

# WEAK MEAN TURBULENT FLOW OF DILUTE POLYMER SOLUTION

**Alexander Liberzon, Michele Guala, Wolfgang Kinzelbach**  
Institute of Hydromechanics and Water Resources Management,  
Swiss Federal Institute of Technology, ETH Zurich,  
CH 8093 Zurich, Switzerland.  
liberzon@ihw.baug.ethz.ch

**Beat Lüthi**  
Risø National Laboratory,  
Frederiksborgvej 399, P.O. 49  
DK-4000 Roskilde, Denmark  
beat.luthi@risoe.dk

**Arkady Tsinober**  
Department of Fluid Mechanics and Heat Transfer,  
Faculty of Engineering, Tel Aviv University,  
Tel Aviv 69978, Israel.  
tsinober@eng.tau.ac.il

## ABSTRACT

We present an experimental, comparative study of the turbulent quantities of a weak mean turbulent flow in water and in a dilute polymer solution with an emphasis on phenomena at small scales. The experiment is performed by using a three-dimensional particle tracking velocimetry (3D-PTV) system. 3D-PTV allows to measure in a Lagrangian manner fields of velocity and velocity derivatives. An interpolation to the Eulerian grid allows to analyze properties of the mean flow and of the fluctuating velocity field, in addition to the small scale quantities such as vorticity, strain and their production terms. The comparison between the experimental results of the water and dilute polymer solution flows provides a direct observation of the influence of polymers on the mean, fluctuating and small scale properties.

## INTRODUCTION

Since the discovery by Toms (1949) of the drag reduction phenomenon, it is well documented that turbulent flows can be strongly modified by additives even in extremely small concentrations (e.g. the bibliography of Nadolink and Haigh, 1995 lists more than 2500 entries). The main aspects of the phenomenon were reviewed by McComb (1990) and Gyr and Bewersdorf (1995) and more recent references could be found in Ptasiniski et al. (2003), Gupta et al. (2004), Dubief et al. (2004), among many others.

In spite of enormous efforts, the physical mechanisms underlying the phenomenon remain poorly understood, partially because of absence of direct experimental evidence on the interaction of diluted polymer solution with turbulence. Drag reduction is the large scale phenomenon, but there is also a consensus that the direct action of the dissolved polymers is on the small scales. Thus, drag reduction is just one aspect of an interaction of polymers with the turbulent field of velocities and velocity derivatives, and any turbulent flow is

expected to be substantially modified in the presence of polymer chains, with or without drag reduction (e.g., Cadot et al., 1998). Hence, it is meaningful to study such an interaction of turbulent flow with dilute polymers in a simple turbulent flow without, or with small mean velocity gradients.

To the best of our knowledge, our first results on the influence of polymers on the field of spatial velocity derivatives, e.g., rate of strain tensor, vorticity and related quantities such as their production and geometrical statistics (Liberzon et al., 2005), are among a few attempts to address the influence of polymers on the field of velocity derivatives. Gyr and Tsinober (1996) have shown that both  $\langle(\partial u_1/\partial x_1)^2\rangle$  and  $\langle(\partial u_1/\partial x_1)^3\rangle$  are an order of magnitude smaller in turbulent flows of polymer and surfactant solutions as compared to those in pure water. Another research is of Crawford et al., (2002), in which the influence of polymers on some statistical properties of Lagrangian accelerations was addressed, and a decrease in the acceleration variance and an increase in the smallest time scales were observed.

The results in the present paper comprise a report on first experimental results on the direct influence of dilute polymers on the field of velocities and spatial velocity derivatives, complemented by information on the evolution of material elements, given in Liberzon et al. (2005). Such an experiment became possible due to the progress made in the three-dimensional particle tracking velocimetry (3D-PTV) technique (Lüthi et al., 2005), which is a non-intrusive and Lagrangian-based technique that allows to assess velocities, velocity derivatives and material elements along the fluid particle trajectories (Guala et al., 2005).

