A NONLINEAR TWO-EQUATION HEAT-TRANSFER MODEL REFLECTING BUOYANCY EFFECT

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ABSTRACT

The main objective of this study is to construct a new two-equation heat-transfer model reflecting buoyant effects in wall-bounded turbulent shear flows. It is now well-known that the turbulent heat-fluxes, which are the key quantities for the prediction of turbulent flows with buoyancy, are not modeled accurately using the traditional turbulence models. In order to appropriately predict wall-bounded turbulent flows with buoyancy, an innovative two-equation turbulence heat-transfer model must be constructed. Consequently, we should improve the modeled expressions for Reynolds stresses and turbulent heat-fluxes reflecting the buoyant effect in wall-bounded turbulent shear flows. The existing two-equation turbulence models were evaluated on the basis of the DNS data of channel flows with buoyancy. Using the results of evaluation, we constructed new modeled expressions for Reynolds stresses and turbulent heat-fluxes, and reconstructed the two-equation heat-transfer model including newly proposed expressions for Reynolds stresses and turbulent heat-fluxes. The proposed two-equation heat-transfer model appropriately predicts wall-bounded turbulent shear flows with buoyancy.

INTRODUCTION

The main objective of this study is to construct a two-equation turbulence heat-transfer model reflecting the buoyant effect. Wall-bounded turbulent shear flows with buoyancy have been encountered in many engineering relevant applications such as a flow in a computer or room. Two-equation turbulence heat-transfer models for analysis of forced convection have been developed to accurately calculate forced convective turbulence flows (e.g., Nagano and Hattori, 2003; Hattori and Nagano, 1998; Abe *et al.*, 1996; Nagano and Shimada, 1996). On the other hand, a two-equation turbulence heat-transfer model with thermal-stratification or buoyant effect is hardly adequate for an analysis of turbulent shear flow with buoyancy. In order to predict adequately wall-bounded turbulent flow with buoyancy, a conventional two-equation heat-transfer model is not appropriate, because the streamwise turbulent heat-flux needed to predict the flow can not be calculated by the model for the model definition. It is well known that only a gradient diffusion type model can predict the turbulent heat flux in a turbulent natural convection flow along a vertical plate (Yin *et al.*, 1991; hereinafter referred to as the YNT model). However, they adopted the empirical equation for the streamwise turbulent heat-flux in their turbulence model due to the principal problem of the eddy diffusivity model. Hence, the YNT model cannot apply to calculate any flow fields. In order to predict the streamwise turbulent heat-flux without the empirical equation, at least a nonlinear two-equation heat-transfer model is necessary. Therefore, we have to modify the expressions of Reynolds shear stress and turbulent heat-flux including the buoyant term based on the nonlinear two-equation heat-transfer model (Abe *et al.*, 1996; Nagano and Hattori, 2003). In this study, in order to determine improvements of these modified expressions for the wall-bounded turbulent shear flows with buoyancy, an *a priori* test method with the aid of DNS result (Nagano and Hattori, 2003) is carried out for the evaluation of model expressions. As a result, we improve the non-linear turbulent heat-transfer model for the wall-bounded turbulent shear flows with buoyancy.

GOVERNING EQUATIONS

The Reynolds-averaged equations for the nonlinear two-equation heat-transfer model in the buoyancy-affected field can be written as follows (Nagano and Hattori, 2003):

$$\bar{U}_{i,i} = 0 \quad (1)$$

$$\frac{D\bar{U}_i}{Dt} = -\frac{\bar{P}_{,i}}{\rho} + \left(\nu \bar{U}_{i,j} - \overline{u_i u_j}\right)_{,j} - g_i \beta \Delta\Theta \quad (2)$$

$$\frac{D\bar{\Theta}}{Dt} = \left(\alpha \bar{\Theta}_{,j} - \overline{u_j \theta}\right)_{,j} \quad (3)$$

$$\overline{u_i u_j} = \frac{2}{3} k \delta_{ij} - 2 C_0 \nu_t S_{ij} + \text{High order terms} \quad (4)$$

$$\overline{u_j \theta} = -\alpha^t_{jk} \bar{\Theta}_{,k} + \text{High order terms} \quad (5)$$

