THE STRUCTURE OF A TURBULENT AXISYMMETRIC JET VIA A VECTOR IMPLEMENTATION OF THE PROPER ORTHOGONAL DECOMPOSITION

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ABSTRACT

In this paper the large-scale structure of a high Reynolds number axisymmetric turbulent jet is characterized experimentally by an implementation of the proper orthogonal decomposition (POD) that utilizes all three fluctuating velocity components. The focus of the work is on the structure in the near-field of the jet due to its importance in numerous technical applications. POD eigenspectra provide the modal energy distribution as a function of both temporal frequency (or Strouhal number) and azimuthal mode number. The paper documents the streamwise evolution of the distribution of energy with azimuthal mode number. Differences arising between scalar and vector implementations of the POD are highlighted. Upstream of the tip of the jet core, the eigenspectra are found to exhibit peak amplitude at a constant Strouhal number based on local mean velocity and momentum thickness for a given azimuthal mode number.

INTRODUCTION AND OBJECTIVES

In this paper the large-scale structure in the near-field of a high Reynolds number axisymmetric jet is studied experimentally by a vector implementation of the proper orthogonal decomposition (POD). This work is focused on extracting POD eigenfunctions and their associated eigenvalues at selected streamwise locations throughout the jet initial region. As was the case in Gordeyev and Thomas (2000, 2002), a summation of the most energetic POD modes is considered synonymous with the term "large-scale structure". The focus on the near field of the axisymmetric jet is motivated by its importance in many technical applications such as propulsion systems, chemical mixers and jet noise.

The proper orthogonal decomposition (POD) proposed by Lumley (1967, 1970) for the investigation of inhomogeneous turbulent shear flows is an example of a non-conditional structure eduction technique that is based on the two-point velocity correlation. The analysis of turbulent shear flows by the POD is the subject of the monograph by Holmes et al. (1996). The POD eigenmodes provide an optimal basis for expansion of the flow in the sense that energy convergence is more rapid than for any other basis. If the energy convergence with mode number is sufficiently rapid that a dominant portion of the flow field energy is captured by only a few modes, then it is reasonable to associate the empirical eigenfunctions with the large-scale, energy-containing eddy structure in the flow.

This investigation is certainly not the first to apply the POD to characterize axisymmetric turbulent jet flow structure. It is, however, the first to provide a full, vector implementation utilizing all three fluctuating velocity components. The first implementation of the POD in the near-field of an axisymmetric jet is reported by Glauser et al. (1987). This scalar implementation based on the streamwise fluctuating component of velocity demonstrated a rapid energy convergence with POD mode number. The first POD mode was found to contain nearly 40% of the turbulent energy, whereas, the first 3 POD modes captured nearly all the resolved turbulent kinetic energy. This work was subsequently extended by Glauser and George (1987) in order to examine the azimuthal mode number dependence of the large-scale jet structure. Citriniti and George (2000) used an impressive polar array of 138 synchronized straight wire probes at a fixed location three diameters downstream of the nozzle exit to acquire simultaneous single-component realizations of the flow in the (r, θ) cross-flow plane. The POD eigenfunction basis was projected onto these instantaneous realizations, thereby capturing the local temporal dynamics of the structures. In a two-component implementation of the POD in the compressible axisymmetric jet, Ukeiley and Seiner (1998) and Ukeiley et al. (1999) examined the azimuthal dependence of the POD eigenspectra at Mach numbers of 0.3 and 0.6. More recently, Jung, Gamard and George (2004) and Gamard, Jung and George (2004) used essentially the same hot-wire array as Citriniti & George (2000) to investigate the evolution of modal energy content in the near field of the jet over the streamwise range $2 \leq x/D \leq 6$ and in the far-field ($20 \leq x/D \leq 69$). As in the work by Citriniti, only the streamwise fluctuating component was measured. The experiments showed that the POD eigenspectra varied significantly in the downstream direction. They demonstrated a progressive shift of the dominant azimuthal mode from $m = 0$ at $x/D = 3$ to azimuthal mode $m = 2$ by $x/D = 6$.

