EXTENDING STEREO P.I.V. TO MEASURE VELOCITY GRADIENTS AT A WALL AND A FREE SURFACE

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ABSTRACT

Velocity gradient is typically estimated in Particle Image Velocimetry (PIV) by differentiating a measured velocity field, which amplifies noise in the measured velocities. If gradients near a boundary are sought, such noise is usually greater than in bulk fluid, because of small tracer displacement, intense deformation of tracer patterns, and laser reflection. We consider here the application of Particle Image Distortion (PID) to *directly* calculate velocity gradients at a fixed wall and at a non-fluctuating free surface. Results from synthetic 2D PIV images suggest this “PIV/IG” technique (“Interface Gradiometry”) achieves higher SNR and accuracy than velocity differentiation. Also, we have developed a procedure to reconstruct three-dimensional velocity gradients from PIV/IG data obtained in stereo views. At a fixed wall, these equations simplify considerably thanks to the no-slip condition. Experimental data from the bottom wall of turbulent open channel flow appear to suffer from a form of pixel locking. As with standard PIV, this shows the importance of adequate tracer diameter in the images, and of sufficient seeding density.

MEASURING INTERFACIAL GRADIENTS

Velocity gradients at a phase boundary determine the viscous boundary stress, and correlate strongly with heat and chemical fluxes, so it is important to measure them accurately in turbulent flow. To validate models for wall-stress and mass transfer in Large Eddy Simulation, our group will apply PIV to measure statistics of wall shear stress, and surface divergence at a free surface, in turbulent channel flow. We further aim to relate boundary gradients to nearby vortical structure. This contribution describes a technique developed for that purpose.

Particle Image Velocimetry (PIV) yields velocities throughout a light sheet. In principle, these velocities might be differentiated to yield the field of velocity gradients, including values at a wall or other interface. There are however numerous difficulties in applying standard PIV near a wall or fluid phase boundary (not to mention taking differences of such a signal!). First, the signal level is low; near a fixed wall, both components approach zero, while for slowly evolving free surfaces, the normal velocity signal is nearly zero. Second, especially at the wall, one must resolve high gradients in thin layers. Where most sensors would yield a deterministically smoothed value of the measurand in the sensing region, Pattern Matching PIV produces a raw signal, the correlation map, which is a *union* of particle matches and mismatches at random locations through the sensing volume. Thus, the effective position of measured PIV velocity vectors, typically dominated by the positions of the brightest tracer particles, is unknown; this is especially unwelcome in a thin shear

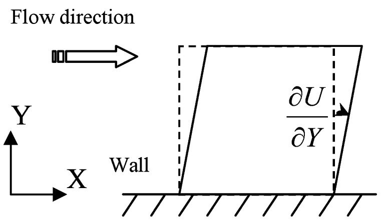

Figure 1. Linear shear deformation adjacent to a fixed wall, as projected onto image sensor.

layer such as found at a wall, when one wants to determine velocity gradients. This communication suggests how to surmount these difficulties with the Pattern Image Distortion (PID) technique (Huang et al., 1993), in this case by shearing PIV image templates parallel to a no-slip wall, or longitudinally deforming templates normal to a free surface. We refer to this enhancement to PIV as "Interface Gradiometry", abbreviated PIV/IG. By simultaneously applying PIV/PID to measure velocity throughout a light sheet and PIV/IG to record instantaneous profiles of velocity gradient along an interface, we hope to extend considerably the capability to visualize interfacial dynamics.

The present paper describes results of validation tests for PIV/IG, using synthetic PIV images near a fixed wall and a free surface. At a fixed wall, templates are anchored at the wall and sheared by varying degrees, and the problem for a single camera is then a *one-dimensional* pattern-matching problem whose only parameter is the projection of wall shear in the viewing direction. At a free surface, assumed herein to be perfectly steady, standard PIV is applied to determine average tracer displacement, following which templates are compressed or extended normal to the surface in search of the deformation that results in the best match. Preliminary measurements of wall gradients in a real Stereo PIV experiment are also presented.