where the Boussinesq approximation is used in Eq. (2), and D/Dt implies a substantial derivative and the Einstein summation convention applies to repeated indices. $\bar{U}_i$ is the mean velocity, $\bar{P}$ is the mean static pressure, g_i is the gravitational acceleration, $\bar{\Theta}$ is the mean temperature, $\Delta\Theta(=\bar{\Theta}-\bar{\Theta}_\infty)$ is the temperature difference, ν is the kinetic viscosity, ρ is the density, β is the coefficient of volume expansion, α is the molecular diffusivity for heat, ν_t is the eddy diffusivity for momentum, α_{ij}^t is the anisotropy eddy diffusivity for heat, $S_{ij}[=(\bar{U}_{i,j}+\bar{U}_{j,i})/2]$ is the strain-rate tensor, k is the turbulence energy, C_0 is the model constant, and τ_m is the hybrid/mixed time scale.

The following transport equations of turbulent quantities making up the expressions of Reynolds shear stress and turbulent heat-flux are given.

$$\frac{Dk}{Dt}=\nu k_{,jj}+T_k+P_k+G_k-\varepsilon \tag{6}$$

$$\frac{D\varepsilon}{Dt}=\nu \varepsilon_{,jj}+T_\varepsilon+\frac{\varepsilon}{k}\left(C_{\varepsilon 1}P_k-C_{\varepsilon 2}f_\varepsilon\varepsilon+C_{\varepsilon 3}G_k\right) \tag{7}$$

$$\frac{Dk_\theta}{Dt}=\alpha k_{\theta,jj}+T_{k_\theta}+P_{k_\theta}-\varepsilon_\theta \tag{8}$$

$$\begin{aligned}\frac{D\varepsilon_\theta}{Dt}=&\ \alpha\varepsilon_{\theta,jj}+T_{\varepsilon_\theta}+\frac{\varepsilon_\theta}{k_\theta}\left(C_{P1}f_{P1}P_{k_\theta}-C_{D1}f_{D1}\varepsilon_\theta\right)\\&+\frac{\varepsilon_\theta}{k}\left(C_{P2}f_{P2}P_k-C_{D2}f_{D2}\varepsilon+C_{P3}f_{P3}G_k\right)\end{aligned} \tag{9}$$

where ε is the dissipation-rate of k, k_θ is the temperature variance and ε_θ is the dissipation-rate of k_θ, respectively. $P_k(=-\overline{u_iu_j}\bar{U}_{i,j})$, $P_{k_\theta}(=-\overline{u_j\theta}\bar{\Theta}_{,j})$ are production terms, and $G_k(=-g_j\beta\overline{u_j\theta})$ is a buoyant term. The turbulent diffusion terms, T_k, T_ε, T_{k_θ} and T_{ε_θ}, are modeled individually using the GGDH modeling (Nagano and Hattori, 2003). $C_{\varepsilon1}$, $C_{\varepsilon2}$, $C_{\varepsilon3}$, C_{P1}, C_{P2} and C_{P3} are model constants, and f_ε, f_{P1}, f_{P2}, f_{P3}, f_{D1} and f_{D2} are model functions.

DERIVATIONS AND EVALUATIONS FOR MODELED EXPRESSIONS OF TURBULENT HEAT-FLUX AND REYNOLDS STRESS

Nonlinear Model of Turbulent Heat-Flux

The transport equation of turbulent heat-flux with the buoyant term is given as follows:

$$\frac{D\overline{u_j\theta}}{Dt}=D_{j\theta}+T_{j\theta}+P_{j\theta}+G_{j\theta}+\Phi_{j\theta}-\varepsilon_{j\theta} \tag{10}$$

where $D_{j\theta}$ is a molecular diffusion term, $T_{j\theta}$ is a turbulent diffusion term, $P_{j\theta}$ is a production term, $G_{j\theta}$ is a buoyant term, $\Phi_{j\theta}$ is a pressure-temperature gradient correlation term, and $\varepsilon_{j\theta}$ is a dissipation term, respectively.

