LARGE-EDDY SIMULATION OF STRAIGHT COMPOUND OPEN-CHANNEL FLOWS

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ABSTRACT

We perform large-eddy simulations of incompressible fully developed turbulent flow in a straight open channel with one flood plain using a dynamic two-parameter model, for depth ratios D_r ($\equiv h/H$) equal to 1/2 and 1/4. The Reynolds numbers based on the hydraulic radius and bulk mean velocity, Re_m, are 5470 (for $D_r = 1/2$) and 4670 (for $D_r = 1/4$). Particular emphasis is placed on a case of low depth ratio; $D_r = 1/4$, where interaction between the main channel and the flood plain is significant and is therefore of practical interest. Overall, the computational results agree quite well with laboratory measurements by Tominaga and Nezu [J. Hydr. Engrg. ASCE 117(1991) 21] in spite of difference in their Reynolds numbers and show that the LES code using a sophisticated SGS model is a promising tool for natural river predictions.

INTRODUCTION

Fully developed turbulent flow in compound open channels is characterized by the interaction between the main channel and the flood plain, which results in a very complicated flow field and hence has stimulated many experimental and numerical studies.

A steady RANS approach has been taken in most of the numerical studies (Krishnappan and Lau, 1986; Kawahara and Tamai, 1988; Larrsson, 1988; Naot et al, 1993a, 1993b). Large-eddy simulation can be expected to be more accurate and reliable for the compound open-channel flows in which horizontal vortices appear due to shear instability at the interface and therefore large-scale unsteadiness is significant. Nevertheless LES has not been widely applied to prediction of the flows. Thomas and Williams (1995) performed LES of flow in a straight compound open channel with one flood plain for depth ratio $D_r = 1/2$. The Reynolds number based on the hydraulic radius and bulk mean velocity, Re_m, is 10400, comparable to the one of the corresponding experiment by Tominaga and Nezu (1991), 13600. The standard Smagorinsky model (Smagorinsky, 1963; Lilly, 1966) was employed along with a wall function near the wall. Satoh et al. (1999) also applied the Smagorinsky model to the flow for a low Reynolds number, $Re_m = 5300$, where they resolved the viscous sublayer instead of introducing wall-function type empiricism. Results obtained from these simulations, however, do not demonstrate the superiority of the unsteady LES over the RANS methods (e.g., Sofialidis and Prinos, 1998, 1999).

Here we present results from LES of the flow using a more sophisticated dynamic two-parameter model, for D_r equal to 1/4 as well as 1/2. Particular emphasis is placed on the former case, where interaction between the main channel and the flood plain is significant and is therefore of practical interest.

PROBLEM FORMULATION

Governing equations and SGS stress model

The governing equations for LES are obtained by applying a spatial filter to the full Navier-Stokes equations to separate the effects of the resolved scale from the subgrid-scale (SGS) eddies. The SGS stress tensor appearing in the filtered equations has been modeled with a dynamic two-parameter model (Salvetti and Banerjee, 1995; Salvetti et al., 1997) defined as:

$$\tau_{ij}^* = -2C\bar{\Delta}^2|\bar{S}|\bar{S}_{ij} + KL_{ij}^{m*} \tag{1}$$

The superscript "*" denotes the trace free operator ($\tau_{ij}^* = \tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk}$), $\bar{\Delta}$ is the grid-filter width, $\bar{S}_{ij} = \frac{1}{2}(\partial\bar{u}_i/\partial x_j + \partial\bar{u}_j/\partial x_i)$ is the resolved strain rate tensor, $|\bar{S}| = (2\bar{S}_{ij}\bar{S}_{ij})^{1/2}$, and $L_{ij}^m = \overline{\bar{u}_i\bar{u}_j} - \bar{\bar{u}}_i\bar{\bar{u}}_j$ is the modified Leonard term (Germano, 1986). The two unknown coefficients C and K are computed dynamically (Germano et al., 1991; Lilly, 1992) and then averaged in the statistically homogeneous direction. In addition the total viscosity $\nu + \nu_T$ is set to be zero whenever it becomes negative (clipping) to prevent solutions from blowing up (Lund et al., 1993). $\alpha = \hat{\bar{\Delta}}/\bar{\Delta}$, the test to grid filter widths ratio, is taken to be 2 and $\bar{\Delta}$ is set to be $(\Delta x_1 \Delta x_2 \Delta x_3)^{1/3}$. We employ Vreman's formulation for the modified Leonard term for the test-scale filter (Vreman et al., 1994). The filtering operations in the dynamic procedure are