## EXPERIMENTAL SETUP

A detailed description of the 3D-PTV technique can be found in Lüthi et al. (2005) and a description of our experimental setup is given in Liberzon et al. (2005). Here we

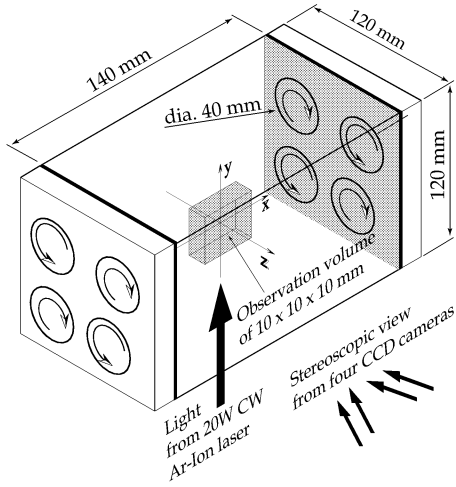


Figure 1: Schematic view of the experiment, including the forcing scheme.

present only its schematic view in figure 1 and few important features. The experiment was performed in a glass tank,  $120 \times 120 \times 140 \text{ mm}^3$ , in which the turbulent flow field is maintained by eight counter-rotating disks of 40 mm in diameter, as it is shown in figure 1. A controlled servo-motor rotates the smooth disks with a constant angular speed of 300 rpm, such to produce in the tank a three dimensional quasi-isotropic turbulent flow with a weak mean flow. The three-dimensional observation volume is  $10 \times 10 \times 10 \text{ mm}^3$ , the flow is seeded with neutrally buoyant polystyrene particles with a diameter of  $30 \mu\text{m}$  and the average concentration of about 1000 particles in the measurement volume. The flow is illuminated with the expanded laser beam from a 20 Watt Ar-Ion laser, and sampled simultaneously with four CCD cameras (progressive scan, monochrome,  $640 \times 480$  pixels, 8 bit per pixel) at a rate of 60 Hz, for a total time of 100 s. The experiment is performed at the same day in water and in dilute polymer solution of 20 wppm of poly(ethylene oxide) (POLYOX WSR 301), prepared in two steps: semi-dilute solution of 1000 ppm is prepared by gentle stirring 24 hours before the experiment, and the dilution to the final concentration is achieved directly in the glass tank. Two types of disks were tested: a) smooth disks, b) disks with 6 baffles each. The results shown here are only for a smooth, frictional forcing, while the results for the the inertial forcing, i.e. disks with baffles are presented in Liberzon et al., (2005). It has been shown that the same effects, though of different magnitude, are observed for the both cases. A thorough comparison of the two energy input cases is a subject of our next publication.

## RESULTS AND DISCUSSION

The direct influence of polymers on the small scale structure of turbulence, as it was expected, is seen in figure 2, in which one observes a suppression of the field of the mean and fluctuating strain is presented by the square of its magnitude  $S^2 = S_{ij}S_{ij}$ ,  $s^2 = s_{ij}s_{ij}$  (a rule of a double index summation is applied everywhere if it not stated otherwise). In all our figures, the legend with the  $\langle \cdot \rangle$  notation presents an average value of a quantity. Obviously, a part of this suppression is also due to the reduced friction at the surface of a smooth rotating disks. However, we have shown in Liberzon et al.

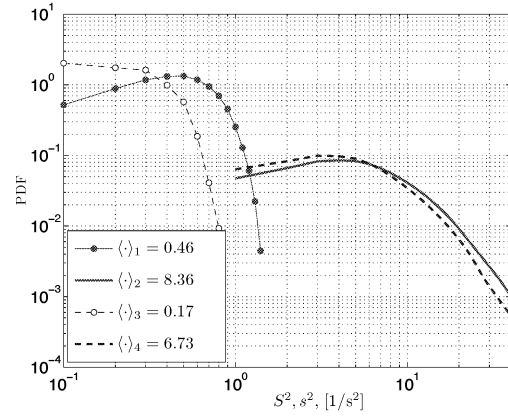


Figure 2: PDF of a square of mean rate of strain,  $S^2$ , for water ( $\bullet$ ) and polymer solution ( $\circ$ ), and fluctuating rate of strain,  $s^2$ , for water (solid line) and polymer solution flow (dashed line).

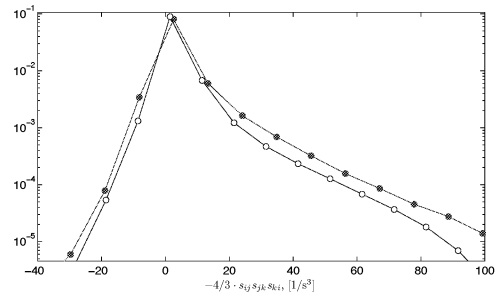


Figure 3: PDF of rate of strain production term,  $-s_{ij}s_{jk}s_{ki}$  for water ( $\bullet$ ) and polymer solution ( $\circ$ ).

