MODELLING THE ENTRANCE REGION IN A PLANE ASYMMETRIC DIFFUSER BY ELLIPTIC RELAXATION

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ABSTRACT

The flow in an asymmetric diffuser is computed with the non-linear v2f model suggested by Pettersson-Reif (2000). Its performance is compared with that of two linear eddy-viscosity models and all results are validated against the LES of Kaltenbach *et al.* (1999). Several modifications to the ε equation are considered and it is shown that this equation indeed plays an important role in the diffuser flow. It is shown that the non-linear relation improves the representation of the individual Reynolds stress components but deteriorates the predictions of the mean flow. Therefore an appreciable effort is spent on identifying the underlying mechanisms that control the evolution of the flow in the diffuser. It is shown that the near-wall resolution of the Reynolds stresses in the entrance region of the diffuser is of essential importance.

INTRODUCTION

Most flows of engineering interest that involve flow separation still defy reliable predictions. The asymmetric diffuser, first considered by Obi *et al.* (1993), constitutes a case that has been studied extensively. It is a particularly challenging test case since the flow exhibits a smooth, adverse pressure gradient (APG) driven separation. The nominally two-dimensional flow in the entrance region of the asymmetric diffuser constitutes a particularly interesting part of this flow configuration that has proven to be very difficult to predict using RANS based closure schemes. The asymmetric geometry causes very different flow developments along the flat and the inclined wall, respectively. The major part of the pressure-rise occurs over a distance less than 5 inlet channel heights (H) downstream the entrance of the diffuser ($x/H \leq 5$), and the maximum APG is reached at $x/H \sim 1$. The adverse pressure-gradient is counteracted by a favourable pressure-gradient eminating from the convexly curved surface on the inclined wall ($-1 \leq x/H \leq 1$). The flow physics is thus very complicated within the narrow entrance region of the diffuser. The combined effects of an adverse pressure gradient, streamline curvature, and the strongly inhomogeneous near-wall region are what makes this portion of the flow field especially challenging.

Apsley & Leschziner (1999) scrutinized the performance of a wide range of models, ranging from linear eddy-viscosity models (EVMs) to differential stress models, in the asymmetric diffuser case. They concluded that linear EVM's are unable to faithfully predict the asymmetric diffuser flow unless they are sensitized using mean strain/vorticity corrections. None of the EVM's considered by Apsley & Leschziner (1999) utilised the elliptic relaxation approach to account for the non-local near-wall effects. In fact, Durbin (1995*b*) demonstrated the importance of near-wall effects by applying his linear v2f-model to this problem – with reasonable success. The objective of the present study is to scrutinize the predictive capability of the v2f-model with focus on the entrance region of the diffuser. The importance of accounting for turbulence anisotropy in the mean momentum equations in this portion of the flow is investigated by applying the non-linear extension of the v2f-model proposed by Pettersson-Reif (2000). Due to the lack of experimental data in the entrance region, the model computations are validated against the carefully performed large-eddy simulation (LES) reported by Kaltenbach *et al.* (1999).

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CLOSURE MODELS

Throughout this study three turbulence models of various closure level have been used to compute the unknown Reynolds stresses in the momentum equations. Our, in terms of modelling approach, least sophisticated closure is the two-equation $k-\varepsilon$ model suggested by Abe *et al.* (1994) (hereafter referred to as the AKN model). The purpose of using this model in addition to the more advanced models was to provide a reference solution of a typical low-Reynolds number $k-\varepsilon$ model. The reason why the AKN model was employed is that this model performs well in other separated flows like the backward facing step and rib-roughened channel flows (Bredberg, 2002).

The linear v2f model used here is a model based on the work of Durbin (1995*b*), which has been slightly modified (e.g. Lien & Kalitzin, 2001) in order to enhance numerical stability. Full details of the model (including model constants) used are given in Cokljat *et al.* (2003). Note that the present computations do not employ the realizability constraint of Durbin (1995*a*) as its effect on the solution was almost negligible.

The above (linear) v2f model also forms the basis to which the non-linear extension of Pettersson-Reif (2000) was added. The extension was formulated explicitly in terms of the mean strain and rotation rate tensors and turbulence quantities available from the linear model.