Table 1: Computational conditions.

	h/H	b/B	B/H	L/H	Re_τ	Re_m	$\Delta t U_\tau/H$	effective # of grid points	Δx_1^+	Δx_2^+	Δx_3^+
Case 1	1/2	1/2	5	6	600	5470	3.0×10^{-4}	2.65×10^{6}	28.125	1.12-12.5	1.12-12.5
Case 2	1/4	1/2	5	6	600	4670	2.5×10^{-4}	2.36×10^{6}	28.125	1.03-14.6	1.12-12.5

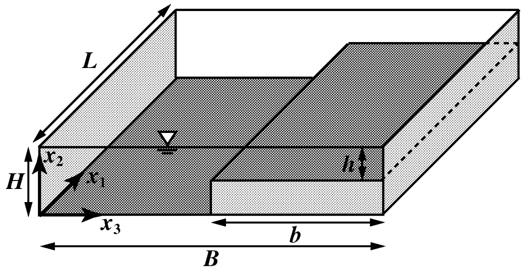

Figure 1: Schematic view of the flow configuration.

done in all three directions as

$$\bar{f}(x_i) = \frac{1}{24}(f(x_{i-1}) + 22f(x_i) + f(x_{i+1})) \qquad (2)$$

$$\hat{f}(x_i) = \frac{1}{6}(f(x_{i-1}) + 4f(x_i) + f(x_{i+1})) \qquad (3)$$

irrespective of if the grid spacing is uniform or not. Hence the filtering and differentiation do not commute in inhomogeneous directions where the grid spacing[the filter width] is nonuniform. We choose to neglect errors due to the noncommutation.

Numerical Methods

The system described in the previous subsection is solved on a Cartesian staggered grid using finite difference methods. The momentum equations are discretized in time with a semi-implicit method that uses the second-order Crank-Nicolson scheme for the viscous diffusion terms and a third-order Runge-Kutta method (Spalart et al., 1991) for the others. Spatial derivatives are discretized with fully-conservative second-order central differences for all terms (Kajishima, 1999). The discretized equations are solved with a fractional step method and the solution of the Poisson equation is obtained using a Fourier transform in the streamwise direction and a SOR method in the other two directions.

Geometry and grid parameters

The filtered Navier-Stokes equations are solved numerically in the domain sketched in Figure 1. The boundary conditions are periodic in the streamwise direction, no-slip on walls, and free slip (zero stress) on the top. The geometry is characterized by the depth ratio D_r, the width ratio (b/B) and the overall aspect ratio (B/H), last two of which are set to 1/2 and 5, respectively. The streamwise domain period (L) is chosen to be $6H$, which is large enough to contain the longest structure present in the flow. The simulations are conducted for a Reynolds number Re_τ of 600, based on the mean-friction velocity U_τ and main-channel flow depth H. It corresponds to a Reynolds number Re_m of about 5470 in Case 1 and 4670 in Case 2.The physical domain is discretized using $128 \times 96 \times 288$ grid points in both cases. The dirscretization is uniform in the streamwise direction, whereas in the spanwise and normal directions points are clustered towards the walls; in particular the first point close to the wall is placed at x_2^+ or $x_3^+ < 1$ and at least 6 points are in the near-wall region (x_2^+ or $x_3^+ \leq 10$). The computational conditions are summarized in Table 1.

After a statistical steady state was reached, data for the statistics were collected for 6 time units (H/U_τ). Statistical quantities are calculated by averaging over time as well as the homogeneous streamwise direction. Simulations were run at the maximum Courant number $\Delta t(\bar{u}_i/\Delta x_i)_{\max}$ of about 1.0.