The modeled expression of turbulent heat-flux including the buoyant term is derived using the following relation:

$$\begin{aligned}\frac{Da_j^*}{Dt}=&\frac{1}{\sqrt{k}\sqrt{k_\theta}}(P_{j\theta}+\Phi_{j\theta}-\varepsilon_{j\theta}+G_{j\theta})\\&-\frac{1}{2}a_j^*\left[\frac{\varepsilon}{k}\left(\frac{P_k}{\varepsilon}+\frac{G_k}{\varepsilon}-1\right)+\frac{\varepsilon_\theta}{k_\theta}\left(\frac{P_{k_\theta}}{\varepsilon_\theta}-1\right)\right]\end{aligned} \tag{11}$$

where $a_j^*(=\overline{u_j\theta}/(k^{1/2}k_\theta^{1/2})$ is the nondimensional turbulent heat-flux and the diffusive effect is neglected.

In local equilibrium state, the following relation holds (Abe *et al.*, 1996):

$$\frac{Da_j^*}{Dt}=0 \tag{12}$$

Regarding $\Phi_{j\theta}$ and $\varepsilon_{j\theta}$, the general linear expression (Launder, 1976) with buoyant effect is employed:

$$\Phi_{j\theta}-\varepsilon_{j\theta}=-C_{t1}\frac{\overline{u_j\theta}}{\tau_u}+C_{t2}\overline{u_k\theta}\bar{U}_{j,k}+C_{t3}\overline{u_k\theta}\bar{U}_{k,j}+C_{t4}g_i\beta k_\theta \tag{13}$$

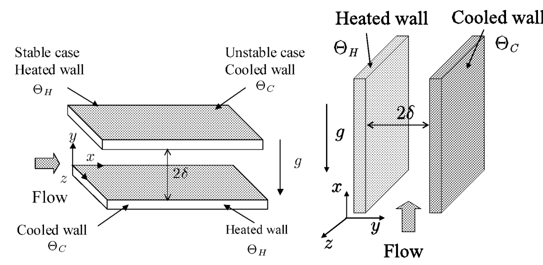

Figure 1: Flow geometry for model evaluation

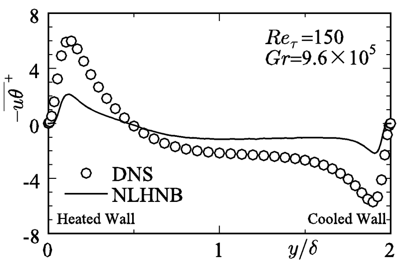

Figure 2: Evaluation result for modeled streamwise turbulent heat-flux (vertical heated plane channel flow)

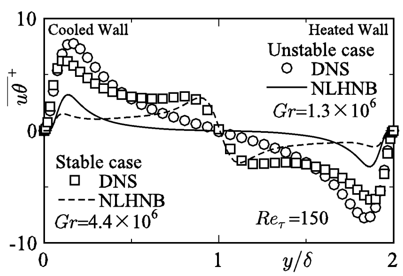

Figure 3: Evaluation result for modeled streamwise turbulent heat-flux (stable/unstable heated plane channel flows)

As a result, the expression of turbulent heat-flux can be derived as follows (Nagano and Hattori, 2003; Abe *et al.*, 1996; hereinafter referred to as the NLHNB model):

$$\begin{aligned}\overline{u_j\theta}=&-\frac{C_{\theta1}}{f_{RT}}\left[\overline{u_ju_k}\tau_m\bar{\Theta}_{,k}-\overline{u_\ell u_k}\tau_m^2\left(C_{\theta2}S_{j\ell}+C_{\theta3}\Omega_{j\ell}\right)\bar{\Theta}_{,k}\right]\\&-\frac{2C_{\theta4}\tau_mg_k\beta k_\theta}{f_{RT}}\left[\delta_{jk}-\tau_m(C_{\theta5}S_{jk}+C_{\theta6}\Omega_{jk})\right]\end{aligned} \tag{14}$$

where $C_{\theta1}\sim C_{\theta6}$ are model constants, τ_m is the characteristic time-scale, and $\Omega_{ij}[=(\bar{U}_{i,j}-\bar{U}_{j,i})/2]$ is the vorticity tensor. f_{RT} is the model function given as follows (Nagano and Hattori, 2003):

$$f_{RT}=1+\frac{1}{2}\tau_m^2\left[C_{\theta2}^2\left(\Omega^2-S^2\right)+\left(C_{\theta3}^2-C_{\theta2}^2\right)\Omega^2\right] \tag{15}$$

where $S^2=S_{mn}S_{mn}$ and $\Omega^2=\Omega_{mn}\Omega_{mn}$.

