A POSSIBLE SOLUTION TO THE LES WALL-MODELING PROBLEM

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ABSTRACT

In this paper, a possible solution to the long-standing problem of near-wall modeling in Large-Eddy Simulation (LES) is presented. There are two components to the approach. First, filtering with homogeneous filters is applied in all spatial directions, including the wall-normal direction. This has the effect of filtering the wall, and introduces an explicit wall term in the equations, which is modeled using a novel optimization technique. Second, an optimal LES model is used for the usual subgrid term in the interior of the flows. To test the validity of this approach, simulations were done with optimal LES models derived from DNS statistical data., with very good results. Research is ongoing to replace the DNS data with theoretical models.

INTRODUCTION

One of the pacing problems in the development of reliable large eddy simulation (LES) models for use in turbulent flows of technological interest is the so-called LES wall-modeling problem (Piomelli & Balaras, 2002). It arises because the length-scale associated with the wall layer of a turbulent wall-bounded shear flow (wall units) gets smaller relative to the shear layer thickness approximately like the inverse Reynolds number (like $Re_\tau^{-7/8}$ in the channel flow). The "large-scale" turbulence in this thin layer also scales in wall units. If the cost of an LES of wall-bounded flows is to remain finite in the limit of infinite Reynolds number, then this wall layer and the large-scale turbulence it supports cannot be represented directly, and so must be modeled. However, current LES models are generally not valid for this near-wall layer because underlying assumptions such as small-scale homogeneity and isotropy are not valid. The alternative is to resolve the near-wall turbulence. The most successful LES of wall-bounded shear flows employ this technique, though this is clearly not viable for arbitrarily large Reynolds number. In this paper, we propose a possible solution to this wall-modeling problem, consisting of two elements.

The first element is motivated by the observation that in an LES, locating anything, including the wall, to more precision than the filter width is inconsistent with the representation. This leads us to a formulation in which the wall is filtered as well as the turbulence. The second element of our proposed solution is the use of optimal LES models for the subgrid turbulence (Langford & Moser, 1999; Volker *et al.*, 2002; Zandonade *et al.*, 2004; Langford & Moser, 2004). In optimal LES, the subgrid force term (or the subgrid stress) is approximated using stochastic estimation. Optimal LES is a formal approximation to what we have called the ideal LES evolution (Langford & Moser, 1999), which can be shown to produce one-time statistics that are exact, and minimum mean-square variation in the instantaneous large-scale evolution. The Optimal LES formalism has the advantage in this context of being valid even in the absence of small-scale isotropy or homogeneity; that is, it is valid for near-wall turbulence. As input, optimal LES requires detailed two-point correlation data. For the purposes of testing the viability of the proposed wall-modeling approach, this data has been obtained from the direct numerical simulation data of Moser *et al.* (1999).

In the remainder of this paper, the filtered boundary formulation is introduced and a test of it's capabilities is presented. The optimal LES models used here are briefly described. The results of filtered boundary LES of the turbulent channel at $Re_\tau = 590$ are then presented in followed by a brief discussion of the implications of this work.

FILTERED WALL FORMULATION

In the filtered boundary LES formulation, the wall-bounded domain is embedded in larger domain, with the Navier-Stokes equations applied to the interior, and $\mathbf{u} = \mathbf{0}$ applied to the exterior domains. A filter is then applied to the larger domain. In this paper, a Fourier cut-off filter was used in all cases. The resulting equations are:

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0$$

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_j}\tilde{u}_i + b_i + M_i \qquad (1)$$

where M_i is the usual LES model term and b_i is the boundary term. The model term is written $M_i = -\partial\tau_{ij}/\partial x_i$, where $\tau_{ij} = \frac{1}{2}(\widetilde{u_i u_j} - \widetilde{\tilde{u}_i \tilde{u}_j})$ is the subgrid stress. If a sufficiently fine filter width is used then M_i is negligible and the only effect is

