and Matsunaga, A., 1995 "Three dimensional particle tracking velocimetry measurement of turbulence statistics and energy budget in a backward facing step flow", Int J. Heat and Fluid flow, 16, 477.

Piomelli U., Balaras E., Pasinato H., Squires K. D., Spalart P. R., 2003. The inner-outer layer interface in large-eddy simulations with wall-layer models. Int. J. Heat and Fluid Flow, 24, p 538-550.

Piomelli, U., Ferziger, J., Moin, P. & Kim, J. 1989. New approximate boundary conditions for large eddy simulations of wall bounded flows. Phys. Fluids A 1, 1061-1068.

Sagaut P., 2002. Large Eddy Simulation for Incompressible Flow: An Introduction (Second edition). Springer.

Schumann, U. 1975. Subgrid scale model for finite difference simulations of turbulent flows in plane channels and annuli. J. Comput. Phys. 18, 376.

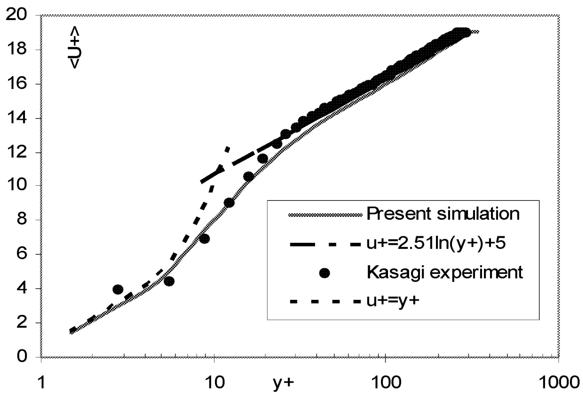

Figure 5 Mean streamwise velocity component distributions (upstream of the step): solid lines: present simulation, circles: Kasagi and Matsunaga's experiment

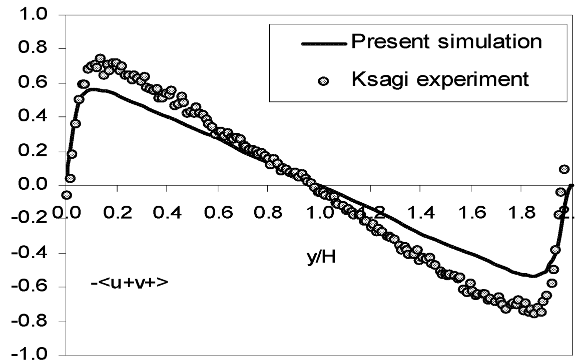

Figure 6 Reynolds stress distributions (upstream of the step): solid lines: present simulation, circles: Kasagi and Matsunaga's experiment

Spalart, P.R., Allmaras, S.R., 1994. A one-equation turbulence model for aerodynamic flows. La Reserche Aerospaciale 1, 5.

Spalart, P.R., Jou, W.H., Strelets, M., Allmaras, S.R., 1997. Comments on the feasibility of LES for wings, and on a hybrid RANS/LES approach. In: Liu, C., Lui, Z. (Eds.). Advances in DNS/LES. Greyden Press, Columbus, pp. 137-148.

Yoshizawa, A., Horiuti, K., 1985. A statistically-derived subgrid model for the large eddy simulation of turbulence. Phys. Fluid 25.

Wang, M., Moin, P., 2002. Dynamic wall modeling for large-eddy simulation of complex turbulent flows. Phys. Fluids 14, 2043-2051.

Wilcox D. C., 2000. Turbulence Modeling for CFD. Second Edition. DCW Industries Inc. p187.