Also worth mentioning here is that the non-linear contribution to $\overline{u_i u_j}$ do not contribute to production of turbulence kinetic energy in two-dimensional computations. Its only effect is to redistribute the available amount of k amongst the individual normal stress components. Therefore the only direct effect the non-linear model has on the results is that caused by the improved modelling of the source terms in the momentum quations. Indirectly, however, the production is altered as the mean strain rate is affected by the Reynolds stresses.

As the non-linear model provides a fairly accurate representation of the individual Reynolds stress components it was decided to investigate the importance of turbulent diffusion by implementing a so called general gradient diffusion hypothesis;

$$D_\phi = \frac{\partial}{\partial x_m}\left(C_\phi \overline{u_m u_n} T \frac{\partial \phi}{\partial x_n}\right) \tag{1}$$

This model was tested for $\phi = k, \varepsilon$ but was found to only have minor influence on the flow in the diffuser. Therefore it was concluded that (modelled) turbulent diffusion is not very important and that the k and ε equation are dominated by their source terms. For the same reason the standard eddy diffusivity model was used for all computations reported here.

Modifications to the ε Equation

Several modifications of the production coefficient $C_{\varepsilon 1}$ in the ε-equation have been considered in the present study. This constant is known to affect the growth rate of shear layers and might therefore be an important feature of the mechanism controlling separation. The $C_{\varepsilon 1}$ expression of the linear v2f model can be written as

$$C_{\varepsilon 1} = A\left(1 + Bg\right) \tag{2}$$

with $g = \sqrt{k/\overline{v^2}}$. As walls are approached the function g grows large ($g \sim 1/y$). As a consequence $C_{\varepsilon 1}$ also grows which

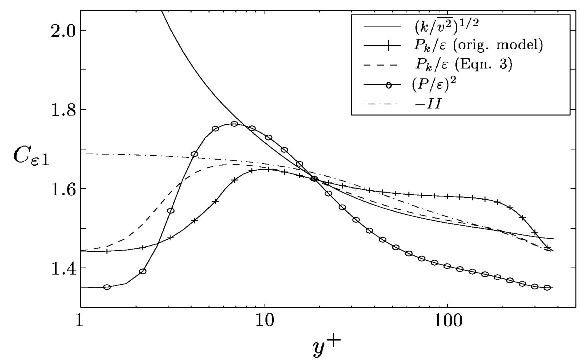

Figure 1: Profiles of $C_{\varepsilon 1}$ computed with different functions g. The functions used are given in the legend. The corresponding model constants A and B (c.f. Eqn. 2) are given in Table 1.

	$\sqrt{k/\overline{v^2}}$	P_k/ε	$(P_k/\varepsilon)^2$	$-II$
A	1.4	1.44	1.35	1.44
B	0.045	0.1	0.13	1.0

Table 1: $C_{\varepsilon 1}$ function constants.

in turn increases the dissipation rate. Therefore, a straightforward way to alter the nearwall dissipation would be to replace g with some other function with a different limiting behaviour.

Different choices of function g were investigated. They were all chosen such that the resulting expression for $C_{\varepsilon 1}$ varies from $C_{\varepsilon 1} \approx 1.3$ in the freestream to $C_{\varepsilon 1} \approx 1.6$ in the nearwall region. The different $C_{\varepsilon 1}$ profiles used are shown i Figure 1. Note that all modifications to $C_{\varepsilon 1}$ used in the diffuser were first calibrated in fully developed channel flow computations.

As it is customary to use $g = P_k/\varepsilon$ within the framework of differential stress models this idea is also adopted here. However, using P_k/ε explicitly available from the linear model, gave a $C_{\varepsilon 1}$ peak too far away from walls (cf. Fig. 1). Instead an expression for P/ε often used in EARSM was tried. It reads

$$\frac{P_k}{\varepsilon} = \frac{\alpha \mathcal{S}^2}{1 - \beta_1 \mathcal{S}^2 + \beta_2 \mathcal{W}^2} \tag{3}$$

where $\mathcal{S} = k/\varepsilon\sqrt{2S_{ij}S_{ji}}$ and $\mathcal{W} = k/\varepsilon\sqrt{2W_{ij}W_{ji}}$. S_{ij} and W_{ij} are the components of the mean rate-of-strain and mean vorticity tensors, respectively. The constants were obtained from the algebraic solution and found to be $\alpha = 0.0567$, $\beta_1 = 0.00255$ and $\beta_2 = 0.0348$ (cf. e.g. Gatski & Speziale (1993)). With this form of g it was possible to find constants A and B that gave good results in the channel flow.