The objective of the present investigation is to experimentally characterize the coherent structure in the near-field of a high Reynolds number axisymmetric jet via a full, three-component implementation of the POD. The POD eigenfunctions and associated eigenvalues are experimentally extracted in a manner similar to that employed by Delville et al. (1999) in the planar mixing layer and Gordeyev and Thomas (2000) in the planar jet. This provides an objective description of the time-averaged coherent structure in the axisymmetric jet flow field in a mixed Fourier-physical domain.

EXPERIMENTAL FLOW FIELD FACILITY

The experiments were performed in a compressible axisymmetric jet facility located at the Center for Flow Physics and

Control at the University of Notre Dame. The facility is operated in the blowdown mode. A three-stage compressor provides air that is dried, filtered and heated before being stored in a large tank. During discharge, the compressor, dryer and electric heater are decoupled from the storage tank and flow is initiated by opening a pneumatically controlled gate valve. The stagnation pressure in the plenum chamber upstream of the axisymmetric nozzle assembly is maintained at a constant, preset value by a pressure controller with pneumatic feedback control. The plenum chamber is lined with acoustic absorbent foam in order to minimize the propagation of noise from upstream piping to the jet nozzle assembly. The axisymmetric jet nozzle has a contraction ratio of 9 : 1 and an exit diameter of $D = 5.06\ cm$. The nozzle contraction is formed from a fifth-order polynomial wall contour with zero-derivative end conditions. Unless otherwise noted, for the measurements presented in this study, the jet facility was operated with a nozzle exit velocity of $U_j = 110\ m/s$ which corresponds to an exit Mach number of $M = 0.3$ and a Reynolds number, $Re_D = 370,000$ (based on nozzle exit diameter). The flow discharges from the nozzle exit into a large anechoic chamber which provides an anechoic free-field environment with a demonstrated low-frequency cut-off of approximately 400 Hz. Measurements show that the mean velocity profile initially possesses a "top-hat" shape which is indicative of a uniform potential core flow bounded by a thin axisymmetric shear layer. Core turbulence intensity levels are less than $u_{rms}/U_j < 0.1\%$ (where the subscript rms denotes a root-mean-square value). The initial jet shear layer momentum thickness was determined to be $\theta_0 = 0.051\ mm$, or $\theta_0/D = 0.001$. The mean velocity variation across the shear layer is well approximated by a hyperbolic tangent profile shape. The jet shear layer widens with streamwise distance and engulfs the irrotational core flow near $x/D = 5.0$. Asymptotic mean velocity decay and spreading rates are in excellent agreement with those reported in the literature for other jet facilities. Although the nascent jet shear layer is laminar, autospectral measurements of the streamwise velocity component confirm that by the first POD measurement station, ($x/D = 3$), the jet shear layer was fully turbulent with a well defined $-5/3$ spectral roll-off.

IMPLEMENTATION OF THE POD

The experimental implementation of the POD requires measurement of the two-point velocity cross-spectral tensor at selected x/D locations and this process is expedited by the use of twin, cross-stream rakes of x-wire probes as shown in Figure 1. This rake arrangement is used to acquire the fluctuating velocities $u_\alpha(r, \theta, t)$ and $u_\beta(r', \theta, t)$. Greek subscripts denote fluctuating velocity components u (streamwise), v (radial) or w (azimuthal) while r and r' denote radial positions on the first and second rakes, respectively. Fourteen miniature x-wire probes fabricated by Auspex Corporation (type AHWX-100) are used. There are seven probes in each rake, equally spaced radially with one rake fixed and the other moveable in the azimuthal direction. The pivot point of the second rake is positioned at the centerline of the jet, i.e., at $r = 0$. The radial separation of the probes on each rake is $\Delta r = 0.635\ cm$. The moveable rake was positioned at intervals of $\Delta\theta = 15°$ in order to form the measurement grid shown in fig. 1. The rake assembly is mounted on a table with a provision to position it at streamwise locations within the range $0 \leq x/D \leq 12$. The required 28-channels of constant temperature hot-wire anemometry and associated anti-alias filters were fabricated in-house. The dynamic response of the transducers was found to be flat to 50 kHz. The anti-alias filtered output voltages from the hot-wire sensors were simultaneously sampled and digitized by means of a 36-channel MSXB data acquisition system made by MicroStar Laboratories.

