THE STRUCTURE OF TURBULENCE IN A SHALLOW WATER WIND-DRIVEN SHEAR CURRENT WITH LANGMUIR CIRCULATION

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ABSTRACT
Large-eddy simulation (LES) of Langmuir circulation in a wind-driven shear current in shallow water is reported. After the introduction and a brief description of the governing equations and the numerical method, we focus on the major differences in the dynamics between wind shear-driven Couette flow and the same flow with Langmuir circulation. This comparison will rely on flow visualizations and diagnostics including mean velocity profiles, invariants of the resolved Reynolds stress anisotropy tensor and balances of the transport equations for resolved mean turbulent kinetic energy and resolved Reynolds stress tensor.

INTRODUCTION
Langmuir circulation (LC), often occurring in the wind and wave driven surface mixed layer of lakes and oceans, consists of pairs of parallel counter-rotating vortices (or cells) oriented approximately in the streamwise direction. Originally characterized by Langmuir (1938), Langmuir cells are thought to be generated by interaction between the wind-driven mean shear current and the Stokes drift current caused by surface gravity waves.

Over the last several decades, numerous field observations of LC have been made using acoustic Doppler current profilers. Most of these works have recorded LC in the ocean surface mixed layer over deep water. Recent observations (Garrett et al., 2004 and Gargett and Wells, 2005) made on the shallow shelf off the southern coast of New Jersey in the presence of strong wind and wave forcing led to the discovery of LC extending throughout the entire water column. Such patterns reaching down to the bottom boundary layer have been termed supercells because of their profound influence on sediment re-suspension and transport.

LES of LC performed up to date have also focused on the surface mixed layer over deep water, far from the bottom boundary layer. Recent simulations include those of Skillingstad and Denbo (1995), McWilliams et al. (1997), and Li et al. (2004). Skillingstad and Denbo found that wave forcing (creating LC) plays a bigger role than convective forcing in generating mixing. McWilliams et al. included the Coriolis force as well as LC forcing and found enhanced vertical turbulent velocity fluctuations due to LC. Li et al. performed a number of simulations and found differences in the turbulence structure between convection-dominated, shear-dominated and Langmuir-dominated turbulence.

We report LES of Langmuir supercells approximating the conditions observed by Gargett et al. (2004) and Gargett and Wells (2005: henceforth GW). To that extent, Couette flow was simulated with a stationary, no-slip plane at the bottom and a constant streamwise, tangential stress boundary condition at the top surface approximating the effect of a constant wind shear stress (see figure 1). The governing equations were augmented with the Craik-Leibovich (CL) force (Craik and Leibovich, 1976) accounting for LC.

![Figure 1: Sketch of Couette flow driven by surface stress, $\tau_{surface}$, due to wind. The total depth is $H$ and $\delta = H/2$. ---, mean velocity profile in a turbulent flow; ---- , linear velocity profile in a laminar flow.](image-url)
to the small-scales.

After a brief description of the governing equations and numerical method, we focus on the major differences in the dynamics between wind shear-driven Couette flow and the same flow with LC under the conditions observed by GW.

GOVERNING EQUATIONS AND NUMERICAL METHOD

Constant density flow is assumed because the shallow water Langmuir supercells described by GW were observed in approximately neutrally stable water. The non-dimensionalized, filtered, incompressible Navier-Stokes equations augmented by the CL vortex force accounting for LC are:

\[
\frac{\partial \tilde{u}}{\partial t} + \nabla \cdot (\tilde{u} \otimes \tilde{u}) = -\nabla p + \frac{1}{Re} \Delta \tilde{u} + \nabla \cdot T + \frac{1}{La_T^2} F_{CL} \tag{1}
\]

\[
\nabla \cdot \tilde{u} = 0
\]

where an over-bar denotes a filtered quantity, \( \tilde{u} \) is the filtered modified pressure (McWilliams et al., 1997) divided by density \( \rho \), and the filtered velocity is \( \tilde{u} = (\tilde{u}_1, \tilde{u}_2, \tilde{u}_3) \) in coordinate system \( x = (x_1, x_2, x_3) \) (figure 1).