Evaluations of the derivative expression (NLHNB model) in Eq. (14) are conducted using DNS databases under wall-bounded, buoyancy-affected turbulent flow in a heated plane channel with unstable or stable stratification as indicated in Fig. 1(a) (Iida and Kasagi, 1997; $Gr=1.3\times10^6$ and $Re_\tau=150$ (unstable case); Iida *et al.*, 2002; $Gr=4.4\times10^6$ and $Re_\tau=150$ (stable case)) and in a vertical heated plane channel shown as Fig. 1(b) (Kasagi and Nishimura, 1997; $Gr=9.6\times10^5$ and $Re_\tau=150$). In the case of a vertical

Table 1: Model constants and functions of turbulent heat-flux model

C_{tm}	R_{tm}	$f_w(\xi)$	$C_{\theta 1}$	$C_{\theta 2}$	$C_{\theta 3}$	$C_{\theta 4}$	$C_{\theta 5}$	$C_{\theta 6}$
1.3×10^2	$\dfrac{C_{tm} n^* R_t^{1/4}}{C_{tm} R_t^{1/4} + n^*}$	$\exp\left[-\left(\dfrac{R_{tm}}{\xi}\right)^2\right]$	$0.14[1 - f_w(40)]$	0.05	0.11	0.3	$0.3[1 - f_w(40)]^2$	$0.2[1 - f_w(20)]$

$C_{\theta 7}$	C_m	$B_{\lambda 1}$	τ_u	τ_θ	R	τ_m
$0.2[1 - f_w(20)]$	$0.25/Pr^{1/4}$	$\dfrac{1 + 2Pr}{20Pr^{0.4}}$	$\dfrac{k}{\varepsilon}$	$\dfrac{k_\theta}{\varepsilon_\theta}$	$\dfrac{\tau_\theta}{\tau_u}$	$\left\{\dfrac{2R}{R + C_m} + \sqrt{\dfrac{2R}{Pr}} \dfrac{30}{R_t^{3/4}} \exp\left[-\left(\dfrac{R_{tm}}{B_{\lambda 1}}\right)^{3/4}\right]\right\} \tau_u$

$n^* = n(\nu\varepsilon)^{1/4}/\nu$, $R_t = k^2/(\nu\varepsilon)$, n: local coordinate normal to wall surface

Table 2: Model constants and functions of transport equations for k_θ and ε_θ

C_{P1}	C_{P2}	C_{P3}	$C_{\varepsilon 2}$	f_{P1}	f_{P2}	f_{P3}	C_{D1}	f_{D1}
$0.85(R + 0.4/Pr^{1/4})$	0.64	1.2	1.9	$1 - f_w(1)$	1.0	1.0	$1 - f_w(4)$	1.0

$C_{D2} f_{D2}$	$C_{D2}^* f_{D2}^*$	f_2
$C_{D2}^* f_{D2}^* \left[1 + C_{D3}^* f_{D3}^* \sqrt{R_t} \left(1 + C_\tau \sqrt{\dfrac{Pr}{R}}\right)\right]$	$(C_{\varepsilon 2} f_2 - 1)\left\{1 - \exp\left[-\left(C_{\varepsilon t} \dfrac{R_{tm}}{5}\right)^2\right]\right\}$	$1 - 0.3 \exp\left[-\left(\dfrac{R_t}{6.5}\right)^2\right]$

C_{D3}^*	f_{D3}^*	C_τ	$C_{\varepsilon t}$	$f_{\theta 1}$	$f_{\theta 2}$	C_h	C_ϕ	f_R
0.025	$f_w(30)$	3.0	$1 + Pr^{1.5}$	$1 + 5f_w(5)$	$1 + 20f_w(10)$	0.20	0.25	$2R/(R + 0.5)$

