A REYNOLDS-STRESS EXPRESSION BASED ON DNS DATA IN NON-LINEAR EDDY-VISCOSITY TURBULENCE MODEL FOR COMPLEX FLOWS

Varangrat Juntasaro, Suabsakul Gururatana, Jirarote Buranarote
Department of Mechanical Engineering, Faculty of Engineering,
Kasetsart University
Bangkhen, Bangkok 10900, Thailand
fengvrj@ku.ac.th

Ekachai Juntasaro
School of Mechanical Engineering, Institute of Engineering,
Suranaree University of Technology
Nakhon Ratchasima 30000, Thailand
junta@sut.ac.th

ABSTRACT

A Reynolds-stress expression used in a non-linear eddy-viscosity turbulence model is developed based on the Direct Numerical Simulation (DNS) data of fully developed turbulent channel flow of Kim et al. (1987) for both $Re_\tau = 180$ and 395. The data are initially used to analyze the accuracy of various Reynolds-stress expressions used in the non-linear turbulence models in order to find the expression that gives the closest Reynolds-stress values to the DNS data. The use of the DNS data directly in the expression is to ensure that the errors in the solutions are not from the modeled transport equations of the turbulent kinetic energy and its dissipation rate. It is found that both Reynolds shear stress and normal Reynolds stresses from the Reynolds-stress expression of Craft et al. (1996) are closer to the DNS data than the other expressions for both Reynolds numbers. The present work aims to further improve the accuracy of the expression of Craft et al. in predicting the Reynolds stresses and consequently the mean velocity profiles. The main objective of the current work is to find the f_μ expression based on the non-linear Reynolds-stress expression of Craft et al. that gives the closest agreement to the values of f_μ extracted from the DNS data. It is found that the Reynolds-stress expression of Craft et al. with the f_μ of Gibson and Dafa'Alla (1994) gives the closer agreement to the DNS data for all Reynolds stresses and mean velocity profile in the case of fully-developed turbulent channel flow. The expression is then evaluated for more complex three-dimensional flow through a straight square duct where there are secondary flows. The DNS data of Gavrilakis (1992) are used for comparison. The present expression shows the improvement in the prediction of the mean spanwise velocity profile at the position near the edge of the duct. The damping function of Gibson and Dafa'Alla is hence the more suitable damping function to be used with the non-linear Reynolds-stress expression of Craft et al.

INTRODUCTION

The accurate simulation of turbulence effects on the flows is important to the prediction of real flows using computational fluid dynamics (CFD). Large eddy simulation (LES) has become popular nowadays due to its accuracy compared to the conventional Reynolds-averaged Navier-Stokes (RANS) turbulence models. However, the RANS turbulence models are still the most suitable models for the engineering applications since the key of industrial competitiveness is to use as less time as possible for the design optimization and the RANS models require much less computational time compared to the LES.

Amongst the RANS models, the non-linear eddy-viscosity turbulence model has shown its higher potential in terms of both accuracy and computational time. It is the only kind of eddy-viscosity models that can predict the anisotropy of the Reynolds normal stresses. However, the Reynolds normal stresses predicted by the existing non-linear models are still far from the direct numerical simulation (DNS) data leading to the inaccuracy of the velocity profiles especially in complex flows.

There are generally two main causes that affect the accuracy of the non-linear eddy-viscosity turbulence models: the constitutive expression for the Reynolds stresses, and the transport equations for the velocity and the length scales. The error from the constitutive expression is mainly due to the damping function and constants used in the expression are based on the linear expression of the Reynolds stresses instead of the non-linear expression itself.

The main objective of the current study is to improve the accuracy of the non-linear Reynolds-stress expression used in the non-linear eddy-viscosity turbulence models via the damping function and model constants. The comparative study of different non-linear Reynolds-stress expressions is initially made. The damping function and constants are then chosen based on the non-linear Reynolds-stress expression by using the DNS data of fully developed turbulent channel flow directly in the expression. This is to ensure that the error in the predicted results are not from the modeled transport equations of the turbulent kinetic energy and its dissipation rate. This has the advantage in the way that the modeled transport equations can be further developed based on the correct Reynolds-stress expression later on. The accuracy of the present Reynolds-stress expression is then assessed using the DNS data of fully developed turbulent channel flow and three-dimensional flow through a straight square duct for comparison.