The filtered vortex force on the right hand side is

\[
F_{CL} = \tilde{u} \times \tilde{\omega} \tag{2}
\]

modeling the mechanism behind LC (Craik and Leibovich, 1976); i.e., the interaction between surface gravity wave Stokes drift velocity, \( \tilde{u} \), and the shear flow represented by the filtered vorticity \( \tilde{\omega} \). The mechanism consists of tilting and stretching of vertical vorticity into the horizontal by the Stokes drift velocity, resulting in enhanced streamwise vorticity.

The equations are non-dimensionalized using the friction velocity \( u_f = (\tau_{surf}/\rho)^{1/2} \) and the channel mid-depth \( \delta = H/2 \), thus leading to a Reynolds number \( Re_f = u_f \delta / \nu \). The non-dimensionalization of the CL force, \( F_{CL} \), gives rise to the turbulent Langmuir number, \( La_T = (u_f/u_c)^{1/2} \), appearing in (1). Note that \( La_T = \infty \) corresponds to \( u_c = 0 \), that is zero Stokes drift velocity and thus no LC. As \( La_T \rightarrow 1 \) LC becomes comparable to the shear flow, while for \( La_T < 1 \) LC dominates. The Stokes drift velocity appearing in (1) is

\[
u_c = u_c \left( \frac{\cosh[2k(x_2 - H)]}{2 \sinh^2(kH)} \right) e_1 \tag{3}
\]

(Phillips, 1967) where \( e_1 \) is the unit vector in the \( x_1 \)-direction. The coefficient \( u_c \), also appearing in the definition of \( La_T \), is defined as \( u_c = \sigma k \alpha^2 \), where \( \sigma \) is the dominant frequency, \( k \) is the dominant wavenumber and \( \alpha \) is the amplitude of the surface gravity waves.

The subgrid-scale (SGS) stress in (1) is defined as

\[
T = \tilde{u} \otimes \tilde{u} - \tilde{u} \otimes \tilde{u} \tag{4}
\]

where the term \( \tilde{u} \otimes \tilde{u} \) gives rise to a closure problem. The deviatoric part of \( T \) is parameterized and the dilatational part is added to \( \tilde{p} \). The following parameterization (Smagorinsky, 1963) is used:

\[
dev(T) = 2 \nu_T \nabla^2 \tilde{u} \tag{5}
\]

where the eddy viscosity is given as

\[
\nu_T = (C_s \Delta)^2 |\nabla^2 \tilde{u}| \tag{6}
\]

and

\[
\nabla^2 \tilde{u} = \frac{1}{2} \left\{ \nabla \tilde{u} + (\nabla \tilde{u})^\text{transpose} \right\} \tag{7}
\]

\[
|\nabla^2 \tilde{u}| = (2 \nabla \tilde{u} \cdot \nabla \tilde{u})^{1/2} \tag{8}
\]

are the filtered strain rate tensor and its norm, respectively. The coefficient \( (C_s \Delta)^2 \), is computed dynamically using the procedure described by Lilly (1992).

In our LES we use a free-slip, rigid-lid approximation for the water surface thus filtering out surface gravity waves. Zhou et al. (1998) performed simulations of LC with a surface wavy boundary layer due to second order Stokes wave and compared results to a simulation with a free-slip, rigid-lid approximation including the CL vortex force. The simulations were in general agreement, demonstrating the validity of the free slip, rigid-lid model with CL vortex forcing.

The numerical method used employs a hybrid spectral/finite difference discretization. Horizontal directions (\( x_1 \) and \( x_2 \)) are discretized spectrally via fast Fourier transforms and the vertical direction (\( x_3 \)) is discretized via high order (fifth and sixth) compact finite difference schemes, allowing for grid stretching in the vertical in order to resolve expected strong vertical gradients in the velocity. Boundary conditions are no-slip velocity at the bottom wall and constant shear stress and zero normal flow (\( \alpha_3 = 0 \)) at the top rigid-lid surface. Periodicity is assumed in the horizontal directions. Time-marching consists of a second order time-accurate fractional step scheme.