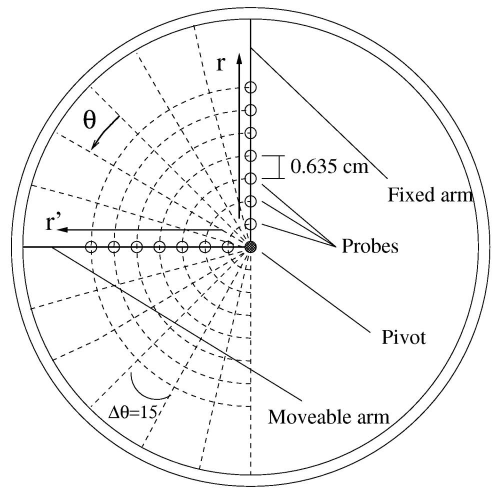

Figure 1: Schematic of the twin rake arrangement.

The cross-spectral matrix $S_{\alpha\beta}(r, r', \Delta\theta, f)$ may be obtained directly from Fourier transformation of the individual velocity-time histories,

$$S_{\alpha\beta}(r, r', \Delta\theta, f) = \lim_{T\to\infty} \frac{1}{T} < \hat{u}^*_\alpha(r, \theta, f)\hat{u}_\beta(r', \theta + \Delta\theta, f) >, \quad (1)$$

where $\hat{u}_\alpha(r, \theta, f)$ denotes Fourier transformation of the velocity vector for each block, T is the total time duration of the data block and the asterisk denotes a complex conjugate. A spatial Fourier transformation of the cross-spectral matrix in the homogeneous $\theta-$direction provides a discrete azimuthal mode number, m, dependent cross spectral tensor in Fourier-physical space.

$$\Phi_{\alpha\beta}(r, r', m, f) = \int S_{\alpha\beta}(r, r', f, \Delta\theta)e^{im\Delta\theta}\, d(\Delta\theta). \quad (2)$$

As shown in Lumley (1970), the spectral correlation tensor $\Phi_{\alpha\beta}(r, r'; m, f)$ is a kernel in the integral equation to find the POD modes for different frequencies f, and azimuthal mode numbers, m,

$$\int \Phi_{\alpha\beta}(r, r'; m, f)\varphi^{(n)}_\beta(r'; m, f)r'dr' = \lambda^{(n)}(m, f)\varphi^{(n)}_\alpha(r; m, f). \quad (3)$$

Here superscript n denotes POD mode number. The solution of eq. 3 gives a complete set of orthonormal eigenfunctions $\varphi^{(n)}_\alpha(r; m, f)$ with corresponding positive eigenspectra $\lambda^{(n)}(m, f)$. The eigenspectra $\lambda^{(n)}(m, f)$ are of particular interest since they represent the energy distribution in frequency-azimuthal mode number space for each of the extracted POD modes. From the above discussion, it is apparent that the problem of finding the POD modes is reduced to solving a number of integral equations (3) with m and f as parameters. In order to extract the POD modes, all components of the $\Phi_{\alpha\beta}$-matrix must be obtained. Unfortunately, the term Φ_{wv} cannot be measured directly using the x-wire rakes. However, it can be obtained from mass conservation requirements and measurements of the other components by using a procedure originally described in Ukeiley and Glauser (1995) and Ukeiley et al. (2001).

EXPERIMENTAL RESULTS

In this section experimental results are presented from the implementation of the POD in the axisymmetric jet. POD eigenmodes and associated eigenspectra were obtained in cross-stream (r, θ) planes for streamwise locations ranging from $x/D = 3$ to 8. These POD modes provide a structural template for the local, time-mean jet coherent structure in a mixed physical-Fourier domain.