NEAR-WALL IMAGE DISTORTION

This section describes how to measure the velocity gradient seen at a fixed wall by a single camera (Nguyen *et al*., 2004). We assume sufficient resolution that the instantaneous velocity profiles near the wall are approximately linear over the height of a template, which we find requires about 3 or more pixels per wall unit, and template heights less than about 4 wall units.

Physical coordinates and velocity will be denoted by (x; streamwise, y; vertical, z; spanwise) and (u, v, w) respectively. We assume no slip at the wall,

$$u = v = w = 0, \qquad (1)$$

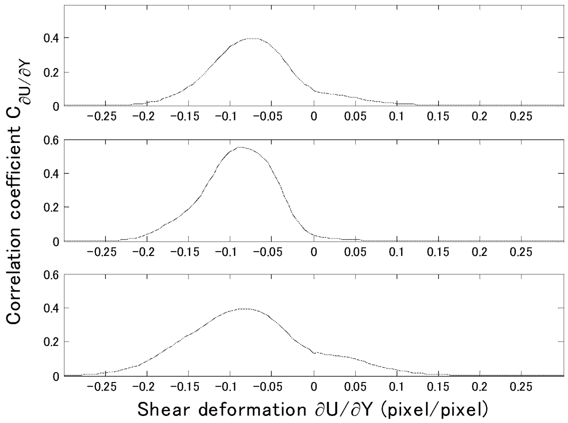

Figure 2. PID correlation curves (eq. 2) at 3 neighboring boundary points, from an image pair in the stereo PIV experiment presented in figs. 4-6. Template size is 80×20 pixels^2 (14.5×3.6 wall units^2), delay between exposures is 5 *ms*.

so tangential velocity derivatives vanish. In particular, ∂u/∂x = ∂w/∂z = 0, and if the flow is incompressible then ∂v/∂y = 0. The only non-zero velocity gradients to be determined are then ∂u/∂y and ∂w/∂y.

Consider 2-component PIV with a single camera viewing parallel to the wall and perpendicular to the light sheet. Pixel coordinates are denoted by *(X,Y)*, and projected components of interimage displacement by *(U, V)*. Assume that the wall appears horizontal on the image, and that magnification m is constant: *(X, Y, U, V) = Δt m(x, y, u, v)*. The rate of shear in ∂u/∂y, is measured through its imaged projection ∂U/∂Y which is estimated by searching for the maximum of the cross- correlation coefficient:

$$C_{\frac{\partial U}{\partial Y}} = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[\left(I_{m+n\frac{\partial U}{\partial Y},n} - \bar{I}_{\frac{\partial U}{\partial Y}}\right)\left(I'_{m,n} - \bar{I}'\right)\right]}{\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{N}\left(I_{m+n\frac{\partial U}{\partial Y},n} - \bar{I}\right)^2}\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{N}\left(I'_{m,n} - \bar{I}'\right)^2}} \qquad (2)$$

$$\bar{I}_{\frac{\partial U}{\partial Y}} = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} I_{m+n\frac{\partial U}{\partial Y},n} \qquad \bar{I}' = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} I'_{m,n}$$

Instead of *translating* templates from the first image before correlating with the second, as in PIV, they are *sheared* parallel to the wall by a range of trial values. Bi-cubic interpolation of pixel intensity is used in resampling the sheared templates, which are then cross-correlated with the second image as in equation (2). The measured ∂U/∂Y corresponds to the peak of the correlation curve (*cf.* fig. 2), as determined by a 3-point Gaussian fit. In practice, digital masking (Gui *et al*., 2003) is applied to cut out boundary reflections, but this refinement is not included in eq. (2).