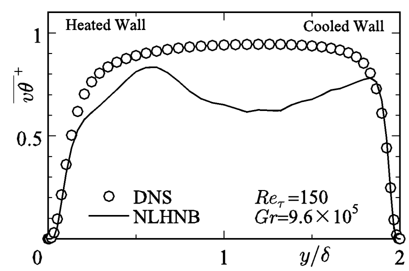

Figure 4: Evaluation result for modeled wall-normal turbulent heat-flux (vertical heated plane channel flow)

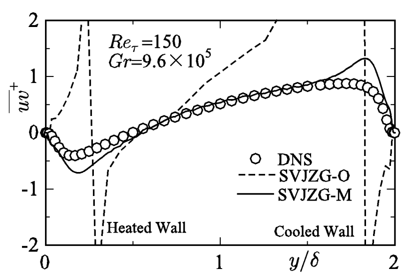

Figure 5: Evaluation result for modeled Reynolds shear stress (vertical heated plane channel flow)

plane channel, the streamwise turbulent heat-flux, $\overline{u\theta}$, appears clearly in both the transport equations of turbulence energy and its dissipation rate. Also, the turbulent heat-flux is included in the modeled expression of Reynolds stress for the buoyancy-affected flow as described later. Therefore, the streamwise turbulent heat-flux should also be predicted exactly by the model. Figure 2 shows the results of assessments for the NLHNB model of the streamwise turbulent heat-flux in the vertical heated plane channel flow. It can be seen that the NLHNB model underpredicts streamwise turbulent heat-flux. Also, in cases of heated plane channel with unstable or stable stratification, obviously, underpredictions of turbulent heat-flux are observed as shown in Fig. 3. On the other hand, the predicted wall-normal turbulent heat-flux by the NLHNB model is shown in Fig. 4. It is clear that the wall-normal turbulent heat-flux is not reproduced accurately in the cases. Consequently, we improve the modeled expression of turbulent heat-flux to predict wall-bounded buoyancy-affected turbulent flows.

Nonlinear Model of Reynolds Stress

The transport equation of Reynolds stress with the buoyant term is given as follows:

$$\frac{D\overline{u_i u_j}}{Dt} = D_{ij} + T_{ij} + P_{ij} + G_{ij} + \Pi_{ij} - \varepsilon_{ij} \tag{16}$$

where D_{ij} is a molecular diffusion term, T_{ij} is a turbulent and pressure diffusion term, P_{ij} is a production term, G_{ij} is a buoyant term, Π_{ij} is a pressure-stain correlation term and ε_{ij} is a dissipation term, respectively.

Introducing the Reynolds stress anisotropy tensor $b_{ij} = \overline{u_i u_j}/2k - \delta_{ij}/3$ and neglecting the diffusive effect, the following relation is derived with Eqs. (6) and (16):

$$\frac{Db_{ij}}{Dt} = \frac{1}{2k}\left(P_{ij} + G_{ij} + \Pi_{ij} - \varepsilon_{ij}\right) - \frac{b_{ij} + \delta_{ij}/3}{k}\left(P_k + G_k - \varepsilon\right) \tag{17}$$

In the local equilibrium state, since the relation $Db_{ij}/Dt = 0$ holds, Eq. (17) yields the following relation.

$$\left(P_{ij} + G_{ij} + \Pi_{ij} - \varepsilon_{ij}\right) = 2(b_{ij} + \delta_{ij}/3)\left(P_k + G_k - \varepsilon\right) \tag{18}$$

So *et al.* (2002) adopted the buoyancy modified nonlinear pressure-strain model (Speziale *et al.*, 1991) as follows:

$$\begin{aligned}\Pi_{ij} = &-\left(C_1 + C_1^* \frac{P_K}{\varepsilon}\right)\varepsilon b_{ij} + C_2 k S_{ij} \\ &+ C_3 k\left(b_{ik} S_{jk} + b_{jk} S_{ik} - \frac{2}{3} b_{mn} S_{mn} \delta_{ij}\right) \\ &- C_4 k(b_{ik}\Omega_{jk} - b_{jk}\Omega_{ik}) + C_5 \varepsilon\left(b_{ik} b_{kj} - \frac{1}{3} b_{mn} b_{mn} \delta_{ij}\right) \\ &- C_6\left(G_{ij} - \frac{2}{3} G\delta_{ij}\right)\end{aligned} \tag{19}$$

where $C_1 \sim C_6$ are model constants.