POD Modal Energy Distribution

Consideration is first given to the rate of energy convergence of the POD modes with mode number n as expressed through their respective eigenspectra, $\lambda^{(n)}(m, f)$. The kinetic energy in each POD mode n, is obtained by summing over all frequencies and azimuthal mode numbers, $\sum_{m,f} \lambda^{(n)}(m, f)$. The energy content in each individual POD mode, $E_r(n)$, relative to the total resolved energy is then given by,

$$E_r(n) \equiv \frac{\sum\limits_{m,f} \lambda^{(n)}(m, f)}{\sum\limits_{n} \sum\limits_{m,f} \lambda^{(n)}(m, f)}. \quad (4)$$

The cumulative energy, $E_c(n)$, provides a direct measure of the rate of energy convergence of the POD modes with n and is given by

$$E_c(n) \equiv \frac{\sum\limits_{k=1}^{n} \sum\limits_{m,f} \lambda^{(k)}(m, f)}{\sum\limits_{n} \sum\limits_{m,f} \lambda^{(n)}(m, f)}. \quad (5)$$

Figure 2 presents both the relative energy content E_r and the cumulative energy E_c of the POD modes at two representative streamwise locations in the jet. At $x/D = 3$ the first POD mode accounts for approximately 41% of the total kinetic energy and this value increases with streamwise distance to 47% at $x/D = 8$. Examination of E_c indicates that the first four POD modes account for 84% of the total kinetic energy at $x/D = 3$ and 86% at $x/D = 8$. The dominance of mode 1 coupled with the rapid energy convergence exhibited in figure 2 supports the notion that a superposition of dominant POD modes is synonymous with the jet coherent structure.

POD Eigenspectra

Figure 3 presents the streamwise evolution of the mode $n = 1$ eigenvalue spectra, $\lambda^{(1)}(m, f)$. In this figure, the frequency is expressed as a Strouhal number based on nozzle diameter, D, and jet exit velocity, U_j. Note that these POD eigenvalue spectra are derived from the full $\Phi_{\alpha\beta}$ matrix. The mode 1 energy content clearly grows with streamwise distance (note the different ordinate scales used in fig. 3). For $x/D = 3$, the spectrum involves a wide range of azimuthal mode numbers but there is a clear tendency for lower azimuthal mode

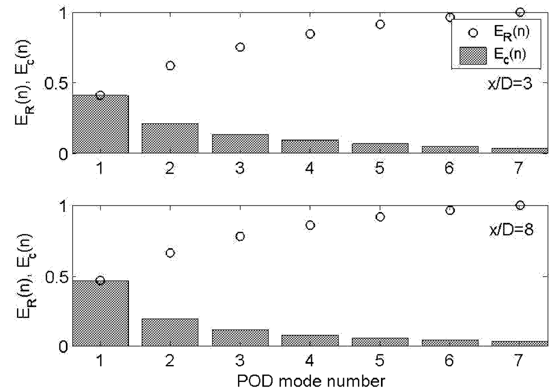

Figure 2: Relative and cumulative energy content of the POD modes at two representative x/D locations.

numbers to dominate with increasing downstream distance. By $x/D = 8$, fig. 3 shows that the peak at azimuthal mode number $m = 2$ dominates the spectrum. These eigenvalue spectra also indicate that significant fluctuation energy is limited to $St_D < 1$ and that a dominant portion of the fluctuation energy occurs for $St_D < 0.5$. The eigenvalue spectra of figure

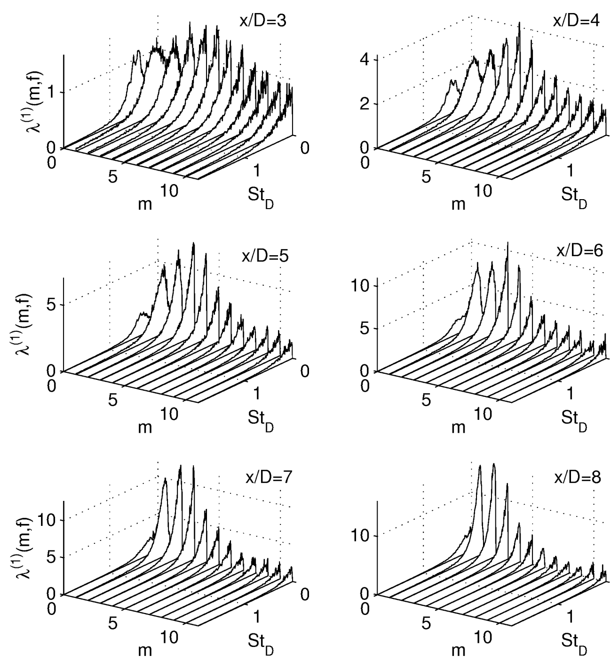

Figure 3: Eigenspectra for the first POD mode in the $(St_D - m)$ domain

3 may be integrated with respect to frequency (or equivalently, St_D) in order to more clearly indicate the streamwise variation in azimuthal mode energy content of the coherent structure. The relative azimuthal energy distribution, $\xi^{(n)}(m)$, is defined as