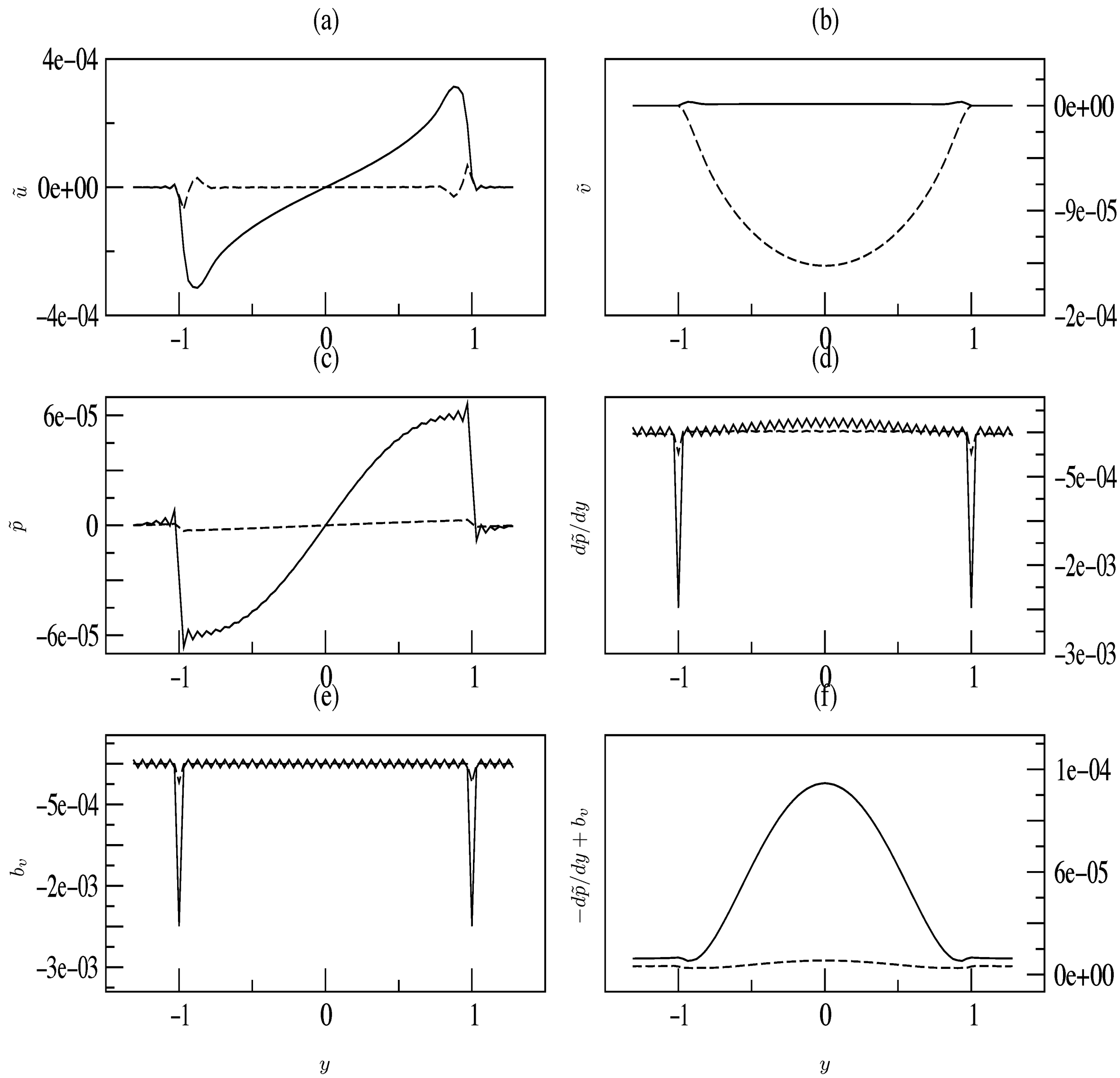

Figure 1: Effect of the boundary terms in the evolution of small disturbances in a channel flow. (a) filtered u velocity, (b) filtered v velocity, (c) filtered pressure, (d) pressure gradient, (e) boundary term for v equation, (f) pressure gradient + boundary term. — real part, - - - imaginary part.

the filtering of the boundary (i.e. a filtered boundary "DNS"). Such a "DNS" was used as a test case (see below).

The boundary term (b_i) can be written

$$b_i(\mathbf{x}) = \int_{\partial R} \sigma_{ij}(\mathbf{x}')n_j G(\mathbf{x}-\mathbf{x}')\,d\mathbf{x}'$$

where σ is the stress at the boundary, including pressure and viscous stress, ∂R is the boundary of the fluid region R and n_j is the unit normal to the surface.

In many LES of wall bounded flows, approximate boundary conditions are used to model the effect of the wall layer (Balaras *et al.*, 1996). The approximate boundary conditions are prescribed in terms of the wall shear stress, so wall stresses must be determined in terms of the resolved velocities. In the present formulation, the unfiltered wall stresses are also required, and for analogues reason.