RESULTS AND DISCUSSION

Computational results are compared with laboratory measurements taken by Tominaga and Nezu (1991) at $Re_\tau \approx 1200$. In the following $\langle f \rangle$ denotes ensemble averaged quantities, and $\bar{f}''$ and f' are defined by $\bar{f} - \langle \bar{f} \rangle$ and $f - \langle f \rangle$, respectively.

Mean velocity

Figure 2 shows mean streamwise velocity contours. The value of averaged friction velocity obtained from the simulations using the linear law are 1.0047 in Case 1 and 0.9970 in Case 2, and are within 0.5% of the nominal value. Despite the difference in their Reynolds numbers, the figure shows that the simulation results agree closely with the experiments. In Figure 2(b) a distortion of the contours near the junction edge is not as pronounced as the one in the experimental data, however, it is much more comparable than the LES results of Thomas and Williams (1995) and Satoh et al. (1999), where the standard Smagorinsky model was employed.

Comparison of the cross-plane velocity vectors is shown in Figure 3. The maximum magnitudes of the vectors in the experiments and in the simulations are $0.043U_m$ and $0.053U_m$ in Case 1, and $0.035U_m$ and $0.058U_m$ in Case 2, respectively. The strong upflow originating from the junction corner along with a pair of vortices is accurately predicted in both cases.

Reynolds stresses

Figure 4 shows the distribution of the turbulent kinetic energy and Reynolds shear stresses which are obtained from Case 2 and normalized by U_τ. The minimum value of the turbulent kinetic energy predicted appears below the free surface as can be seen in the experimental data. It appears on the free surface in the prediction of Sofialidis and Prinos (1998) using a nonlinear $k - \varepsilon$ model.

Comparison of the Reynolds shear stresses is also highly satisfactory. High positive and negative values of $-\langle u_1' u_2' \rangle$ near the junction edge are accurately reproduced, which is in accordance with the strong bulging of the velocity contours.

Model coefficients

Figure 5 shows contours of the ensemble-averaged model constants, $\langle C \rangle$ and $\langle K \rangle$, obtained from Case 2. The value of ;$\langle C \rangle$ ranges from -0.0037 to 0.0061. $\langle C \rangle$ is negative only