$$\xi^{(n)}(m) \equiv \frac{\sum\limits_{f} \lambda^{(n)}(m, f)}{\sum\limits_{n} \sum\limits_{m} \sum\limits_{f} \lambda^{(n)}(m, f)}. \quad (6)$$

Figure 4 presents the streamwise variation of the azimuthal energy distribution $\xi^{(n)}(m)$ for POD modes $n = 1$ and $n = 2$. This figure shows an initially broad distribution of energy over multiple azimuthal modes. The figure also clearly shows the preferred growth of lower, non-zero azimuthal mode numbers with increasing streamwise distance. However, the azimuthal mode $m = 1$ is observed to dominate both POD modes at all streamwise locations $3 \leq x/D \leq 8$ and this relative dominance clearly grows with streamwise distance. In contrast, the axisymmetric mode $m = 0$ exhibits little streamwise growth.

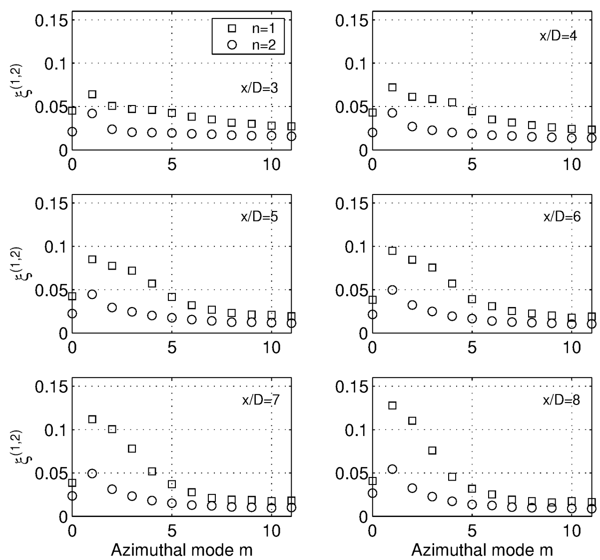

Figure 4: Azimuthal mode energy distribution $\xi^{(n)}(m)$ for POD modes $n = 1$ and $n = 2$.

As noted previously, Citriniti and George (2000) and Jung et al. (2004) utilized the POD to examine the large-scale structure in the axisymmetric turbulent jet. Both implementations involved measurement of POD eigenspectra based on the streamwise fluctuating component. In order to form a basis for comparison with these previous studies, eigenfunctions and corresponding eigenspectra based on only the streamwise fluctuating component can be obtained from the diagonal element Φ_{uu} by means of a scalar implementation of the POD,

$$\int \Phi_{uu}(r, r'; m, f)\varphi_u^{(n)}(r'; m, f)r'dr' = \lambda_u^{(n)}(m, f)\varphi_u^{(n)}(r; m, f), \tag{7}$$

where $\varphi_u^{(n)}(r; m, f)$ and $\lambda_u^{(n)}(m, f)$ are the u-component eigenfunction and associated eigenvalue spectrum, respectively. Similarly, eigenfunctions and corresponding eigenspectra based on the v and w fluctuating velocity components can be obtained from equation 7 with Φ_{vv} and Φ_{ww} the respective kernels.