In the current description, in which the unfiltered velocity is zero in the buffer domain, the wall stress is the surface forcing required to ensure that momentum and energy are not transfered to the buffer domain. That is, that the velocity remains zero. This suggests a technique for determining the wall stress. Instead of defining a force to make the velocity zero at the boundary as in embedded boundary numerical methods (Verzicco *et al.*, 1998; Mohd-Yusof, 1998), we choose σ_{wall} to minimize the transport of momentum to the exterior domain. To this end, the wall stresses at each time step are defined by minimizing

$$E = \int_{\mathcal{B}} |\tilde{\mathbf{u}}|^2 + \alpha \left|\frac{\partial \tilde{\mathbf{u}}}{\partial t}\right|^2 d\mathbf{x} \tag{2}$$

where the integral is over the buffer domain. The $|\tilde{\mathbf{u}}|^{\mathbf{2}}$ term

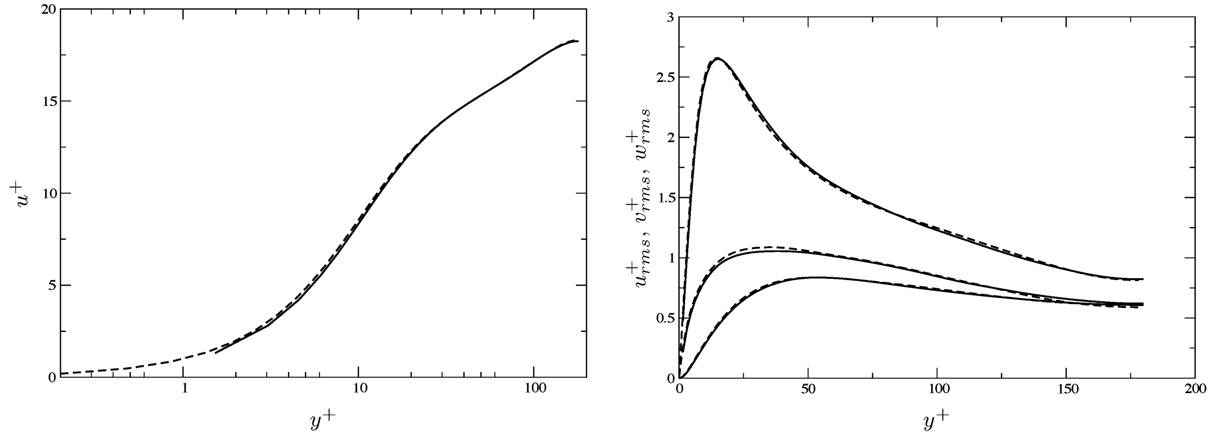

Figure 2: Mean (left) and rms (right) velocities in turbulent channel flow at $Re_\tau = 180$. ——— present using filtered boundary formulation, — — — Moser et. al. (1999)

forces the energy in the buffer domain to be small, and the $\alpha \left| \frac{\partial \tilde{\mathbf{u}}}{\partial t} \right|^2$ term ensures that the transfer of energy into the domain is small. The constant α controls the balance between these two competing requirements and is set to a value of order Δt^2. In the Fourier spectral method employed here, this minimization is straight forward since it can be done independently for each (k_x, k_z) wavenumber, resulting in a 6-parameter optimization in $(\sigma_{xy}, \sigma_{yy}, \sigma_{zy})$ Das & Moser (2001).

To evaluate this approach, we consider two test cases: propagation of an Orr-Sommerfeld wave and low Reynolds number turbulence in a channel. In both cases, the Fourier cut-off filter is fine enough to make the model term M_i negligible.

In the Orr-Sommerfeld case, the simulated growth rate was within 0.25% of the exact value for the case considered. More interesting is the role the boundary term plays. Consider the exact unfiltered pressure fluctuations. They are formally zero in the exterior, resulting in a discontinuity in pressure, and the resulting Gibbs phenomenon in the filtered pressure is shown in figure 1c. The wall normal pressure gradient appears in the v-momentum equation, and this quantity is dominated by the filtered delta function at the boundary and the resulting Gibbs phenomenon (figure 1d). Yet the Gibbs phenomenon in velocity perturbations in figure 1a and b is imperceptible. The reason is that the term b_v (figure 1e) has exactly the same structure as the pressure gradient and cancels the Gibbs phenomenon (figure 1f). The role of the boundary terms in the momentum equation is thus to regularize the stress discontinuities at the wall (both pressure and viscous stresses).