NUMERICAL RESULTS

In this section, we compare the following two cases: 1) Couette flow driven by a surface wind stress with CL vortex forcing and 2) Couette flow driven by a surface wind stress without LC. In both cases, the constant surface wind stress is applied such that \( Re_f = 180 \). Case 1) is characterized by \( La_T = 0.7 \) and \( \lambda = 6H \). These values for \( La_T \) and \( \lambda \) are representative of the coastal shelf shear flow with Langmuir supercells observed by CW. Case 2) is characterized by \( La_T = \infty \), thus no LC. Note that \( Re_f \) representative of the observations is much greater than that of the present simulation (\( Re_f = 180 \)). Although not reported here, simulations with LC at higher Reynolds numbers were performed without great changes in the results, demonstrating that the simulation at \( Re_f = 180 \) is not adversely affected by low Reynolds number effects.

Following the direct numerical simulations of traditional Couette flow at about \( Re_f = 170 \) of Lee and Kim (1991), the domain dimensions for both cases, 1) and 2), were chosen as \((L_1/\delta, L_2/\delta, L_3/\delta) = (4\pi, 8\pi/3, 2)\). Here \( L_1, L_2 \) and \( L_3 \) are the lengths of the domain in the \( x_1, x_2 \), and \( x_3 \)-direction, respectively. The computational grid for the Couette flow with no LC contained 33 points in \( x_1 \), 65 points in \( x_2 \) and 65 points in \( x_3 \) \((33 \times 65 \times 65)\). A greater number of points in the \( x_3 \)-direction was required for Couette flow with LC in order to resolve stronger vertical gradients, thus the grid had \((33 \times 65 \times 97)\) points. The non-dimensionalized time step was chosen as 0.0025 and 0.005 for the cases with and without LC, respectively, in order to yield temporal accuracy and not violate the CFL condition.
Visualizations

Figure (2) shows contours of instantaneous streamwise fluctuating velocity, \( u'_1 \), on the horizontal plane at the middle of the channel. In both cases of the Couette flow (with LC and without LC) there is at least one pair of high- and low-speed regions or streams highly elongated in the streamwise direction \( (x_1) \) and alternating in the spanwise \( (x_3) \) direction. In the flow with no LC shown in figure (2b) the spanwise length of each region is approximately equal to the depth, \( H \). Animations reveal that when the CL vortex force is turned on, the high-speed regions merge, as the flow transitions from two pairs of streaks to one pair. Figure (2a) shows the one-pair structure characterizing Couette flow with LC.

![Simulation with LC](image1)

![Simulation without LC](image2)

Figure 2: Instantaneous contours of \( u'_1 \) on horizontal \( (x_1-x_2) \) plane at middle of channel \( (x_3 = H/2) \) in Couette flow with and without LC. Fluctuations \( u'_1 \) are normalized by the mean centerline streamwise velocity, \( U_c \). \( \cdots \cdots \cdots \), \( u'_1 / U_c > 0 \); \( \cdots \cdots \cdots \), \( u'_1 / U_c < 0 \).

Figures (3) and (4) show the mean vertical structure of the fluctuating velocity components in the flow with and without LC, respectively. Overall, both flows exhibit positive and negative spanwise cell structures in each of the fluctuating velocity components; the flow with LC has a spanwise one-cell structure while the flow without LC has a spanwise two-cell structure. Furthermore, in the mean, flow with LC has much stronger maxima and minima in all fluctuating velocity components: extrema of \( \langle u'_1 \rangle_{x_1} \), \( \langle u'_2 \rangle_{x_1} \), and \( \langle u'_3 \rangle_{x_1} \) are approximately 3, 2.5 and 10 times greater, respectively.

As seen in figure (3a), the flow with LC is characterized by intensification of positive \( \langle u'_1 \rangle_{x_1} \) near the surface and near the bottom, in close agreement with the observations of GW. In the case of the flow without LC, there is no such intensification as the magnitude of \( \langle u'_1 \rangle_{x_1} \) is roughly uniform (either positive or negative) in most of the water column. Note that in the observations, quantities are averaged over time only and not over time and streamwise direction or over time and horizontal directions, as is done here.