CONSTITUTIVE RELATION OF REYNOLDS STRESSES

The accuracy of various Reynolds-stress expressions used in the non-linear turbulence models is initially analyzed using the DNS data for the fully developed turbulent channel flow of Kim et al. (1987) at both $Re_\tau = 180$ and 395 in order to find the expression that gives the Reynolds-stress values closest to the DNS data. The Reynolds-stress expression considered include the expression of Pope (1975), Nisizima and Yoshizawa (1987), Speziale (1987), Myong and Kasagi (1990), Rubinstein and Barton (1990), Shih et al. (1993), Lien et al. (1996), Craft et al. (1996), Apsley and Leschziner (1998), and Abe et al. (2003). The Reynolds-stress values are calculated from each expression by substituting the values of the turbulent kinetic energy (k), the dissipation rate of turbulent kinetic energy ($\tilde{\varepsilon}$) and the mean velocity ($\bar{U}$) appeared in the expression with the DNS data. The calculated Reynolds stresses are then compared with the DNS data of Reynolds stresses to assess the accuracy of each Reynolds-stress expression. It is found that the expression of Craft et al. (1996) predicts the Reynolds stresses closer to the DNS data than the other expressions at both Reynolds numbers (Figures 1 to 4).

However, the Reynolds shear stress and especially the Reynolds normal stresses predicted by the expression of Craft et al. are still far from the DNS data. The present study therefore aims to improve the accuracy of the expression of Craft et al. by optimizing the damping function f_μ and the empirical constants for the non-linear expression of Craft et al.

Following the method of Patel et al. (1985), the distribution of f_μ based on the expression of Craft et al. can be obtained from Eq. (1) for the fully developed turbulent channel flow as follows:

$$f_\mu = -\frac{\overline{uv}}{C_\mu \frac{k^2}{\tilde{\varepsilon}} \frac{\partial \bar{U}}{\partial y}} \tag{1}$$

where

$$C_\mu = \left(\frac{0.3}{1+0.35\left(\frac{k}{\tilde{\varepsilon}}\frac{\partial \bar{U}}{\partial y}\right)^{1.5}}\right)\left(1 - exp\left[\frac{-0.36}{exp\left(-0.75\frac{k}{\tilde{\varepsilon}}\frac{\partial \bar{U}}{\partial y}\right)}\right]\right) \tag{2}$$

The DNS data for the fully developed turbulent channel flow of Kim et al. at $Re_\tau = 180$ and 395 are substituted in the above equation in order to obtain the DNS distribution of f_μ. The DNS distribution of f_μ is then re-plotted in terms of turbulent Reynolds number Re_t. The f_μ expression in terms of Re_t is preferred to that in terms of the distance from the wall y^+. This is because y^+ is indefinable in separation regions due to its definition that depends on the friction velocity U_τ. The current work aims to find the f_μ expression for the non-linear Reynolds-stress expression of Craft et al. (1996) that gives the closest agreement with the DNS distribution of f_μ. The different expressions for f_μ are compared with the DNS data of f_μ. The f_μ expression of Gibson and Dafa'Alla (1994) shows the closest agreement with the DNS data and therefore it is used in collaboration with the Reynolds-stress expression of Craft et al. to model the Reynolds shear stresses (Figures 5 and 6).

The accuracy of the Reynolds normal stresses are further improved by calibrating the empirical constants used in the Reynolds-stress expression with the DNS data using the chosen f_μ (Figures 7 and 8).

The present Reynolds-stress expression is written as follows:

$$\overline{uv} = -C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}}\left(\frac{\partial \bar{U}}{\partial y}\right) \tag{3}$$

$$\overline{uu} = \frac{2}{3}k + C_\mu f_\mu \frac{k^3}{\tilde{\varepsilon}^2}\left(\frac{\partial \bar{U}}{\partial y}\right)^2 (0.51) \tag{4}$$

$$\overline{vv} = \frac{2}{3}k + C_\mu f_\mu \frac{k^3}{\tilde{\varepsilon}^2}\left(\frac{\partial \bar{U}}{\partial y}\right)^2 (-0.34) \tag{5}$$

$$\overline{ww} = \frac{2}{3}k + C_\mu f_\mu \frac{k^3}{\tilde{\varepsilon}^2}\left(\frac{\partial \bar{U}}{\partial y}\right)^2 (-0.17) \tag{6}$$

where

$$C_\mu = \left(\frac{0.3}{1+0.35\left(\frac{k}{\tilde{\varepsilon}}\frac{\partial \bar{U}}{\partial y}\right)^{1.5}}\right)\left(1 - exp\left[\frac{-0.36}{exp\left(-0.75\frac{k}{\tilde{\varepsilon}}\frac{\partial \bar{U}}{\partial y}\right)}\right]\right) \tag{7}$$

and

$$f_\mu = exp\left(\frac{-6}{\left(1+\frac{Re_t}{50}\right)^2}\right)\left(1 + 3exp\left(\frac{-Re_t}{10}\right)\right) \tag{8}$$