For POD modes $n = 1$ and $n = 2$, eigenspectra $\lambda_u^{(n)}(m, f)$ were obtained and subsequently integrated over all frequencies to obtain $\xi_u^{(n)}(m)$ which is defined exactly as in equation 6 except that $\lambda_u^{(n)}(m, f)$ is used. The numerator is the integral of the u-component eigenvalue spectrum over all frequencies and the denominator is the total resolved u-component energy. Figure 5 compares the azimuthal distribution of energy $\xi_u^{(n)}(m)$ as determined in this study with corresponding values obtained by Jung et al. (2004). The comparison is made at several streamwise locations in the near-field of the jet. Except for the axisymmetric mode $m = 0$, this comparison shows remarkable agreement at each measurement location. The probe array used in this study can resolve $m \leq 11$ while that used by Jung et al. (2004) resolves $m \leq 15$. The comparison provides a direct assessment of the degree of spatial aliasing of higher azimuthal modes. Fig. 5 shows that disparities do occur for the highest mode numbers ($m \geq 7$) and this is clearly a manifestation of the spatial aliasing of higher wavenumber modes in the current study. More significant, however, is the very good agreement between the dominant eigenvalues at each location. All show a shift of peak energy from $m = 4, 5$ near $x/D = 3$ to lower azimuthal mode numbers with increasing streamwise distance and this leads to $m = 2$ dominating beyond the tip of the jet core ($x/D = 6$). Note also that the Reynolds number for the current study is over twice that in the study by Jung et al. (2004). The favorable agreement in the azimuthal distribution of energy $\xi_u^{(n)}(m)$ shown in Fig. 5 is consistent with the assertion of Glauser (1987) that once Re_D is sufficiently high then there is little dependence of the modal energy distribution on Reynolds number. As noted above, the

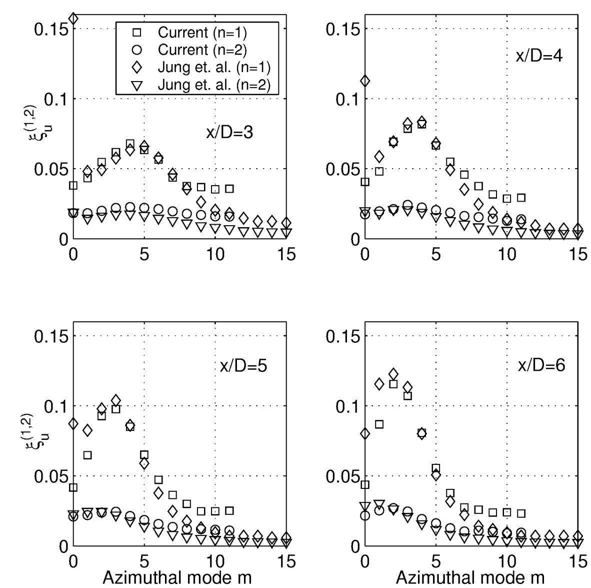

Figure 5: Comparison of the azimuthal mode number dependence of $u-$component eigenspectra.

largest disparity between the measurements occurs for the axisymmetric mode $m = 0$. Its modal energy content is initially quite high in relation to the helical modes in the experiment of Jung et al. (2004) but decays rapidly with streamwise distance. In the current experiment, the $m = 0$ mode is at low level and is essentially neutral, exhibiting virtually no growth with x/D. This disparity in behavior is likely associated with differences in initial conditions between the jet flow field facilities. In particular, the initial momentum thickness in the facility used by Jung et al. is estimated to be twelve times larger than in the current experiment. Hence, in terms of x/D, the shear layer development in the experiment of Jung et al. will be delayed relative to the current facility. In addition, extraordinary precautions were taken in the current experiment to avoid facility-dependent acoustic forcing of the nascent jet shear layer (which would be axisymmetric). This is not the case in the study by Jung et al. and it is therefore possible that the energy content of the $m = 0$ mode is artificially high near the jet lip. The key point to note, however, is that $m = 0$ is in decay in the measurements of Jung et al. and is tending towards values similar to those observed in the current experiment.

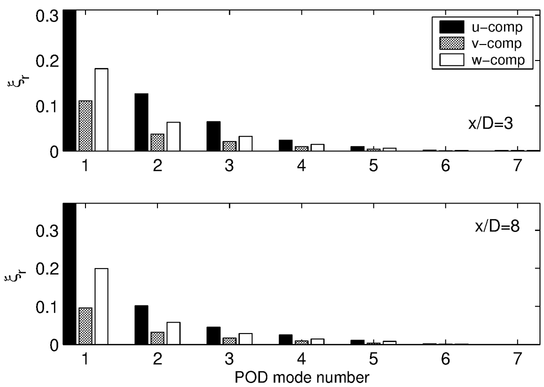

Figure 6: Relative energy in streamwise, radial and azimuthal components of the POD modes