To avoid having a model sensitized to rotation only in the ε equation $\mathcal{W}$ was replaced with $\mathcal{S}$. The effect this had on the solution in our case was negligible. With $g = P_k/\varepsilon$ (from Eq. 3) the predicted mean velocity field was improved but still not as good as the field of the linear model. As the non-linear model predicts too high levels of turbulent kinetic energy (shown later in the result section), P_k/ε (from Eq. 3) was replaced by its square in an effort to further increase the rate of dissipation in regions where $P_k > \varepsilon$.

Finally, the potential of using the second invariant of the anisotropy tensor $b_{ij} = \overline{u_i u_j}/q^2 - 1/3\delta_{ij}$, defined as $II = -0.5 b_{ij} b_{ji}$, was examined. This quantity has a finite wall value and enables us, together with the quantities discussed above, to give $C_{\varepsilon 1}$ a fairly arbitrary shape that fulfills our farfield and nearwall 'bounds' of about 1.3 and 1.6, respectively.

NUMERICAL CONSIDERATIONS AND TEST CASE

The in-house code used, CALC-BFC (Boundary Fitted Coordinates), is a structured code using SIMPLEC and a co-located grid arrangement with Rhie and Chow interpolation (Davidson & Farhanieh, 1995). The momentum equations were discretised using the central differencing scheme, whereas the van Leer scheme was used for the turbulence quantities.

All inlet boundary conditions were obtained from a separate computation of a fully developed channel flow. At the outlet Neumann boundary conditions were applied for all quantities except for the pressure. The boundary condition for P was $\partial^2 P/\partial +^2 = 0$ at all boundaries, where $+$ is a surface normal vector.

The mesh consisted of 256 cells in the streamwise direction with 64 cells covering the height of the diffuser. y^+ values were always below 1.1 for the first wall adjacent cells. The extension of the numerical domain was $-10 \quad x/H \quad 40$.

The addition of the non-linear terms to the original linear v2f model introduces a rather strong coupling between the mean velocity field and the source terms in the momentum equations – the Reynolds stress derivatives. This coupling tend to make the non-linear model unstable. In order to reduce the instabilities to some extent 'numerical smoothing' of certain computed quantities was introduced. This was done by weighting cell derivatives of both the Reynolds stresses in the momentum equations and the mean velocity derivatives appearing in the non-linear $\overline{u_i u_j}$ expression with corresponding values from adjacent cells.

RESULTS

As mentioned above the overall aim of this study is to investigate the possible effects of introducing the non-linear extension to the standard v2f model suggested by Pettersson-Reif (2000). Previous numerical computations, e.g. Apsley & Leschziner (1999) and Kaltenbach *et al.* (1999), suggest that the immediate vicinity of the diffuser throat is the most crucial region to capture in order to correctly predict the large separated zone some distance into the diffuser. The reason is that in the throat the flow turns towards the inclined lower wall as near-wall fluid accelerates around the corner causing a suction peak. It turns out that the level of (near-wall) flow turning in this region, i.e. the magnitude of the suction peak, largely determines the size and location of the downstream separated region.

That is, if the predicted flow follows the inclined wall too closely (the x-momentum flux towards the inclined wall being large) the tendency of separation further downstream will be small, whereas less turning in the throat (low x-momentum flux) will cause strong downstream separation. Therefore, the analysis of the results presented herein will focus on the entrance region of the diffuser.

Figures 2 and 3 display the predicted mean velocity field and the Reynolds stress components that appear in the mean momentum equations, i.e. $\overline{uv}$, $\overline{v^2}$ and $\overline{u^2}$. The performance of the linear and non-linear v2f models are compared with the AKN $k-\varepsilon$ model and the large-eddy simulation of Kaltenbach *et al.* (1999). Also included are the results obtained with $g = (P_k/\varepsilon)^2$, P_k/ε evaluated using Eq. 3.