To assess the applicability of this technique in simulating turbulent flow, a fully developed channel flow is computed on a $128 \times 256 \times 128$ grid with 20 point in the buffer region. The friction Reynolds number is $Re_\tau = 180$ and the domain size is the same as in Moser *et al.* (1999). The mean and rms velocities from this simulation are in excellent agreement with those of Moser *et al.* (1999) (see figure 2), and the near wall turbulence exhibits the familiar structures, such as streaks and inclined shear layers.

OPTIMAL LARGE EDDY SIMULATION

Optimal LES is based on the observation that there is an ideal LES model, which guarantees correct single time statistics and minimum error is short-time dynamics (Langford & Moser, 1999; Pope, 2000). This ideal model given by

$$m_i = \langle M_i | \tilde{u} = w \rangle, \tag{3}$$

where m_i is the model for the term M_i in (1), w is the LES field, and $\tilde{u}$ is the filtered real turbulence. In essence, this is the average of M_i over all turbulence fields that map to the LES field through the filter. Unfortunately this model is intractable, so in optimal LES we approximate this model using stochastic estimation (Adrian, 1977; Adrian *et al.*, 1989; Adrian, 1990). In the LES performed here, the stochastic estimation formulation is simplified by the homogeneity of the channel flow in directions parallel to the wall, and the formulation must be further simplified to avoid problems of over generalization (Volker *et al.*, 2002). The linear stochastic estimate used here can thus be written:

$$\hat{m}_i(y) = \langle \hat{M}_i \rangle + K_{ij}(y)\hat{E}_j(y) \tag{4}$$

$$\langle \hat{M}'_i(y)\hat{E}^*_k(y) \rangle = \hat{K}_{ij}(y)\langle \hat{E}_j(y)\hat{E}^*_k(y) \rangle \tag{5}$$

where $\hat{}$ indicate the Fourier transform, and the event vector E_j is a vector consisting of the fluctuating LES velocities w'_j and their y derivatives. The correlations appearing in (5) must be determined to complete the model. For the purposes of the test described below, the correlations were evaluated using the DNS data from Moser *et al.* (1999) at $Re_\tau = 590$. Using DNS data allows the optimal LES formulation to be evaluated without uncertainties introduced by further modeling of the correlations.

It should be noted that this is the simplest model form of those proposed by Volker *et al.* (2002) for the channel, and that in Volker *et al.* (2002) models of this form performed poorly. The reason was that this form does not properly represent the wall-normal transport of energy and Reynolds stress. As pointed out by Härtel & Kleiser (1998), in the absense of wall-normal filtering, the contribution of the subgrid term to the resolved-scale energy equation is positive near the wall (see figure 3), which is due to the subgrid contribution to the transport of energy from the production peak to toward the wall. However, when coarse wall-normal filtering is employed as in the LES considered here, this structure is eliminated, and the subgrid energy contribution is nowhere positive. Volker *et al.* (2002) found that a more complicated form that did represent the wall normal transport produced a model that performed

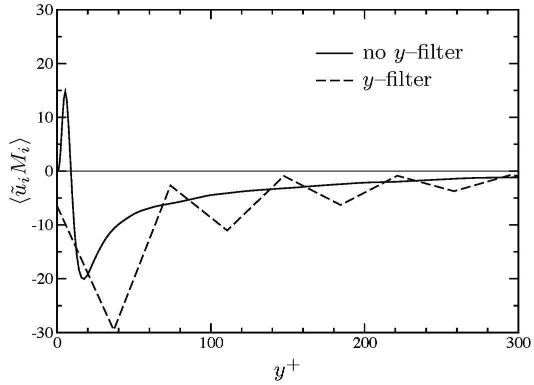

Figure 3: Subgrid energy transfer $\langle \tilde{u}_i M_i \rangle$ in turbulent channel flow, with and without wall-normal filtering.

very well. But with coarse wall-normal filtering, since the subgrid term is everywhere dissipative, the simple model given above should perform very well, as indeed it does (see below).

FILTERED-WALL LES RESULTS

The filtered boundary formulation and the optimal LES model were used to perform an LES of turbulent channel flow with bulk Reynolds number $Re_b = 10,950$ corresponding to a channel with $Re_\tau = 590$. Periodic boundary conditions were used in streamwise (x) and spanwise (z) directions, with domain sizes $L_x = 2\pi h$ and $L_z = \pi h$ (h is the channel half width). DNS of this case was performed by Moser *et al.* (1999), and optimal LES were performed by Volker *et al.* (2002).