(2005) that the reduction of fluctuating strain is observed as well in the case of disks with baffles, where no drag reduction occurs. Therefore, we can see that even at relatively low concentrations (20 wppm), far from the boundaries, the field of strain is reduced substantially as compared to the flow of pure water. This reduction is especially strong at large strain values, where it is up to two orders of magnitude. Strain reduction in the presence of polymers could be seen as a result of reduction of production of strain  $-s_{ij}s_{ij}s_{ij}$ , shown in figure 3, which again exhibits a strong reduction of the largest values of  $-s_{ij}s_{ij}s_{ij}$ . Again, the same effect was shown in the case of inertial forcing, i.e. disks with baffles (Liberzon et al., 2005).

Similar but weaker reduction occurs for enstrophy  $\omega^2$  and its production  $\omega_i\omega_j s_{ij}$  (we remind that in homogeneous turbulent flows  $\langle \omega^2 \rangle = 2 \langle s^2 \rangle$  and  $\langle \omega_i\omega_j s_{ij} \rangle = -\frac{4}{3} \langle s_{ij}s_{ij}s_{ij} \rangle$ ). These results, along with the results related to the influence of dilute polymers on the evolution of material lines  $l_i$ , such as their stretching  $l_i l_j s_{ij}$ , stretching rate  $l_i l_j s_{ij}/l^2$ , evolution of Cauchy-Green  $W_{ij}$  and stretching tensor  $T_{ij}$ , all shown in Liberzon et al. (2005), and are not repeated here for a sake of clarity. Instead, we demonstrate the effect of polymers on the field of velocity fluctuations, Reynolds stresses and production of turbulent kinetic energy. Drag reduction effect in wall-bounded flows was found to be associated with a significant decrease of the Reynolds stresses, without a substantial reduc-

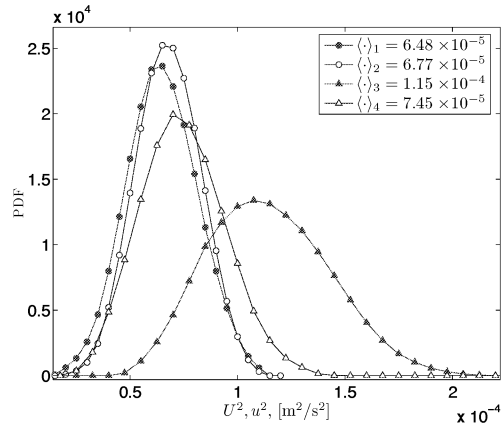


Figure 4: PDF of the mean (circles) and turbulent kinetic energy (triangles),  $U^2$  and  $u^2$ , respectively, for water (full symbols) and polymer solution (open symbols) flow (open symbols).

tion of the r.m.s values of the velocity fluctuations (Warholic et al. 1999). In addition, the turbulent kinetic energy production, similar to the dissipation, was measured and found to be strongly reduced in the drag reduced flow (see e.g., Tsinober, 1990). These phenomena are commonly associated with the "decorrelation effect" of the velocity vector components.

In our experiment, the turbulent kinetic energy  $u^2$  in the bulk flow (i.e., quasi-isotropic turbulent flow, far from the walls) is reduced, as it is shown in figure 4. The decorrelation effect is seen in figure 5, in which the ratio of the Reynolds stresses to the turbulent kinetic energy, is depicted. We observe that the off-diagonal components of the  $\langle u_i u_j \rangle$  tensor decreased more, (but not significantly more) than the diagonal terms,  $u_i^2$ . The reduction of the Reynolds stresses has an instant implication on the production of turbulent kinetic energy,  $P = -\langle u_i u_j \rangle S_{ij}$ , which is shown in figure 6. Although the influence of reduced Reynolds stresses is masked by the normalization with the turbulent kinetic energy,  $u^2$ , we can elucidate the effect by means of invariant quantities. The first one is an alignment between the Reynolds stress tensor  $-\langle u_i u_j \rangle$  and the mean rate of strain tensor,  $S_{ij}$ . Statistically, this alignment is described by the probability density function (PDF) of the cosine of the angle between two tensors (i.e., dot product of two tensors, normalized with their norms), as it is shown in figure 7, and it is independent of the magnitude of the fluctuating velocity field, Reynolds stresses, or of the mean rate of strain.