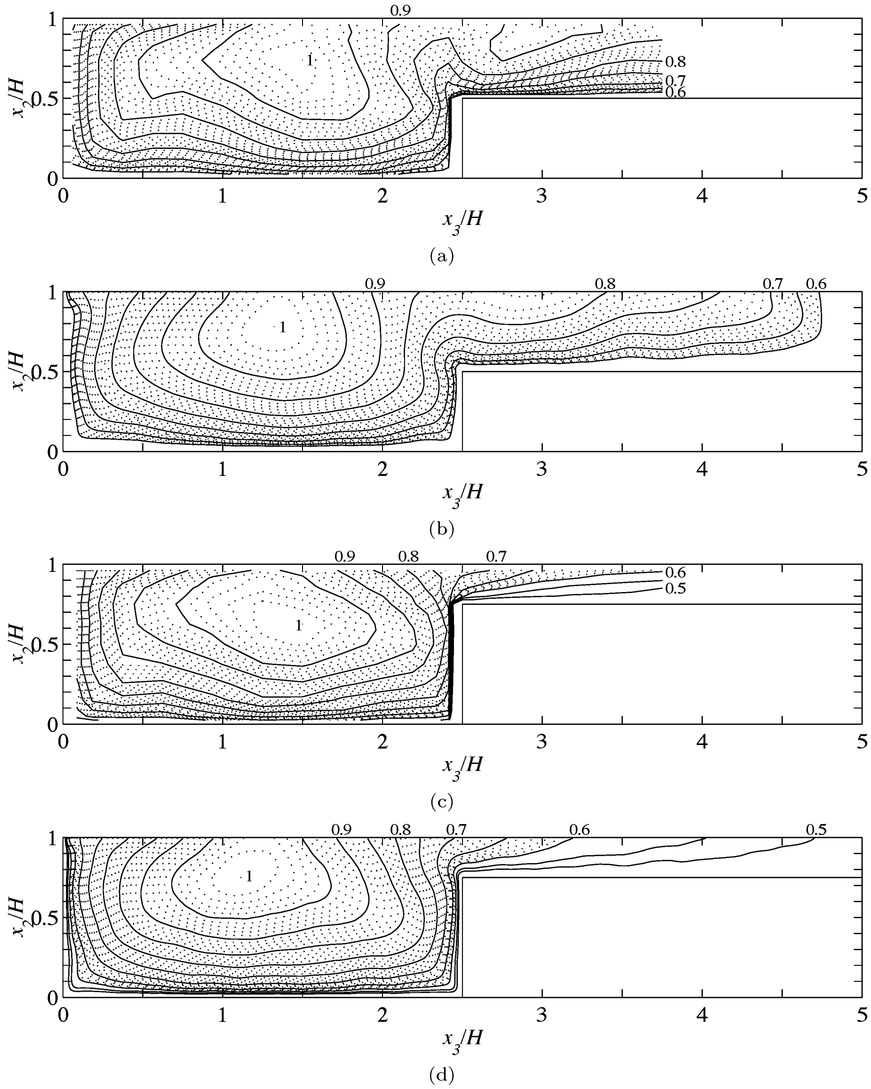

Figure 2: Contours of streamwise mean velocity normalized by its maximum. (a) Case 1, experiment ($U_{max}/U_\tau = 23.7$). (b) Case 1, present LES ($U_{max}/U_\tau = 22.4$). (c) Case 2, experiment ($U_{max}/U_\tau = 25.4$). (d) Case 2, present LES ($U_{max}/U_\tau = 23.7$).

at about 8% of the effective 8192 grid points in the cross-stream plane, most of which are seen near the rigid shear free boundary. The average and maximum numbers of grid points where the clipping was executed are about 44 points/step and 333 points, respectively. They correspond to about 0.0019% and 0.0141% of the grid points used. This agrees well with the findings of Zang et al. (1993), where the dynamic mixed model was proposed and applied to lid-driven cavity flows.

The value of $\langle K \rangle$ ranges from 0.92 to 2.50 and is large near the wall. This indicates that the modeled cross term contributes to the SGS stress only near the no-slip boundaries, where it is comparable to L_{ij}^m.

CONCLUSIONS

Large-eddy simulations of fully developed turbulent flows in a straight compound open channel were performed for two depth ratios, $D_r = 1/2$ and $1/4$. The free surface was approximated by rigid shear-free condition. The SGS stress tensor appearing in the filtered Navier-Stokes equations are modeled by a dynamic two-parameter model.

The results are compared quite well with laboratory measurements by Tominaga and Nezu (1991) in spite of difference in their Reynolds numbers.

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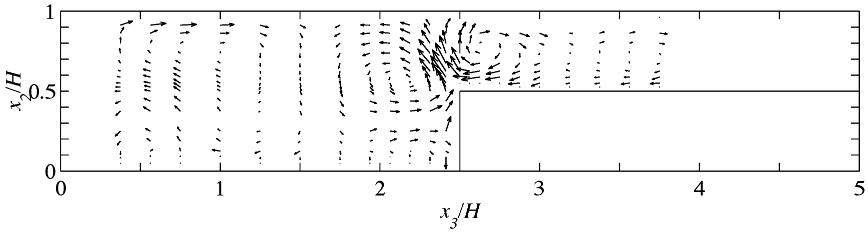

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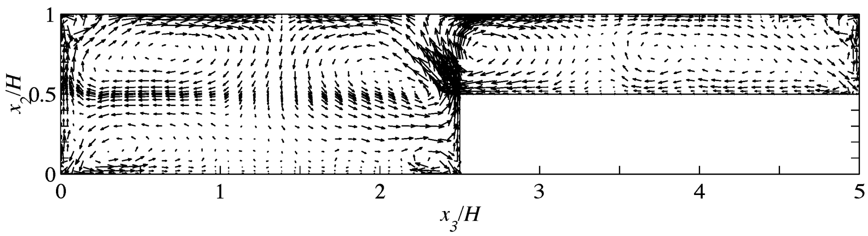