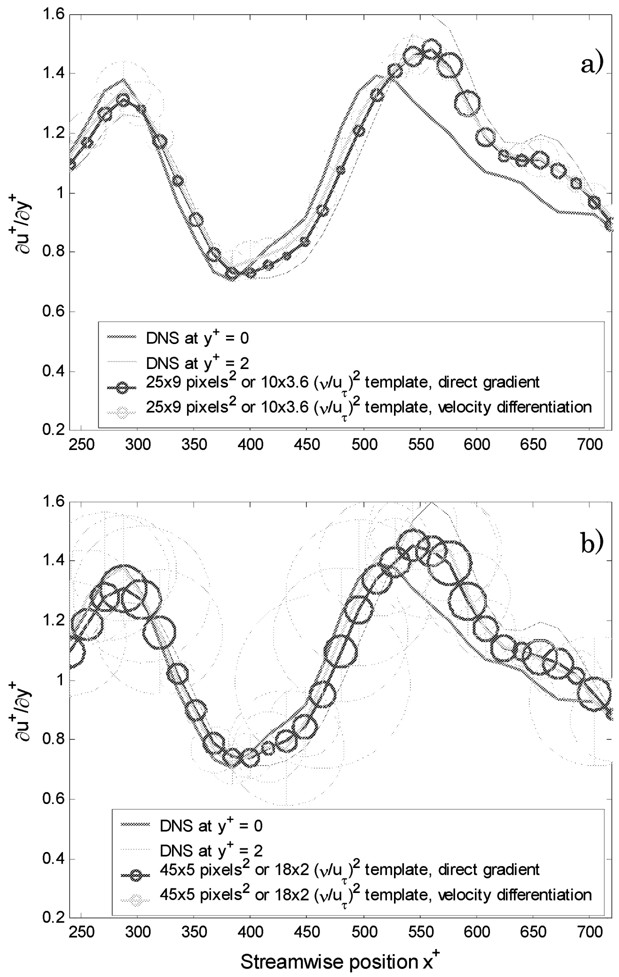

Figure 3. Instantaneous streamwise profiles of wall gradient $\partial u^+/\partial y^+$ component from DNS, with PIV/IG, and differentiation of PIV/PID velocities obtained from synthetic images from DNS velocity field. Template sizes are: a) 25×9 pixel^2 (10×3.6 wall units^2), b) 45×5 pixel^2, (18×2.0 wall units^2). Average tracer count in a template is 14.2.

VALIDATION BY 2D SYNTHETIC IMAGES

Synthetic PIV images were produced based on a single known velocity field from a Navier-Stokes simulation of turbulent open-channel flow ($Re_\tau = 300$), using a 64^3 grid on a domain with a height:width:length ratio of 1:1.92:3.84. Although this grid does not qualify the simulation as fully resolved, it suffices for the purpose of PIV/IG validation; we refer this velocity field as "DNS data". Figure 3 shows velocity gradient profiles from DNS which varies widely in both the streamwise and wall-normal directions. A set of 20 synthesized image pairs were generated from the same DNS velocity field in a streamwise-vertical plane. Tracer image are taken to have a Gaussian profile with diameters from 3 to 6 pixels, and peak intensities from 100 to 200. Seeding density is 0.064 tracer/pixel^2, about the same as the normal tracer density used for PIV-STD Project's images (Okamoto *et al*., 2000). Particle displacements are about 10 pixels far from the wall, and about 2 pixels at $y^+ = 5$.

Profiles of wall gradient in each image pair were calculated directly by PIV/IG. For comparison, wall velocity gradient was also obtained by differentiating velocity previously obtained from PIV/PID (cross-correlation enhanced by Particle Image Distortion) with the same image templates used for PIV/IG. When differentiating, the distance was taken from the template center to the wall. This method of differentiation produces only half the error of normal velocity differentiation. The same trial values of shear deformation were applied in PIV/IG and PIV/PID. For each method, the measurements vary from one synthetic image pair to another. The radii of circles in the figures represent the RMS deviation, and the continuous lines the local sample means of the 20 measurements at each location. Figure 3.a shows a near-optimal selection of template size, 10 wall unit wide × 3.6 wall unit high, yielding 14.2 tracers per template on average. In fig. 3.b, template height is reduced, with template width increased to keep the same area and number of tracers. This makes the mean profiles of both methods approach the DNS profile for $y^+ = 0$, but it increases the RMS deviation of the measurements. Conversely, increasing template height may reduce the RMS fluctuation but, when increased above 4 wall units, the profiles of sample means differ considerably from the DNS gradients at $y^+ = 0$. In both figures, PIV/IG is more accurate than the velocity differentiation method.