In both flows, a region of positive \( \langle u'_1 \rangle_{x_1} \) coincides with a region of negative \( \langle u'_2 \rangle_{x_1} \) and vice-versa. Regions of positive \( \langle u'_2 \rangle_{x_1} \) are referred to as upwelling limbs and regions of negative \( \langle u'_3 \rangle_{x_1} \) are referred to as downwelling limbs. At mid-depth in the flow with LC, the ratio of spanwise length of the upwelling limb to spanwise length of the downwelling limb is 1.6, in close agreement with the observational value of 1.5. In the flow without LC this ratio is approximately 1. Additionally, the downwelling limbs in the observations and in the the flow with LC possess greater intensity (magnitude) of \( \langle u'_1 \rangle_{x_1} \) than their adjacent upwelling limbs. In the the flow without LC, the magnitude of \( \langle u'_1 \rangle_{x_1} \) in the downwelling limbs is nearly the same as in the upwelling limbs.

![Contours of averaged fluctuating velocity components](image3)

Figure 3: Contours of averaged fluctuating velocity components (normalized by \( U_c \)) on \( x_2-x_3 \) plane for Couette flow with LC. \( \langle \cdot \rangle_{x_1} \) denotes averaging in time and over \( x_1 \). \( \langle \cdot \rangle \) corresponds to averaging in time and over \( x_1 \) and \( x_2 \).

Furthermore, as seen in figures (3c) and (3d), in both flows a region of upwelling coincides with a region where negative spanwise \( (x_2) \) gradient of \( \langle u'_1 \rangle_{x_1} \) occurs near the surface. The opposite trend is seen near the bottom; a region of upwelling coincides with a region where positive spanwise gradient of \( \langle u'_1 \rangle_{x_1} \) occurs near the bottom. Finally, in the flow with LC, extrema of \( \langle u'_1 \rangle_{x_1} \) occur at the surface (similar to the field observations), in contrast to the flow without LC where they occur in the upper third of the water column.

Color versions of the previous figures may be found in www.ccpo.odu.edu/~tejada.

Mean Profiles, Resolved Reynolds Stresses and Invariants

Figures (5a) and (6a) show mean streamwise velocity for Couette flow with LC and without LC, respectively. In both cases the mean spanwise velocity \( u_2 \) and the mean vertical velocity \( u_3 \) (not shown) are nearly zero. A major difference between the two flows occurs in \( u_1 \). In the flow with LC, the LC serves to homogenize \( u_1 \) throughout most of the water column. This behavior can be seen in figure (6a) where \( u_1 \) is roughly constant throughout most of the water column. In the flow without LC, \( u_1 \) has non-zero slope throughout the water column.
of the water column.

Figures (5c) and (6c) show resolved shear Reynolds stress components for the flow with and without LC, respectively. $\langle u_1' u_4' \rangle$ components for both cases are close to each other in magnitude throughout most of the water column; for the LC case, $\langle u_1' u_4' \rangle$ attains slightly greater values, especially near the middle of the water column. The normalized magnitude of $\langle u_1' u_4' \rangle$ in both cases is in close agreement to that recorded in the field observations. Additionally, for both cases, $\langle u_1' u_3' \rangle$ is nearly zero throughout the entire water column, similar to the observations. For Couette flow without LC, this also occurs with $\langle u_1' u_3' \rangle$. In Couette flow with LC, $\langle u_1' u_2' \rangle$ is nearly zero throughout most of the water column, except for the bottom part where the CL vortex force induces slight variations. Of importance is the fact that the data recorded in the field observations exhibit a $\langle u_1' u_2' \rangle$ component which is far from zero, perhaps due to a non-zero mean velocity in $x_3$, $\langle u_3 \rangle$ (see GW). This is in contrast to the current simulations for which $\langle u_2 \rangle$ is nearly zero.

Figures (7) and (8) show maps of the Lumley invariants ($\Pi = b_{ij} b_{ik}$ and $\Pi = b_{ij} b_{ik} b_{ik}$ (Pope, 2000)) for Couette flow with and without LC. The quantity $\Pi^{1/2}$ serves as a measure of the magnitude of the anisotropy, while the location of the coordinate ($\Pi^{1/2}, \Pi^{1/3}$) serves as a measure of the shape of the anisotropy. Figures (7) and (8) show the trajectories of the Lumley invariant maps varying from $x_3/H = 0$ (bottom wall) to $x_3/H = 1$ (top surface). For both flows with and without LC, the fluctuating motion is two-component (near the top bounding curve of the Lumley triangle) very close to the bottom wall because $\langle u_1' u_2' \rangle$ is much smaller than the other two normal Reynolds stress components. In the case of Couette flow without LC, the fluctuating motion moves close to a cigar-shaped axisymmetric state (near the right hand side edge of the triangle) as the distance away from the bottom wall increases. The reason for this behavior is that $\langle u_1' u_2' \rangle$ is larger than $\langle u_1' u_2' \rangle \approx \langle u_1' u_3' \rangle$, especially in the middle region of the water column. In the upper-half region of the channel, the fluctuating motion moves back towards the two-component state.