From Figures 3b-d it is evident that the linear v2f model is unable to quantitatively reproduce any feature of the primary stress components. However, as earlier shown by Durbin

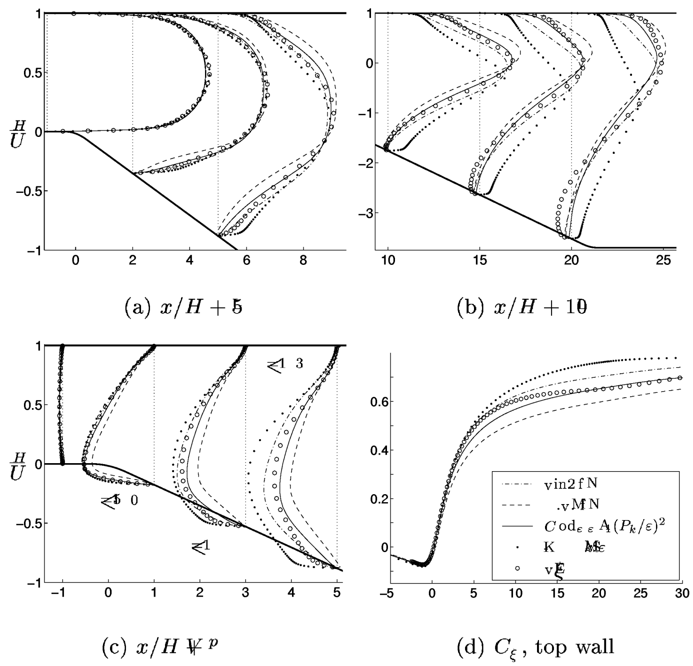

Figure 2: Various quantities related to mean flow properties.

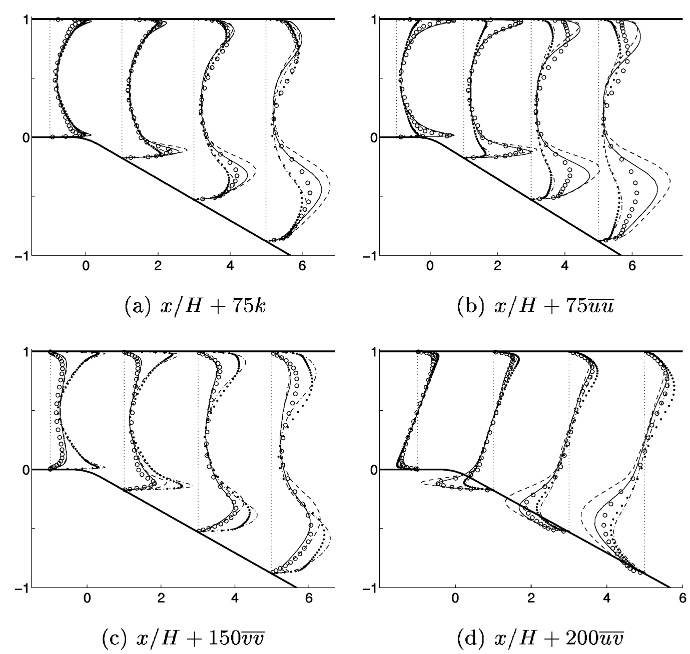

Figure 3: Profiles of various turbulence quantities. Symbols as in Figure 2d.

(1995*b*), the model is still able to predict the mean flow evolution reasonably well, especially just downstream the diffuser throat (Fig. 2a and c). Further into the diffuser (Fig. 2b) the agreement is not that good as the strength of the recirculation and the velocity at the top wall are both underpredicted.

The performance, in terms of the mean flow, of the non-linear model in the vicinity of the throat is worse. This model produce too a small flow turning, which is clearly illustrated by the velocity profiles (Fig. 2c). As the flow curvature is not that strong with the non-linear model the mass flux in the upper part of the diffuser becomes too large, which in turn gives room for a large separated region. The $k-\varepsilon$ model on the other hand severely overpredicts the flow turning in the diffuser throat. The result is that this model produce no separation at all.