Substituting Eq. (19) into (18), the introducing the dissipation-rate anisotropy $d_{ij} = (\varepsilon_{ij} - 2\varepsilon\delta_{ij}/3)/2$, and solving as for b_{ij} explicitly (Gatski and Speziale 1993) using the integrity basis $\mathbf{b} = \sum_{n=1}^{K} Q^{(n)}\mathbf{T}^{(n)}$ (Pope 1975), we can obtain

Table 3: Model constants and functions of Reynolds stress model

C_μ	C_D	C_η	C_{v1}	C_{v2}	C_g	C_λ	τ_{mg}	f_μ
0.12	0.8	5.0	0.4	1.0×10^2	-0.7	0.1	$C_\lambda f_\lambda \frac{k}{\varepsilon}$	$[1-f_w(32)]\left\{1+\frac{40}{R_t^{3/4}}\exp\left[-\left(\frac{R_{tm}}{35}\right)^{3/4}\right]\right\}$

f_λ	τ_{Rw}	f_B
$[1-f_w(25)]\left\{1+\sqrt{\frac{2R}{Pr}}\frac{15}{R_t^{3/4}}\exp\left[-\left(\frac{R_{tm}}{30}\right)^{3/4}\right]\right\}$	$\sqrt{\frac{1}{6}\frac{f_{Ro}/C_D}{f_{S\Omega}}\left(1-\frac{3C_{v1}f_{v2}}{8}\right)f_{v1}^2}$	$1+C_\eta(C_D\tau_{Ro})^2(\Omega^2-S^2)$

$f_{S\Omega}$	S_f	f_{v1}	f_{v2}
$\frac{\Omega^2}{2}+\frac{S^2}{3}-\left[\left(\sqrt{\frac{S^2}{2}}-\sqrt{\frac{\Omega^2}{2}}\right)f_w(1)\right]^2$	$1+\sqrt{S^2}\tau_u$	$\exp\left[-\frac{(R_{tm}/52)^2S_f}{(R_{tm}/52)^2+S_f}\right]$	$1-\exp\left(-\frac{\sqrt{R_t}}{C_{v2}}\right)$

Table 4: Model constants and functions of transport equations for k and ε

$C_{\varepsilon1}$	$C_{\varepsilon2}$	$C_{\varepsilon3}$	C_S	C_ε	f_ε	f_{t1}	f_{t2}
1.45	1.9	1.2	1.4	1.8	$\left\{1-0.3\exp\left[-\left(\frac{R_t}{6.5}\right)^2\right]\right\}[1-f_w(3.7)]$	$\frac{1+15f_w(10)}{[1-f_w(32)]^{1/2}}$	$\frac{1+10f_w(10)}{[1-f_w(32)]^{1/2}}$

the following the nonlinear model of Reynolds stress (So *et al.*, 2002; hereinafter referred to as the SVJZG model):

$$\mathbf{b}^* = Q^{(1)}\mathbf{S}^* + Q^{(2)}\left(\mathbf{S}^*\boldsymbol{\Omega}^* - \boldsymbol{\Omega}^*\mathbf{S}^*\right) + Q^{(3)}\left(\mathbf{S}^{*2} - \frac{1}{3}\{\mathbf{S}^{*2}\}\mathbf{I}\right) + Q^{(4)}\mathbf{f}^* + Q^{(5)}\left(\mathbf{f}^*\boldsymbol{\Omega}^* - \boldsymbol{\Omega}^*\mathbf{f}^*\right) \quad (20)$$

where, $\mathbf{S}^*(=C_s g\tau S_{ij})$, $\boldsymbol{\Omega}^*(=C_\omega g\tau\Omega_{ij})$ and $\mathbf{f}^*(=G_{ij}/G-\delta_{ij}^{(2d)})$ are nondimensional tensors, respectively, $\tau=k/\varepsilon$ and $g=\left(\frac{1}{2}C_1+\frac{P}{\varepsilon}+\frac{G}{\varepsilon}-1\right)^{-1}$ are given, and $Q^{(1)}\sim Q^{(5)}$ are derivative model functions, which can be given as follows:

$$Q^{(1)} = \frac{1}{D_1}\left[1-\frac{\mathbf{G}^*}{3D_2}(\eta_8+2\eta_{14})-\frac{\mathbf{G}^*}{3}\right],\; Q^{(2)}=Q^{(1)} \quad (21)$$

$$Q^{(3)} = -\frac{2}{D_1}Q^{(1)}-\frac{\mathbf{G}^*}{D_2\eta_1}(\eta_8+2\eta_{14})+\frac{\mathbf{G}^*}{\eta_1} \quad (22)$$

$$Q^{(4)} = \frac{1}{2D_2}\mathbf{G}^*,\quad Q^{(5)}=\frac{1}{2D_2}\mathbf{G}^* \quad (23)$$

$$D_1 = -\left(1-\frac{2}{3}\eta_1-2\eta_2\right),\quad D_2=-(1-2\eta_2) \quad (24)$$

where $\eta_1=\{\mathbf{S}^{*2}\}$, $\eta_2=\{\boldsymbol{\Omega}^{*2}\}$, $\eta_8=\{\mathbf{f}^*\mathbf{S}^*\}$, $\eta_{14}=\{\boldsymbol{\Omega}^*\mathbf{S}^*\mathbf{f}^*\}$, $\mathbf{G}^*=gC_g(G/\varepsilon)$ and $G=G_{ii}/2$.

The derived expressions in Eq. (20) are found to be identical with cases for thermal field using DNS databases. Figure 5 shows a result of evaluation in the vertical heated plane channel flow. Note that the dashed line indicates the result of the original model and the solid line shows the modified model. Since Eq. (24) often gives a negative value in parentheses at large $\eta 2$, the model overpredicts remarkably near the wall as shown in Fig. 5. In other cases, the tendency of the model prediction gives a similar state (figures not shown). Thus, to avoid the overprediction, the functions are modified as $D_1=-\left(1-\frac{2}{3}\eta_1+2\eta_2\right)$ and $D_2=-(1+2\eta_2)$. Thus, the prediction is improved in most part of channel, but the near-wall behavior of model prediction should be carefully corrected.

NEW PROPOSED MODELS

Thermal Field

So far, the modeling for the dissipation term, $\varepsilon_{j\theta}$, in the transportation of turbulent heat-flux has been omitted, because the dissipation does not influence the prediction of wall-normal heat-flux in the wall-bounded flow. However, the dissipation affects predicting the streamwise heat-flux in the wall-bounded flow. Therefore, we introduce the effect of dissipation-rate into the modelling for Φ_{ij} and $\varepsilon_{j\theta}$ as follows:

$$\Phi_{j\theta}-\varepsilon_{j\theta} = -C_{t1}\frac{\overline{u_j\theta}}{\tau_u}+C_{t2}\overline{u_k\theta}\bar{U}_{j,k}+C_{t3}\overline{u_k\theta}\bar{U}_{k,j}+C_{t5}g_i\beta k_\theta - C_{t4}\overline{u_iu_j}A_{ik}\bar{\Theta}_{,k}-C_{t6}A_{jk}g_k\beta k_\theta \quad (25)$$

where $A_{ik}(=\overline{u_iu_k}/k)$ is the nondimensional Reynolds stress tensor. The $\bar{\Theta}_{,k}$-related term is included, because the production of $\varepsilon_{j\theta}$ is strongly affected by the temperature gradient. Note that the adopted buoyant term is proposed by Craft *et al.* (1996).