STEREO RECONSTRUCTION OF GRADIENTS

The equation for stereo velocity reconstruction has the form $[\boldsymbol{U}] = [\boldsymbol{M}][\boldsymbol{u}]$, where $[\boldsymbol{U}] = (U^1,V^1,U^2,V^2)^T$ is comprised of two 2D velocity fields (in image space) from stereo cameras 1 and 2, $[\boldsymbol{u}]$ is the physical velocity to be found, and $[\boldsymbol{M}]$ is magnification matrix. $[\boldsymbol{u}]$ can be obtained from this equation by least-squares. Differentiating this equation produces the reconstruction equation of velocity gradient. At a fixed wall, the no-slip condition reduces the gradient reconstruction equation to a very simple form: $\partial[\boldsymbol{U}]/\partial \mathbf{y} = [\boldsymbol{M}]\ \partial[\boldsymbol{u}]/\partial \mathbf{y}$. The term $\partial[\boldsymbol{U}]/\partial \mathbf{y}$ can be converted from $\partial[\boldsymbol{U}]/\partial \mathbf{Y}$ (velocity gradients in image space) along the wall boundary (aligned with X-axis) in particle images. Finally $\partial[\boldsymbol{u}]/\partial \mathbf{y}$ is determined by least-squares.

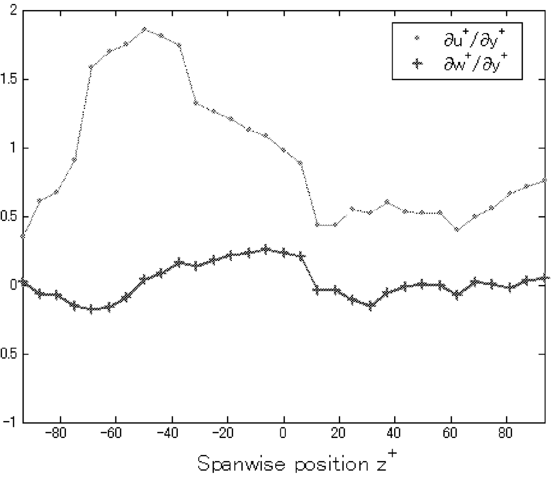

Figure 4. Instantaneous profiles of streamwise and spanwise velocity gradient from stereo experiment.

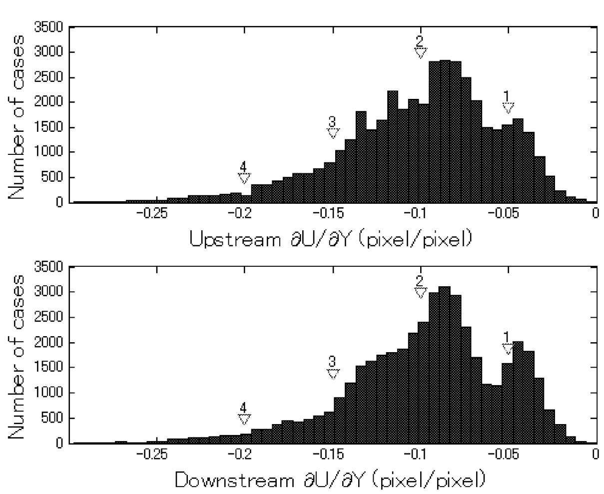

Figure 5. Histograms of wall velocity gradient in images of upstream and downstream cameras.