The pressure distribution along the upper wall provides an *indication* of how large the separated region is. If the pressure does not rise rapidly enough the flow is not retarded as expected in the upper part of the diffuser and the recirculating region is large. If the pressure on the other hand rises too fast it indicates that the mass flow is evenly distributed throughout the diffuser and that the separated region is too small. In Figure 2d the pressure coefficient C_ξ is plotted for the different computations. As expected the linear v2f model gets the pressure distribution about right in the first half of the diffuser, whereas for $x/H > 10$ the extent of the separated region is somewhat underpredicted. The non-linear version that did not capture the flow turning must then also fail in reproducing the pressure. Clearly the increase in C_ξ is too small. The reason is the underprediction of the flow turning in the throat that has concentrated the mass flux towards the upper part of the diffuser. This is also supported by the velocities displayed in Figures 2a-b, which also suggests that the size of the separated region not necessarily related to C_ξ (as the recirulating motion is underpredicted by all models). Instead it is the extent of the region of 'main' streamwise flow that couples with C_ξ.

The failure of the non-linear v2f model appears puzzling given the quality of the turbulent stress predictions. While the reattachment point of the main separation region is fairly well predicted (cf. Fig. 4b), the separation occurs far too early. This is illustrated in Figure 5a, where the predicted friction coefficients just downstream the throat are plotted along the lower wall. In the following sections we will try to find the underlying reason to why the non-linear model produces less flow turning in the entrance region than the linear model does.

The Effect of the Individual Stress Components

In order to illustrate how sensitive the flow in the diffuser is to the predicted Reynolds stresses (their derivatives) the results of a 'numerical experiment' are shown in Figure 4. The plots show streamlines in and around the separated region. Figures 4a and b display results of the linear and the original non-linear v2f model (the latter was time-averaged as the non-linear model predicted an unsteady separation bubble in the very entrance of the diffuser). The latter predicts a slightly larger separated region than the linear version and in an effort to find out why, the non-linear contributions to the Reynolds stress tensor were modified in two different ways. Firstly, as it has earlier been proposed (Apsley & Leschziner, 1999) that a correct representation of the shear stress, $\overline{uv}$, is crucial in order to capture the flow in the diffuser throat, we used the linear expression for $\overline{uv}$ of the original v2f model, which gives a reasonable velocity field, simply by switching of the non-linear contribution to $\overline{uv}$. Secondly, the opposite was tried, i.e. the non-linear terms were used only when computing $\overline{uv}$ and the nearly isotropic relation of the linear model was adopted for the normal components. These modifications do only affect the flow in regions affected by the diffuser part of the domain, not in the plane channel upstream of it.

The effect of using the non-linear contribution to $\overline{uv}$ is shown in Figure 4c. Obviously it promotes the growth of the

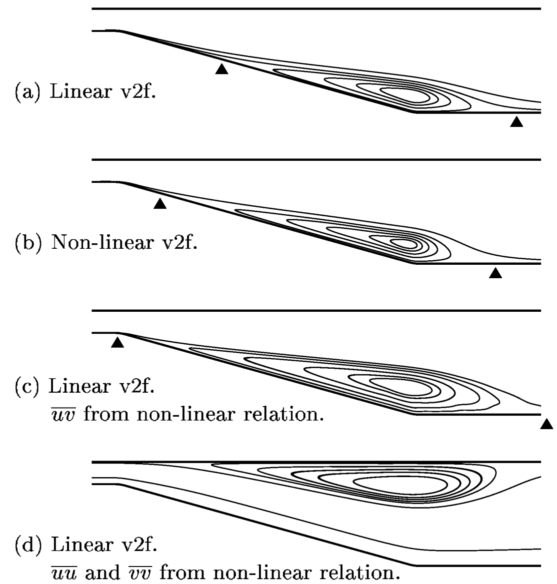

Figure 4: Streamlines illustrating separated regions in the diffuser. Separation and reattachment points marked by triangels.

separated region, which now completely covers the inclined wall. Finally, the corresponding effect of using the non-linear terms only when computing $\overline{uu}$ and $\overline{vv}$ is shown in Figure 4d. The effect is surprisingly strong. These terms do not just affect the size of the separated region but, which is even more interesting, also cause the separated region to move from the lower inclined wall to the opposit straigth wall. Although not shown here the separate effect of the two normal components was aslo investigated by switching either of the non-linear contributions to $\overline{uu}$ and $\overline{vv}$ off. It turned out that if either of the non-linear contributions were active the separation bubble erroneously moved towards the straight wall.