In addition, it is of special interest to analyze the phenomena in the invariant frame of reference, e.g., the eigenframe of the mean rate of strain tensor. We follow the idea of Gurka et al., (2004), in which the most significant contribution was observed, both experimentally and numerically, to be associated with the compressing eigenvalue (eigenvector) of the mean rate of strain tensor,  $\Lambda_3^S$  ( $\lambda_3^S$ ). It means, that the production is decomposed into the three contributions:  $-\langle u_i u_j \rangle S_{ij} = -\langle u^2 \Lambda_1^S \cos^2(\mathbf{u}, \lambda_1^S) \rangle - \langle u^2 \Lambda_2^S \cos^2(\mathbf{u}, \lambda_2^S) \rangle - \langle u^2 \Lambda_3^S \cos^2(\mathbf{u}, \lambda_3^S) \rangle$ , and the only significant positive values are added by the third term on the right hand side. In this equation,  $u^2 = u_1^2 + u_2^2 + u_3^2$ , and  $\Lambda_i^S$  are the eigenvalues and  $\lambda_i^S$  are the corresponding eigenvectors of the mean rate of strain tensor  $S_{ij}$  (by definition,  $\Lambda_1^S > \Lambda_2^S > \Lambda_3^S$ , thus  $\Lambda_1^S > 0, \Lambda_3^S < 0$ ).

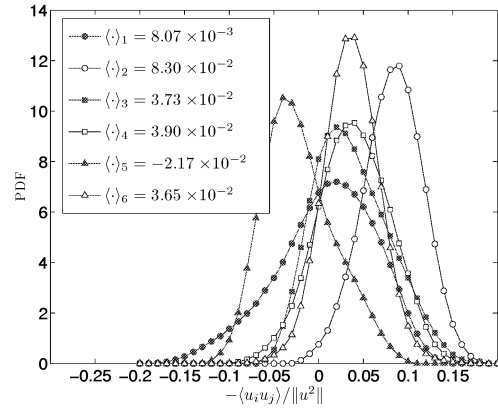


Figure 5: PDF of the Reynolds stresses  $-\langle u_i u_j \rangle$ ,  $i \neq j$ , normalized with mean turbulent kinetic energy  $\|u^2\|$ , for water (full symbols) and polymer solution flow (open symbols). The notations are:  $\langle u_1 u_2 \rangle$  circles,  $\langle u_1 u_3 \rangle$  squares, and  $\langle u_2 u_3 \rangle$  triangles.

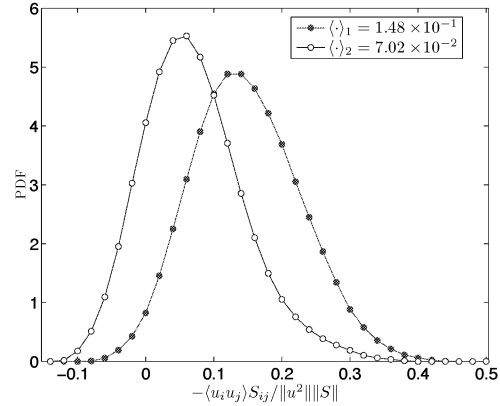


Figure 6: PDF of the non-dimensional turbulent kinetic energy production,  $P = -\langle u_i u_j \rangle S_{ij}$ , normalized with the mean turbulent kinetic energy  $\|u^2\|$  and mean rate of strain  $\|S\|$ , for water (●) and polymers (○).

We demonstrate the effect of dilute polymers on each one of the contributive terms in figure 8. Again, a part of the effect is hidden in the reduced mean rate of strain, (see e.g., figure 2). But, the picture is not that simple at all - we identify the influence of polymers in invariant and non-dimensional quantities, such as an angle between the fluctuating velocity vector and the eigenframe of the mean rate of strain tensor,  $\cos(\mathbf{u}, \lambda_i^S)$ . This quantity, shown in figure 9, is independent of the magnitude of the strain and of the turbulent kinetic energy and demonstrates an intrinsic effect of dilute polymers. All the results express in this or another way the effect of dilute polymers on the Reynolds stresses and production of turbulent kinetic energy. The fundamental physical mechanism is not revealed yet, however, the demonstrated results give us more direct and unbiased evidence of the phenomena through the invariant quantities and geometrical statistics.