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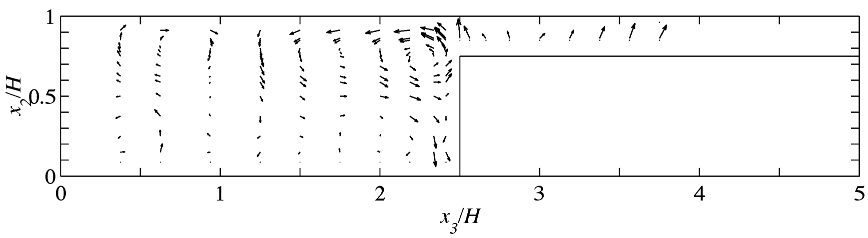

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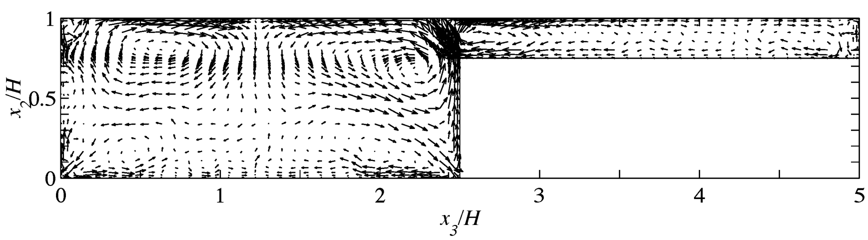

(d)

Figure 3: Vectors of secondary currents. (a) Case 1, experiment. (b) Case 1, present LES. (c) Case 2, experiment. (d) Case 2, present LES. The maximum magnitudes occur $0.043U_m$ at $(x_2/H, x_3/H) = (0.59, 2.41)$, $0.053U_m$ at $(0.55, 2.45)$, $0.035U_m$ at $(x_2/H, x_3/H) = (0.86, 2.50)$, and $0.058U_m$ at $(0.99, 0.55)$, in (a), (b), (c), and (d), respectively. Every fourth vector is plotted in (b) and (d) for clarity.