All in all, it seems that the non-linear terms of the *normal* stress components push the tendency for separation towards the upper straigth wall. This tendency is counteracted by the non-linear contribution to the shear stresses that amplifies the separated region along the lower wall. The combined effect of the non-linear terms is to increase the extent of the separated region along the inclined wall.

The Effect of Modifying $C_{\varepsilon 1}$

Our attention will now turn to the effect of the modifications to the $C_{\varepsilon 1}$ coefficient suggested above. Recall from previous sections that the intention of modifying $C_{\varepsilon 1}$ was to change the near-wall behaviour of the non-linear model. As already mentioned using the production to dissipation ratio available from the original linear model in the diffuser was never an option as this form of $C_{\varepsilon 1}$ gave poor results in the fully developed channel flow. If instead the P_k/ε expression given in Eqn. 3 was used the predictions of the Reynolds stresses were improved (this result is not shown here). Unfortunately this improvement had only minor effects on the mean flow that was somewhat more accurately predicted. Varying the constants A and B had little influence on the mean flow results for this form of g. Another more interesting effect this modification had on the behaviour of the non-linear model was that it seems to make the model numerically more stable.

This has to be due to the fact that the lower levels of $C_{\varepsilon 1}$ near walls (cf. Fig. 1) decrease the dissipation rate, which in turn increases the predicted levels of eddy viscosity.

Using $g = -II$ neither gave any substantial improvement in terms of mean velocity profiles. However, when $g = (P_k/\varepsilon)^2$ was employed it was found that the extent of the separated region could be controlled by manipulating the constants A and B. Decreasing A, which leads to an increase in B in order to preserve the behaviour in the 1D channel flow, proved to reduce the size of the separated region. Values of $A = 1.3$ and $B = 0.17$ gave almost no separation at all, whereas $A = 1.35$ and $B = 0.13$ gave a 'bubble' size comparable to that of the LES. The latter set of constants is the one that has been used in all plots that include results of a modified $C_{\varepsilon 1}$ expression, and are always represented by solid lines. Increasingly larger values of A gave larger regions of separation.

With this form of g all results in Figures 2 and 3 are substantially improved. Even when the mean flow results of the non-linear model are compared with those of the linear one it is no longer certain that it is the latter that produce the overall best agreement (e.g. Fig. 2b and d). Also note that with this form of $C_{\varepsilon 1}$ the non-linear model is able to accurately reproduce the dip in friction coefficient associated with the tiny separation bubble seen in the LES (Fig 5a). Without the modification the extent of this bubble is overpredicted.

Also worth to emphazise here is that the smaller separated region predicted with the original non-linear model was not stationary in time. It produced a 'flapping' motion that did shed off small vortex like structures. As the resolved fluctuating energy was very small, only about half a percent of the modelled fluctuating energy, it is the authors' belief that the effect of this transient on the averaged solution is negligible.

However, two major concerns remain. The first one is the behaviour in the immidiate vicinity of solid walls. As seen in Figure 5b the friction coefficient along the upper wall is overpredicted by some 50-100%. The linear model does a far better job in this respect. This is particulary worrying considering that the velocity component, at least some distance away from the upper wall, is better represented by the modified non-linear model than by the linear model (Fig 2a and b). Our second concern is that the improved numerical stability found when using $g = P_k/\varepsilon$ was lost when this expression was replaced with $g = (P_k/\varepsilon)^2$. With the latter choice of function g there exists an undesired transient located around the position of the reattachment point. Again this transient is small but large enough to prevent a fully converged solution (the momentum residuals scaled with the inlet momentum flux reached a level of approximately 0.01, 0.001 is regarded as being fully converged).

An Explanation to the Different Model Behaviours

As the development of the non-linear model constitutes an ongoing effort it would be highly desireable to isolate the flow features that are responsible to why the non-linear addition to the v2f model can have such a negative impact on the mean flow. A related question of equal interest is why the linear v2f model produces a much more realistic velocity field than the AKN model does. Note that the stresses predicted by the two linear models investigated here (Fig. 3) are surprisingly similar.