In contrast, for the case with LC, as distance from the wall increases the turbulence moves towards a pancake-shape state as the trajectory of the map goes into the interior of the triangle towards the left hand side edge. The reason for this behavior is that $\langle u_1' u_2' \rangle$ and $\langle u_1' u_3' \rangle$ are much greater than $\langle u_1' u_4' \rangle$.
serves to balance viscous dissipation (denoted by squares).

Near the top surface, the transport of mean TKE follows different dynamics in the two cases, as depicted by figure (10). In flow with LC, the CL vortex force acts as a source of TKE, reaching a maximum at the top surface and is mostly balanced by a negative pressure transport. In the case with no LC, the pressure transport is nearly zero given that the CL force is zero. The rest of the terms in the two cases only show slight differences.

Figure 9: Near bottom wall budget terms in transport equation for turbulent kinetic energy. Terms are normalized by viscous scales (Pope, 2000); location of wall is at $x_3^+ = 0$. ——, turbulent transport; ——, pressure transport; ——, SGS transport; ⋄ ⋄ ⋄, viscous diffusion; □, viscous dissipation; ○, SGS dissipation; ⋅ ⋅ ⋅, production by shear; +, production by CL force (Langmuir); ○, sum of all terms.

Figure 10: Near top surface budget terms in transport equation for resolved turbulent kinetic energy. Terms are normalized by viscous scales; location of top surface is at $x_3^+ = 0$. Symbols follow same convention as in figure (9).

Figures (11) and (12) contrast the budget terms for transport of $- <u'_1 u'_3>$ (the dominant component of the shear Reynolds stress) in Couette flow with and without LC. In the case with LC, near the wall (figure (11a)) pressure transport serves to balance CL vortex forcing and pressure-strain redis-

Budgets

Figure (9) contrasts the budget terms for transport of mean resolved turbulent kinetic energy (TKE) in both flows close to the bottom wall. The main difference here is the presence of the CL vortex force (denoted by plus symbols in figure (9a)) acting as a sink in the flow with LC. This sink is partially balanced by pressure transport (denoted by the dashed line). In the flow without LC (figure (9b)), the CL vortex force is zero and thus the pressure transport is practically dormant. At the wall, in both cases, viscous diffusion (denoted by dots)
tribution (denoted by stars). At the wall, in the case with no LC (figure (11b)) pressure transport balances pressure-strain redistribution, as the CL vortex force is zero. For the region $x_a^+ > 5$ appearing in the figure, production by shear (denoted by the x-marks) plays a bigger role than pressure transport as they both serve to balance pressure-strain redistribution. In the simulation with LC, production by shear is negligible as most of the source is provided by turbulent transport.

It is remarkable that the near-wall dominant terms in the flow without LC (i.e. pressure transport and pressure-strain redistribution) are an order of magnitude smaller than the near-wall terms in the flow with LC. This same disparity is also noted between dominant terms near the top surface (figure (12)). In the case with LC, the dominant terms are once again pressure transport, the CL vortex force, and pressure-strain redistribution. Meanwhile in the case without LC, the dominant terms are production by shear, pressure transport, turbulent transport, and pressure-strain redistribution.

CONCLUSIONS
Stress driven Couette flow with LC is very different from that without LC. The differences are present in all of the diagnostics shown. Also, the presence of LC results in greatly enhanced vertical mixing. Finally, basic features of the flow with LC are in general agreement with the field observations of GW.

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REFERENCES

Figure 11: Near bottom wall budget terms in transport equation for $-\langle u'_i u'_j \rangle$. Terms are normalized by viscous scales. *, pressure-strain redistribution; the rest of the symbols follow the same convention as in figure (9).

Figure 12: Near top surface budget terms in transport equation for $-\langle u'_i u'_j \rangle$. Terms are normalized by viscous scales. Symbols follow the same convention as in figure (11).