In order to provide a measure of the relative energy content contributed by each fluctuating velocity component towards the total kinetic energy of the jet coherent structure, the ratio $\zeta_r(n;\alpha)$ is defined as,

$$\zeta_r(n;\alpha) \equiv \frac{\sum\limits_{m,f} \lambda_\alpha^{(n)}(m,f)}{\sum\limits_{n}\sum\limits_{\alpha}\sum\limits_{m,f} \lambda_\alpha^{(n)}(m,f)}, \quad (8)$$

where α represents any fluctuating velocity component (u, v, or w). The numerator is the total modal energy content for a given fluctuating velocity component. This is normalized by the total energy contained in the u, v, and w- component eigenvalues as obtained from their respective scalar implementations of the POD. Hence, the denominator is a metric that is at least related (but not necessarily equal) to the total energy of the coherent structure. Figure 6 presents the relative energy ζ_r for each velocity component versus POD mode number as obtained at two representative streamwise locations in the jet. Note that the energy content in the streamwise component is dominant followed, in turn, by the azimuthal and radial components. For POD mode 1 the azimuthal energy content is $53-59\%$ of that in the streamwise component while the radial component accounts for $30-36\%$. It is apparent that the sum of the modal energy contained in the azimuthal and radial fluctuating components is nearly equal to that in the streamwise component. Since the azimuthal and radial components contain significant modal energy relative to the streamwise component, there is a possibility of losing flow physics by neglecting the v and w-components and this provides one motivation for the vector implementation of the POD presented in this paper.

Strouhal Number Dependence of the POD Eigenspectra

Figure 7 presents POD mode $n = 1$ eigenspectra for azimuthal mode $m = 1$ as obtained at several streamwise locations throughout the near-field of the jet. The abscissa is a Strouhal number, St_D, based on the jet exit velocity, U_j, and the nozzle diameter, D. The measured eigenspectra of Figure 7 each feature a broad peak and the St_D associated with this peak decreases with x/D as shown in the inset diagram of the figure. It is expected that the temporal frequency associated