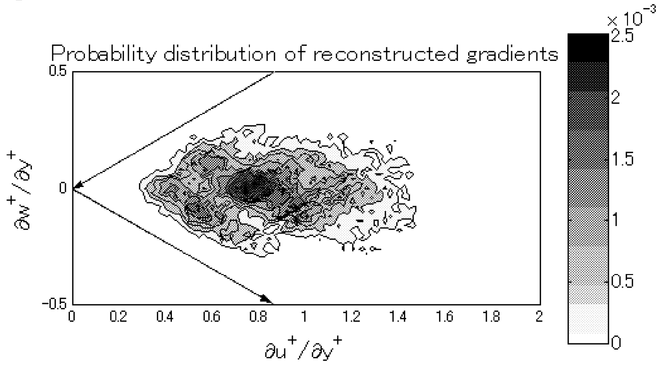

Figure 6. 2D histogram of wall-velocity gradient. Above and below arrows show viewing directions of the down- and upstream cameras.

STEREO PIV/IG EXPERIMENTS

Stereo PIV experiments have been performed in a 0.5*m* wide, 8*m* long water channel. In the runs presented here, flow depth was 60*mm*, water temperature was 11^{o}C and discharge was 3.324 *liters/s*. The laser illuminates a cross-stream sheet about 1.7-2*mm* thick. Two $1K^2$ cameras view horizontally, at about 30^{O} from the upstream and downstream directions, through prisms on the channel wall. Frame rate is 30*Hz*; frame straddling with a 5.0 *ms* delay yields velocity fields at 15 *Hz*. A glass bottom wall greatly reduces laser reflection, and any remaining reflection line is eliminated by masking (Gui *et al* 2003). Adjacent to the wall, where tracer displacements are 0.5 pixels or less, the resulting loss of tracer information is expected to be negligible. The Reynolds shear stress profile was extrapolated to obtain a friction velocity of 0.765*cm/s*, yielding $Re_\tau = 358$.

Wall velocity gradient was calculated by PIV/IG using 80×20 pixels^2 templates, equivalent to 14.5×3.6 wall units^2. This is a little larger than the templates applied to the synthetic images above, because the tracer density of 0.008 tracers/pixel^2 was 8 times lower than for the synthetic images. Instantaneous profiles of the reconstructed wall velocity gradients are shown in figure 4.

Based on a sample of 1400 image pairs recorded at 1Hz, figure 5 shows histograms of the apparent rate of shear seen by the two cameras. The range of the graph coincides with that of PIV/IG interrogation with the step of 0.005 pixel/pixel. The graphs show a very clear peak in the bin just to the right of –0.05, and other clear biases towards values near –0.085, -0.13, -0.17. This effect strongly suggests some type of pixel locking, which in standard PIV is the bias toward integer values of tracer displacement that occurs when tracer images have an intensity distribution less than about two pixels wide. We argue that this occurs in PIV/IG when tracer density and reflected intensity is low, as follows. First, consider the correlation peak in the extreme case in which only one tracer particle lies somewhere in the template. Shearing the template shifts the tracer's intensity profile in proportion to its height, and correspondingly the width of the 1D correlation curve (*cf.* fig. 1), will be inversely proportional to the tracer's height. Thus, tracer particles near the top of the template, because of their narrower correlation peaks, will dominate the measurement of shear rate. Accordingly, we hypothesize that the biases in the histogram are toward pixel values of displacement for tracer particles near the top edge of the template. In Figure 5, the triangles indicate rates of shear giving displacements of 1, 2, 3, and 4 pixels in the top row of the template, near the "biased bulges" in the histograms. Although

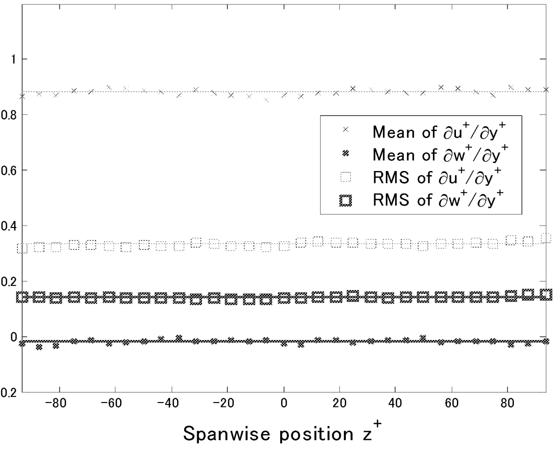

Figure 7. Local mean and RMS fluctuation of wall gradients from 1400 measurements. Straight lines are spanwise averages.