## CONCLUDING REMARKS

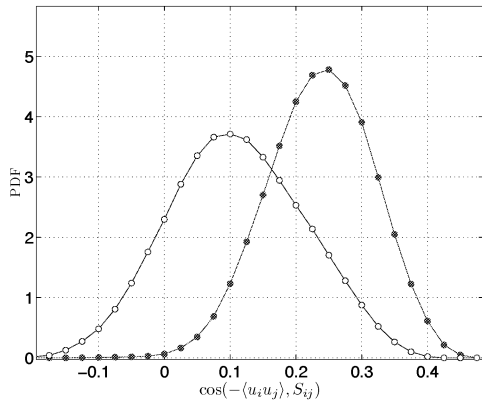


Figure 7: PDF of the cosine of the angle between the Reynolds stress tensor  $-\langle u_i u_j \rangle$  and the mean rate of strain tensor,  $S_{ij}$ , for water ( $\bullet$ ) and ( $\circ$ ).

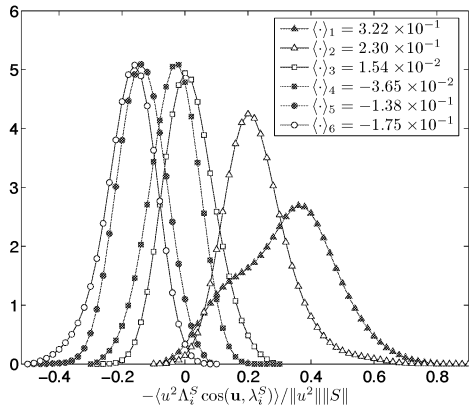


Figure 8: PDFs of the three contributive terms  $P_i = -\langle u^2 \Lambda_i^S \cos^2(\mathbf{u}, \lambda_i^S) \rangle$  to the turbulent kinetic energy production,  $i = 1, 2, 3$ , for water ( $\bullet$ ) and ( $\circ$ ). All terms are normalized the mean turbulent kinetic energy  $\|u^2\|$  and the mean rate of strain  $\|S\|$

We applied three-dimensional particle tracking velocimetry (3D-PTV) method, which allowed to obtain direct experimental comparison of the fields of velocities, i.e. turbulent kinetic energy, Reynolds stresses, and velocity derivatives, i.e. strain, strain production in a weak mean turbulent flows of water and dilute polymer solution. We also were able to observe concomitant changes in the production of turbulent kinetic energy in a turbulent bulk region, far from the boundaries, of dilute polymer solution and modifications of the associated geometrical statistics.

Our results were obtained at rather small value of the Reynolds number. This is mainly due to the necessity of obtaining the tensor of velocity derivatives ( $\partial u_i / \partial x_j$ ) along fluid particle trajectories. Our belief is that, at least qualitatively the obtained results should be true at larger values of the Reynolds number. The reported results are the first ones out of a broader ongoing research and we believe that the described approach is promising in further elucidating at least of some of the basic issues of turbulent flows of dilute polymer solution until it will become possible to handle both

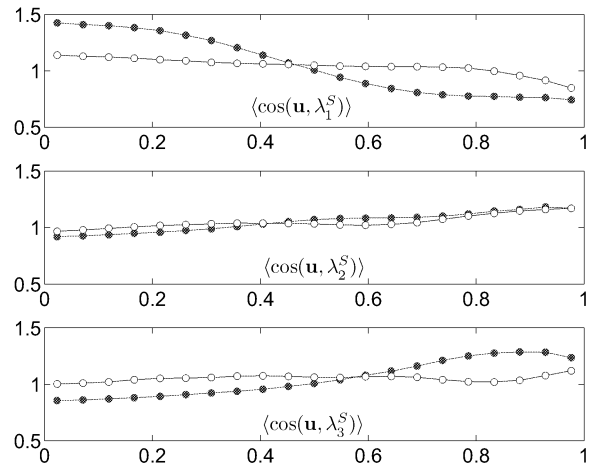


Figure 9: PDF of the cosine of the angle between the fluctuating velocity vectors,  $\mathbf{u}$ , and the eigenframe of the mean rate of strain tensor,  $\lambda_i^S$

larger Reynolds number flows and direct access to polymer molecules and/or aggregates conformation.

This research is supported by ETH research fund, under TH-18/02-4.

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