To accommodate the filtered boundary formulation, a buffer region is added outside the channel and periodic boundary conditions are used in the extended wall-normal (y) domain. Fourier cut-off filters in each direction are used to define the large scales, with effective filter widths of $\Delta x^+ = 116$, $\Delta y^+ = 37$ and $\Delta z^+ = 58$ in the three spatial directions. In x and z, these are the same filters used in Volker *et al.* (2002). Note that these filter widths are sufficiently large to eliminate the structure of the near-wall viscous and buffer layers.

The filtered boundary model and the optimal LES model were used to perform an LES of the channel flow. The statistical correlations required as input to the optimal LES formulation were determined from the DNS of Moser *et al.* (1999). Sample results from this simulation are shown in figure 4. Note that despite the fact that the wall layer was not resolved, both the mean velocity and the rms velocities are in remarkably good agreement with the filtered DNS.

DISCUSSION

The results described above are intriguing because they suggest that it is not necessary to resolve the near-wall layer in an LES to obtain an accurate simulation of a wall-bounded flow. However, because the simulations reported here were based on knowledge of of statistical correlations obtained from DNS, the work presented here does not constitute a practical broadly-applicable LES model. For this, the need for DNS statistical data must be overcome. None-the-less, the current results do demonstrate the value of the wall-filtering approach, and the optimization model for the wall stresses. It would appear that this approach may form the foundation of a solution to the well-known LES wall modeling problem.

To relieve the need for DNS data, we are pursuing research on the theoretical and phenomenological representation of the near-wall multi-point velocity correlations. A combination of similarity scaling minimal empirical input may be sufficient to determine the required correlations. However, it is not clear whether the involved and sophisticated modeling of optimal LES is needed to take advantage of the filtered wall-formulation. The fact that the subgrid contribution to the energy equation is everywhere dissipative suggests that more standard models (e.g. Smagorinsky may applicable). This is currently being explored.

Finally, we note that the tests performed here were particularly arduous for the filtered boundary formulation because discontinuities in derivatives (as in the velocity at the wall are poorly represented by Fourier spectral methods, with Gibbs phenomena as the result. It is particularly remarkable, then, that the wall stress model used here is able to treat and largely cancel this Gibbs phenomenon.

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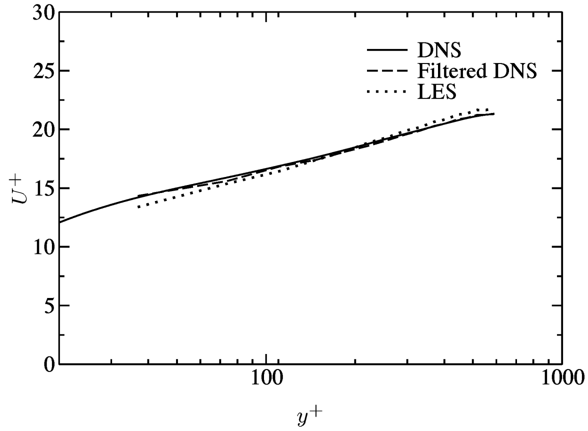

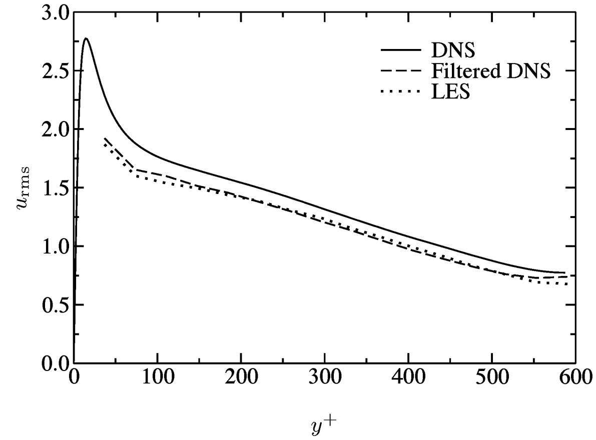

Figure 4: Mean velocity (left) and rms streamwise velocity fluctuations (right) in a turbulent channel at $Re_\tau = 590$. The LES was performed using the filtered boundary optimal LES formulation.

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