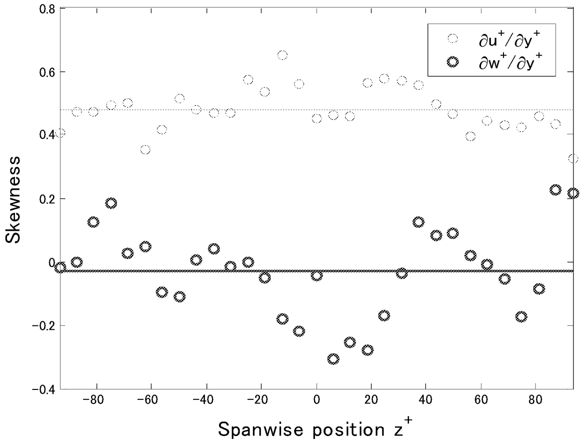

Figure 8. Local skewness of wall velocity gradient. Straight lines are spanwise averages.

we would expect, from the above discussion, for such bulges to lie outboard of the triangles, rather than slightly "inboard" as observed, the overall correspondence leaves little room for doubting our hypothesis.

Figure 6 shows the two-dimensional histogram of the rate of bed shear. The biases observed in the 1D histograms yield streaks along the viewing directions of the two cameras, and such bias clearly limits the value of this experimental frequency distribution.

In view of this bias, our eyeballed error estimates for the low-order moments of the distribution in figure 6 are 3% for the mean, 8% for the r.m.s. fluctuation of ∂u/∂y, and 12% for the r.m.s. fluctuation of ∂w/∂y. Figs. 7 and 8 show the profiles of the moments of the 1400 gradient data at each point along the wall; the mean and r.m.s. fluctuation appear well converged, but not the skewness.

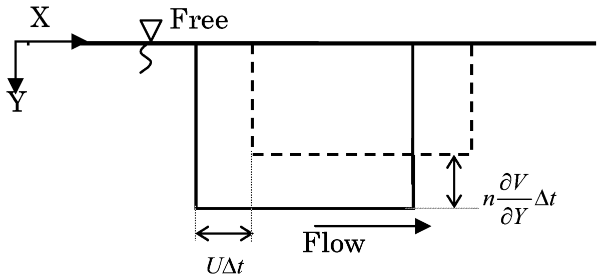

Figure 9: Translation of template in first image (solid line), followed by deformation by surface-normal gradient ∂V/∂Y.

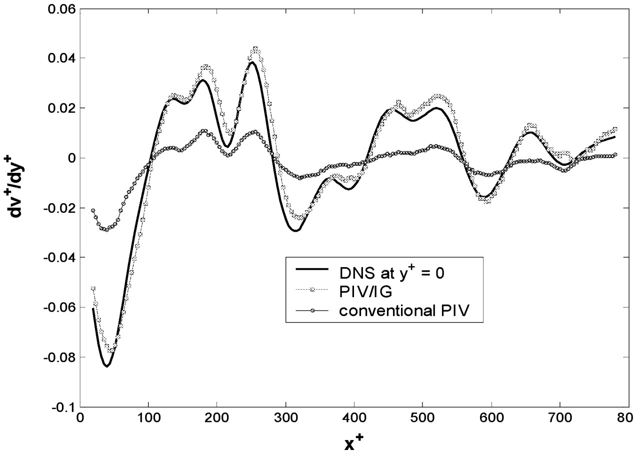

Figure 10: $\partial V^+/\partial Y^+$ derived from a DNS velocity field, compared with local sample averages from standard PIV and PIV/IG applied to 20 synthetic image pairs. Template size of 45x25pixels^2, equivalent to 18x10 wall units at Re_τ=240.