As it is the author's belief that the downward turning of the flow in the diffuser throat is important the equation for the

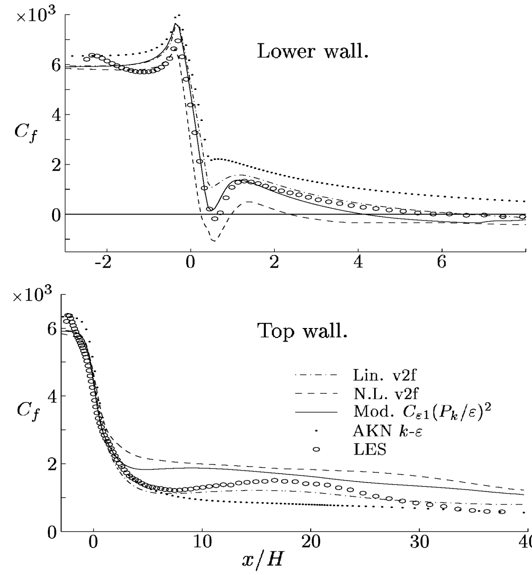

Figure 5: C_f along the lower and upper wall.

component velocity was further investigated. As shown in Figure 2c there is a substantial difference in the predictions of this component among the different models. Therefore there was reason to believe that the equation source term, $\partial \overline{v^2}/\partial y$, could explain some of the differences in the results as the predictions of $\overline{v^2}$ using the linear models (the original v2f or the AKN model) are very different from that of the non-linear one. However, it seems that, just as in the fully developed region of the flow, the poor prediction of $\partial \overline{v^2}/\partial y$ to a large extent is balanced by the pressure derivative $\partial P/\partial y$. It was soon also realized that it is not an error in an individual 'local' stress component that makes the difference seen in Fig. 2c. Such a distributed difference across the entire height of the diffuser must be due to differences in the predicted pressure field.

Another conclusion drawn was that the stresses in the regions shown in Figure 3 is of no use explaning the large differences between the models. The reason is twofold. Firstly, the linear v2f model and the AKN model give almost identical results, which are very different from the LES, and only the former gives a reasonably accurate prediction of the velocity field. Secondly, the non-linear v2f model resolves the anisotropy in rather good agreement with LES, but cannot provide a reliable estimate of the mean flow. Therefore we turned our attention to the nearwall behaviour of the models.

Figure 6 displays the Reynolds stress derivative source terms that appear in the equation. The two top plots show the individual terms, -$\partial \overline{uu}/\partial x$ and -$\partial \overline{uv}/\partial y$, whereas the bottom one shows their sum. The different terms were evaluated at a constant height above the lower wall. In the region upstream the diffuser this height corresponded to $y^+ \approx 25$. In order to assure that the effect of the non-linear constitutive relation was isolated from that of a poorly predicted velocity field the source terms of the non-linear model were computed from a velocity field of the linear model. We thus see explicitly where the non-linear relation alter the momentum equation source terms. Also added are the results of the AKN model and the LES.

It is clear that none of the models is able to predict the rapid variations of the near-wall Reynolds stresses. A trend is that

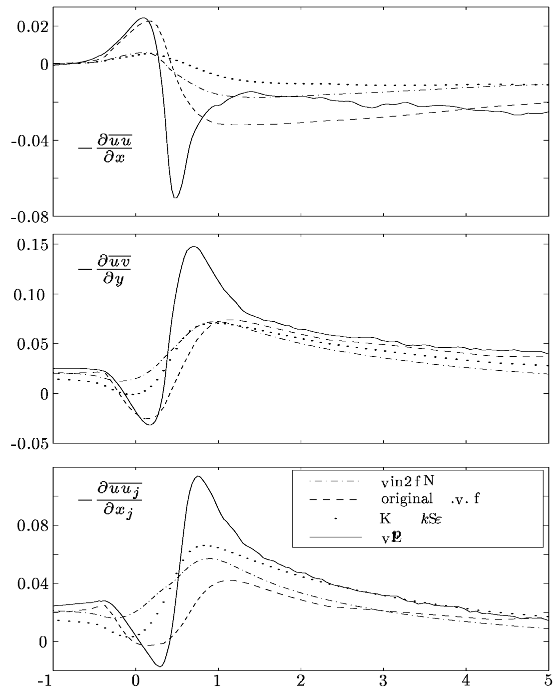

Figure 6: The Reynolds stress derivative source terms in the -equation. All quantities are evaluated a distance $0.025H$ above the inclined lower wall. This height corresponds to $y \approx 25$ in the region upstream the diffuser.

the non-linear model seems to initially (x/H 0.25) produce stresses in agreement with the LES but, as the other models, fails completely in capturing the strong increase in Reynolds stresses where $\partial\overline{uv}/\partial y$ dominates the -equation source term. This is believed to be the reason why the velocity field of non-linear model is not as accurate as that of the linear version. As the linear v2f does not capture the initial source term minimum that decelerates the near wall fluid it does not suffer severely from failing to capture also the downstream source term maximum as the effect of these errors seem to cancel each other. Note that a source term of positive sign, as predicted by the linear v2f model, corresponds to a force in the streamwise (x) direction.