RESULTS AND DISCUSSION

The accuracy of the present Reynolds-stress expression in predicting the mean velocity profile is evaluated using the fully developed turbulent channel flow as the first test case. The Reynolds shear-stress term in the channel flow equation (Eq. 9) is first substituted by the present Reynolds-stress expression. The DNS data of Kim et al. (1987) is then substituted to each variable appearing in the channel flow equation to obtain the values of the mean velocity. The use of the DNS data to directly obtain the mean velocity profile has the advantage that the only source of the error is clearly from the Reynolds-stress expression since all the values except the mean velocity values are obtained from the DNS data. It is found that the mean velocity profile using the current Reynolds-stress expression shows the closer agreement to the DNS data than the Reynolds-stress expression of Craft et al. (1996) in general at both Reynolds numbers (Figures 9 and 10).

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial \bar{U}}{\partial y}\right) - \frac{\partial}{\partial y}(\rho \overline{uv}) = \frac{\partial P}{\partial x} \tag{9}$$

The present Reynolds-stress expression is further evaluated for three-dimensional flow through a straight square duct. The DNS data of Gavrilakis (1992) are again used directly to obtain the values of the Reynolds shear stresses, the Reynolds normal stresses, the mean streamwise and the mean spanwise velocity profiles. It is found that the present expression gives the close agreement to the original expression of Craft et al. in general (Figures 11, 12 and 14). However, the present expression predicts the mean spanwise velocity profile (w/u_0) closer to the DNS data than the original expression at the position near the edge of the duct ($z/h = 0.1$) as can be seen in Figure 13.

CONCLUSIONS

The Reynolds-stress expression has proved to have significant effect on the accuracy of the predicted results using the eddy-viscosity turbulence models. The non-linear Reynolds-stress expression modified in the present study via the optimized damping function f_μ and the empirical constants results in the higher accuracy of the predicted Reynolds stresses and the mean velocity profile in the case of fully-developed turbulent channel flow. Furthermore, it is found that the present expression predicts the mean spanwise velocity profile closer to the DNS data at the position near the edge of the duct then the original expression of Craft et al. in the case of three-dimensional flow through a straight square duct. This is because the more suitable damping function is used in the present non-linear Reynolds-stress expression.

It can be concluded that the method to find the damping function by using the DNS data directly in the non-linear Reynolds-stress expression itself is the suitable method to improve the prediction in the near wall region.

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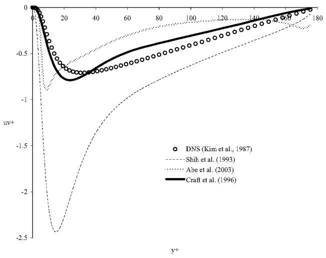

Figure 1: Comparison of the Reynolds shear stress for different Reynolds-stress expressions for the fully-developed turbulent channel flow at $Re_\tau = 180$.

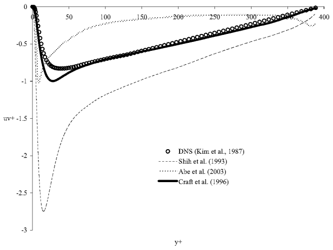

Figure 2: Comparison of the Reynolds shear stress for different Reynolds-stress expressions for the fully-developed turbulent channel flow at $Re_\tau = 395$.

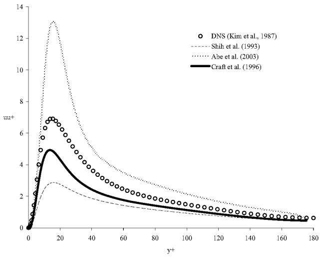

Figure 3: Comparison of the Reynolds normal stress uu^+ for different Reynolds-stress expressions for the fully-developed turbulent channel flow at $Re_\tau = 180$.

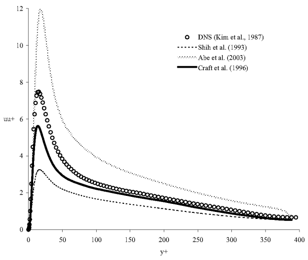

Figure 4: Comparison of the Reynolds normal stress uu^+ for different Reynolds-stress expressions for the fully-developed turbulent channel flow at $Re_\tau = 395$.