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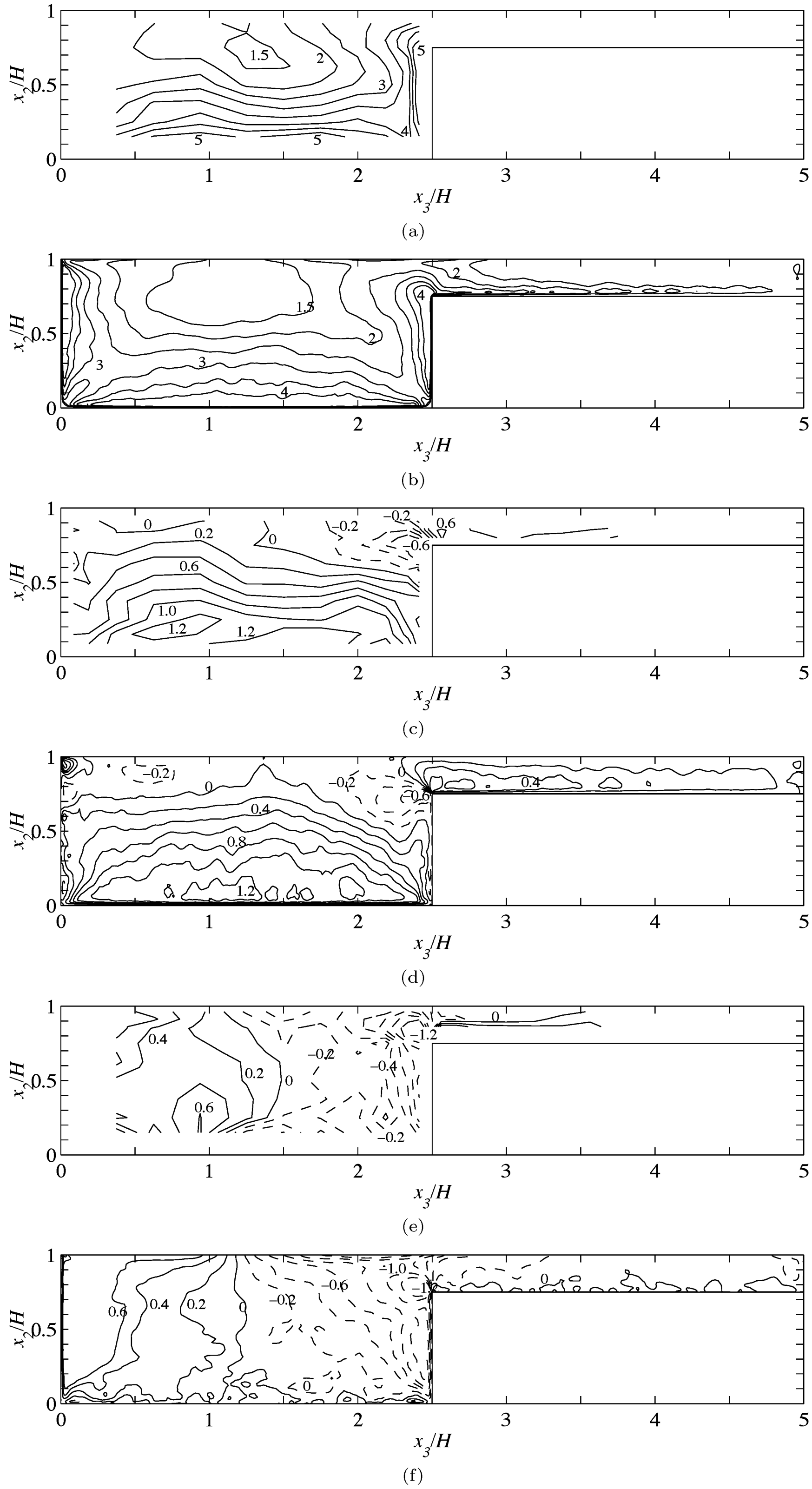
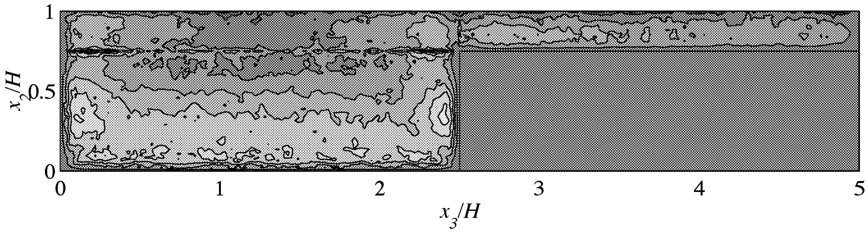


Figure 4: Contours of Reynolds stresses in Case 2 normalized by the averaged friction velocity. (a) $\langle u_i' u_i' \rangle/2$, experiment. (b) $(\langle \bar{u}_i'' \bar{u}_i'' \rangle + \tau_{ii}^*)/2$, present LES. (c) $-\langle u_1' u_2' \rangle$, experiment. (d) $-(\langle u_1'' u_2'' \rangle + \tau_{12})$, present LES. (e) $-\langle u_1' u_3' \rangle$, experiment. (f) $-(\langle u_1'' u_3'' \rangle + \tau_{13})$, present LES. Negative values are shown as dashed lines.



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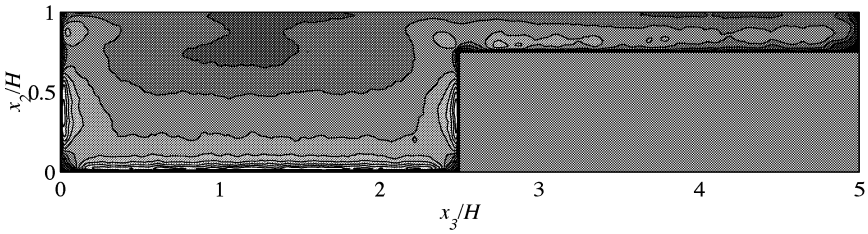

(b)

Figure 5: Contours of emsemble averaged model coefficients in Case 2. (a) $\langle C \rangle$. (b) $\langle K \rangle$. Darker [lighter] in gray means lower [higher] in value. $\langle C \rangle$ and $\langle K \rangle$ range from -0.0037 to 0.0061, and from 0.92 to 2.50, respectively.

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