GRADIOMETRY NEAR A FREE SURFACE

This section, abstracted from Phan *et al* (2005), presents the extension of the PIV/IG technique to measure the surface-normal compressive velocity gradient at a free surface, and tests the method with synthetic images from simulated open channel flow for the two-component ("2C") case. We assume zero Froude number, *i.e.* fluctuations of the water surface are considered negligible. The free surface is accordingly taken to satisfy free-slip with, additionally, zero stress at the surface:

$$\frac{\partial u}{\partial y} = \frac{\partial w}{\partial y} = v = 0 \qquad (3)$$

Consider 2C PIV images in which the water surface is horizontal. In this implementation of PIV/IG, standard PIV is first applied to determine the average translation of each template. Then, applying the principle of the PID technique, the template of the first image is deformed by many trial values of the surface-normal velocity gradient ∂V/∂Y over a search range, with re-sampling again performed by bi-cubic interpolation. Translated and deformed templates are cross-correlated with the template of the second image, analogously to PIV/IG at a fixed wall. Each trial value of vertical velocity gradient in the search range gives a correlation value, and by determining the location of the peak on the resulting correlation curve, we can estimate vertical velocity gradient at a position near the free surface.

Specifying the same tracer properties as for the wall velocity gradient measurement, we synthesized images from a single known velocity field to test the accuracy of the 2-component PIV/IG technique. We used a single instantaneous velocity field generated by well-resolved (192 grid points in the vertical) Direct Numerical Simulation (DNS) of turbulent open channel flow at $Re_\tau=240$ and zero Froude number. Tracer particle velocities within a streamwise-vertical virtual light sheet were interpolated by cubic-spline interpolation from the DNS grid. The size of a synthetic image is 2000x250 pixels^2, corresponding to a field of view of 800x100 wall units^2, *i.e.* 2.5 pixels per wall unit. (Note that the finest turbulence scales at the free surface are several times larger than those at the wall in such a flow).

Figure 10 shows longitudinal profiles of the sample averages of $\partial v^+/\partial y^+$ calculated from 20 pairs of synthetic PIV images using standard PIV and PIV/IG with template sizes of 45x25pixels^2, as compared with those derived from DNS in open channel flow at $Re_\tau=240$. This figure suggests that the velocity gradients measured by PIV/IG are much more accurate than those measured by simple finite-differencing of standard PIV. For these synthetic images, this new technique achieves higher S.N.R. in high gradient regions, as found for the fixed wall.

Figure 10 only shows the local sample averages of the PIV/IG results over 20 image pairs. For this template height, 25 pixels, local standard deviations over the 20 sample points average 0.0077 in wall units. Increasing the template height to 35 and 45 pixels reduced the averaged local r.m.s. deviation to 0.0044 and 0.0029 respectively. In this case the smaller random error obtained with a template height of 45 pixels more than compensates the slightly greater systematic error induced by the increased spatial filtering. Furthermore, some local bias occurs at extremal points of the curve for the smallest template (Fig. 10). One likely reason is that when template height is very small, the vertical displacement of tracers between images is too low, about 0.1 pixels in this case, to yield accurate values. To minimize total error in the current conditions, template height should be chosen larger than 25 pixels.

CONCLUSION

The "PIV/IG" technique proposed here is an extension to PIV for direct measurement of shear rate at a fixed wall, and of surface-normal velocity gradient at a free surface. It has been validated by synthetic PIV images in a standard 2-component PIV configuration. The results suggest that this technique is superior to differentiation of conventional PIV data for measuring velocity gradients at an interface. Like standard PIV, the accuracy and resolution depend on the choice of template size, particularly its height. In general, as height gets smaller, the velocity gradient approaches the exact value at the interface if there are sufficient tracer images in each template. However, excessively small template height yields larger random error, and bias due to low tracer displacement.

Preliminary data from stereo PIV/IG at the bottom wall of open-channel flow show that pixel locking remains as much an issue as with standard PIV. In future work, we hope to improve our experimental technique so as to mitigate the effects of pixel locking. We also plan to carry out stereo reconstruction of the wall-normal velocity gradient at the free surface, and thence to measure statistics of normal velocity gradient in open channel flows.

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