It also seems that the difference between the linear v2f and the AKN models can be explained with the same mechanism. The latter model, that produced no separation, predicts higher values of the source term sum in the interval 0.5 x/H 4. The main source term difference between these models origins from the $\partial\overline{uu}/\partial x$ term.

CONCLUSIONS

The performance of a non-linear v2f model in separated flows has been examined by computing the flow in an asymmetric diffuser. The model has been compared with a linear v2f model, the AKN k-ε model and data from a LES. It was found that for any model to correctly predict this flow capturing the flow evolution in the region 0 x/H 2 is crucial. Here exists a delicate balance between Reynolds stresses in the near-wall region. Failing to predict this balance has dramatic effects of the downstream flow. It was also found that the dissipation rate equation plays an important role. A few modifications to this equation that improved the performance of the non-linear model were suggested but need to be studied further. Rapid changes in turbulence quantities exists in the diffuser. Still, it seems that the choice of diffusion model has only limited effect on the predicted mean velocity field.

Finally, an explanation to the differences in mean flow predictions amongst the models was sought. Our candidate is to be found in the near-wall region momentum equation source terms. It suggests that the non-linear model fails partly because it captures the very first part of the entrance region well and that the linear model works better because of two errors cancelling each other.

References

Abe, K., Kondoh, T. & Nagano, Y. 1994 A new turbulence model for predicting fluid flow and heat transfer in separating and reattaching flows–1. Flow field calculations. *Int. J. of Heat and Mass Transfer* **37**, 139–151.

Apsley, D. & Leschziner, M. 1999 Advanced turbulence modelling of separated flow in a diffuser. *Flow, Turbulence and Combustion* **63**, 81–112.

Bredberg, J. 2002 Turbulence modelling for internal cooling of gas-turbine blades. PhD thesis, Dept. of Thermo and Fluid Dynamics, Chalmers University of Technology, Göteborg, Sweden.

Cokljat, D., Kim, S., Iaccarino, G. & Durbin, P. 2003 A comparative assessment of the $\overline{v^2}-f$ model for recirculating flows. AIAA-2003-0765.

Davidson, L. & Farhanieh, B. 1995 CALC-BFC: A finite-volume code employing collocated variable arrangement and cartesian velocity components for computation of fluid flow and heat transfer in complex three-dimensional geometries. Rept. 95/11. Dept. of Thermo and Fluid Dynamics, Chalmers University of Technology, Gothenburg.

Durbin, P. 1995*a* On the $k-\varepsilon$ stagnation point anomaly. *International Journal of Heat and Fluid Flow* **17**, 89–90.

Durbin, P. 1995*b* Separated flow computations with the $k-\varepsilon-v^2$ model. *AIAA Journal* **33**, 659–664.

Gatski, T. & Speziale, C. 1993 On explicit algebraic stress models for complex tubulent flows. *Journal of Fluid Mechanics* **254**, 59–78.

Kaltenbach, H., Fatica, M., Mittal, R., Lund, T. & Moin, P. 1999 Study of flow in a planar asymmetric diffuser using large-eddy simulation. *Journal of Fluid Mechanics* **390**, 151–185.

Lien, F. & Kalitzin, G. 2001 Computations of transonic flow with the $\overline{v^2}$-f turbulence model. *International Journal of Heat and Fluid Flow* **22**, 53–61.

Obi, S., Aoki, K. & Masuda, S. 1993 Experimental and computational study of turbulent separating flow in an asymmetric plane diffuser. In *Ninth Symposium on Turbulent Shear Flows, Kyoto, Japan*, p. 305.

Pettersson-Reif, B. A. 2000 A nonlinear eddy-viscosity model for near-wall turbulence. AIAA paper 2000-0135.