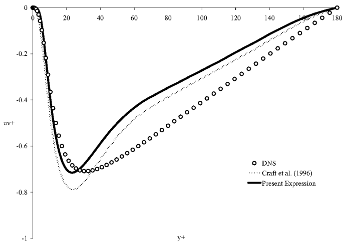

Figure 5: Reynolds shear stress of the fully developed turbulent channel flow using the present Reynolds-stress expression at $Re_\tau = 180$.

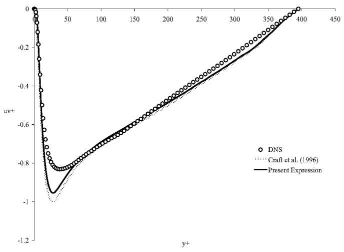

Figure 6: Reynolds shear stress of the fully developed turbulent channel flow using the present Reynolds-stress expression at $Re_\tau = 395$.

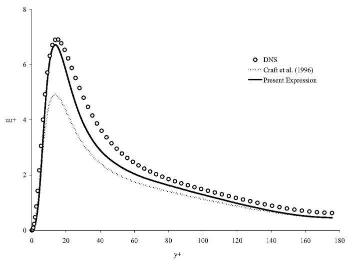

Figure 7: Reynolds normal stress uu^+ of the fully developed turbulent channel flow using the present Reynolds-stress expression at $Re_\tau = 180$.

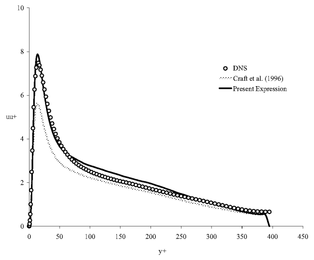

Figure 8: Reynolds normal stress uu^+ of the fully developed turbulent channel flow using the present Reynolds-stress expression at $Re_\tau = 395$.

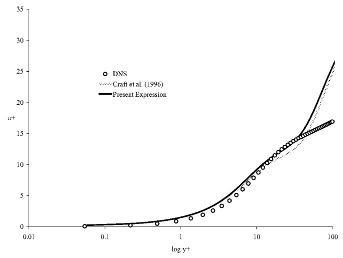

Figure 9: Mean velocity profile of the fully developed turbulent channel flow using the present Reynolds-stress expression at $Re_\tau = 180$.

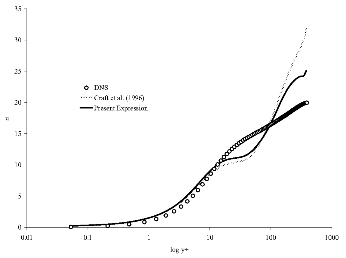

Figure 10: Mean velocity profile of the fully developed turbulent channel flow using the present Reynolds-stress expression at $Re_\tau = 395$.

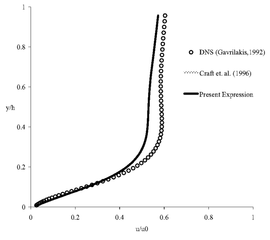

Figure 11: Mean streamwise velocity profile of the three-dimensional flow through a straight square duct using the present Reynolds-stress expression at $z/h = 0.1$.

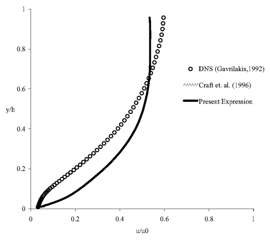

Figure 12: Mean streamwise velocity profile of the three-dimensional flow through a straight square duct using the present Reynolds-stress expression at $z/h = 0.5$.

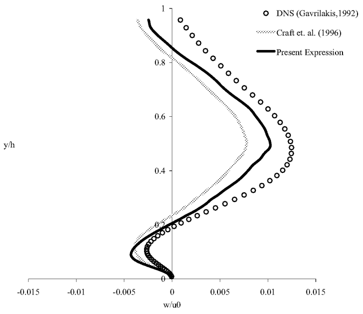

Figure 13: Mean spanwise velocity profile of the three-dimensional flow through a straight square duct using the present Reynolds-stress expression at $z/h = 0.1$.

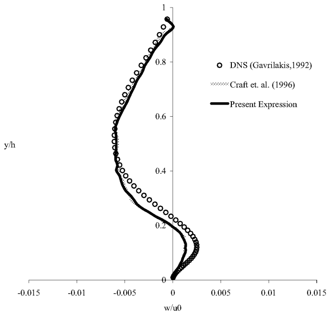

Figure 14: Mean spanwise velocity profile of the three-dimensional flow through a straight square duct using the present Reynolds-stress expression at $z/h = 0.5$.