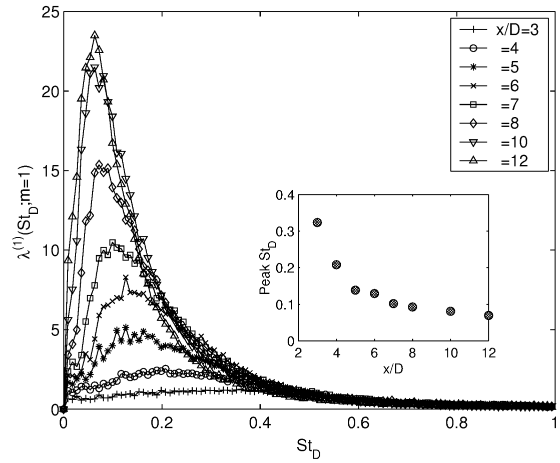

Figure 7: POD mode 1 eigenspectra for azimuthal mode number 1

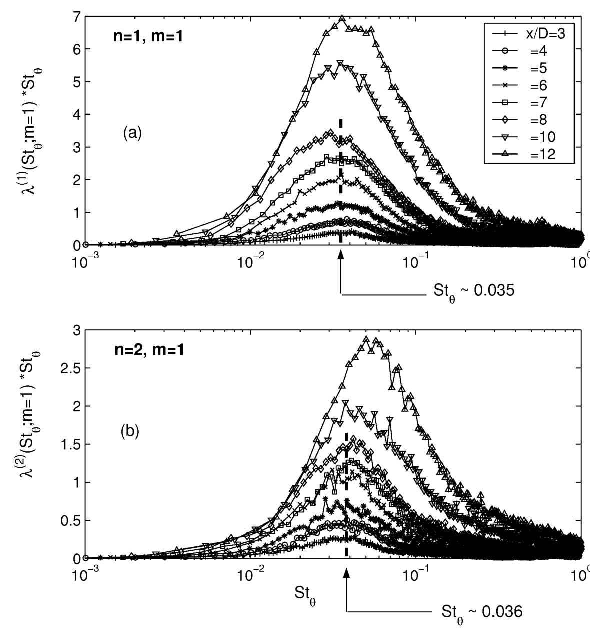

Figure 8: POD mode 1 and 2 eigenspectra for azimuthal mode number 1

with a particular POD mode is proportional to the convective speed of the structure past the fixed measurement location and inversely proportional to a local characteristic length scale associated with the structure. The convective speed will be some fraction of the local jet centerline velocity, U_{max}. Upstream of the jet potential core, it is reasonable to expect the relevant length scale to be a characteristic axisymmetric shear layer length scale like the local momentum thickness, θ. Figure 8 presents the same eigenspectra as shown in Figure 7 but with the frequency scaled in terms of Strouhal number $St_\theta = f\theta/U_{max}$. In order to properly emphasize the spectral peak, the eigenspectra are equivalently presented by plotting the product $St_\theta * \lambda^{(1)}(St_\theta; m = 1)$ versus St_θ on a logarithmic abscissa. This figure clearly shows that the peaks in the eigenvalue spectra occur at a constant value of $St_\theta \approx 0.035$. This scaling of the eigenspectra peaks with St_θ is consistent with the notion that the large-scale structure represented by the POD modes is a manifestation of jet shear layer modes of instability.

Figure 8b presents similar results for the POD mode $n = 2$, $m = 1$ eigenspectra where $St_\theta * \lambda^{(2)}(St_\theta; m = 1)$ is also presented as a function of $log(St_\theta)$. Again, for locations upstream and near the tip of the jet core, the eigenspectra peaks occur at a constant $St_\theta \approx 0.036$ which is approximately the same value noted for $\lambda^{(1)}(St_\theta; m = 1)$. The scaling of the spectral peak with St_θ is observed to break down beyond the jet potential core as expected. This scaling of the eigenspectra with St_θ is not restricted to the $m = 1$ azimuthal mode. Similarly scaled eigenspectra, $\lambda^{(1)}(St_\theta; m = 2)$, (not presented here) also exhibit collapse upstream of the jet potential core with the spectral peak occurring at a lower value $St_\theta \approx 0.025$. This serves to illustrate the general finding that eigenspectra for higher azimuthal modes numbers also exhibit collapse but are associated with lower St_θ values at peak amplitude.

DISCUSSION

In this paper, the azimuthal mode and Strouhal number energy distribution of the large-scale structure in an axisymmetric turbulent jet is examined via the POD. Disparities exist between the azimuthal mode energy distribution resulting from vector and scalar $u-$component implementations of the POD as illustrated by comparison of figures 4 and 5 at corresponding streamwise locations. Insight regarding the origin of this disparity comes from examination of eigenspectra based on the fluctuating $w-$component velocity which were obtained from equation 7 with Φ_{ww} the kernel. The eigenspectra $\lambda_w^{(n)}(m, f)$ were subsequently integrated over all frequencies to obtain $\xi_w^{(n)}(m)$. Figure 9 presents $\xi_w^{(n)}(m)$ for POD modes $n = 1$ and $n = 2$, at two representative streamwise locations in the near-field of the jet. The azimuthal mode $m = 1$ is observed to be dominant at each streamwise location. In fact, this was the case for all streamwise locations investigated in the near-field of the jet. Since the energy content of the $w-$component (relative to u) has been shown to be quite significant (see figure 6), it is conjectured that this is one reason for the dominance of azimuthal mode $m = 1$ in the vector implementation of the POD.

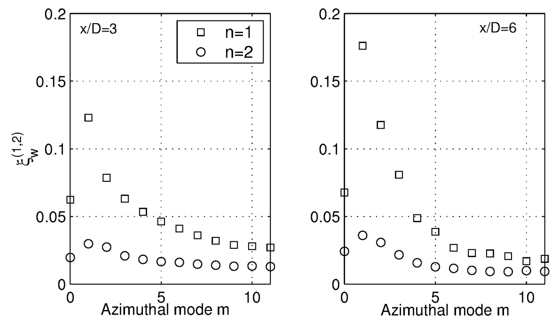

Figure 9: Comparison of the azimuthal mode number dependence of $w-$component eigenspectra.

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