Using Eq. (25) and the same manner of deriving the explicit algebraic model of turbulent heat-flux, we can obtain the following modeled expression for the turbulent heat-flux:

$$\overline{u_j\theta} = -\alpha_{jk}^t\bar{\Theta}_{,k}+\frac{\tau_m^2}{f_{RT}}(C_{\theta1}\overline{u_\ell u_k}+C_{\theta5}\overline{u_\ell u_i}A_{ik}) \times(C_{\theta2}S_{j\ell}+C_{\theta3}\Omega_{j\ell})\bar{\Theta}_{,k} - \frac{2C_{\theta6}\tau_m g_k\beta k_\theta}{f_{RT}}\left[\delta_{jk}-\tau_m(C_{\theta2}S_{jk}+C_{\theta3}\Omega_{jk})\right] - \frac{2C_{\theta7}\tau_m A_{\ell k}g_k\beta k_\theta}{f_{RT}}\left[\delta_{j\ell}-\tau_m(C_{\theta2}S_{j\ell}+C_{\theta3}\Omega_{j\ell})\right] \quad (26)$$

$$\alpha_{jk}^t = \frac{\tau_m}{f_{RT}}(C_{\theta1}\overline{u_ju_k}+C_{\theta4}\overline{u_iu_j}A_{ik}) \quad (27)$$

$$f_{RT} = 1+\frac{1}{2}\{\tau_m[1-f_w(40)]\}^2(C_{\theta3}\Omega^2-C_{\theta2}S^2) \quad (28)$$

where the model constants and functions in Eq. (26) are indicated in Table 1.

To model the transport equation for thermal field, the turbulent diffusion terms in Eqs. (8) and (9) are modeled by GGDH modeling (Nagano and Hattori, 2003):

$$T_{k\theta} = \left(C_h f_{\theta1}\frac{k}{\varepsilon}f_R\overline{u_ju_\ell}k_{\theta,\ell}\right)_{,j} \quad (29)$$

$$T_{\varepsilon_\theta} = \left(C_\phi f_{\theta2}\frac{k}{\varepsilon}f_R\overline{u_ju_\ell}\varepsilon_{\theta,\ell}\right)_{,j} \quad (30)$$

Also, the model constants and functions in Eqs. (8) and (9) are indicated in Table 2.

Velocity Field

We reconstruct the expression of Reynolds shear stress considering the evaluation results. Since the modeling of Π_{ij}

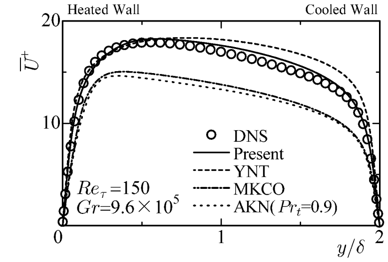

Figure 6: Distributions of mean velocity in vertical heated plane channel flow

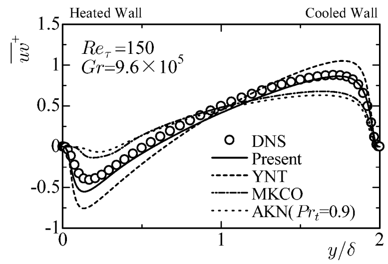

Figure 7: Distributions of Reynolds shear stress in vertical heated plane channel flow

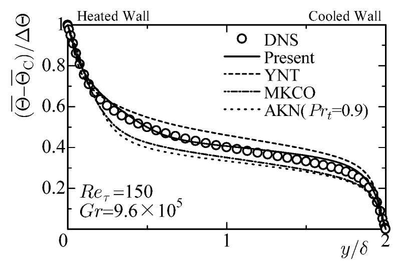

Figure 8: Distributions of mean temperature in vertical heated plane channel flow

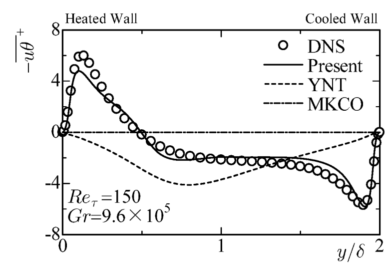

Figure 9: Distributions of streamwise heat-flux in vertical heated plane channel flow

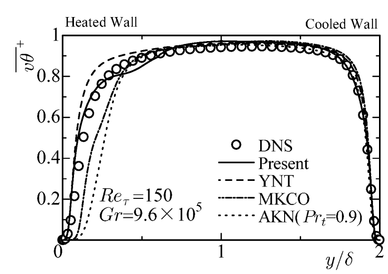

Figure 10: Distributions of wall-normal heat-flux in vertical heated plane channel flow

mentioned above (So *et al.*, 2002) is adopted accordingly IP model ($C_5 = 0$), the proposed model also employs the IP model with buoyant effect. When the same procedure for the

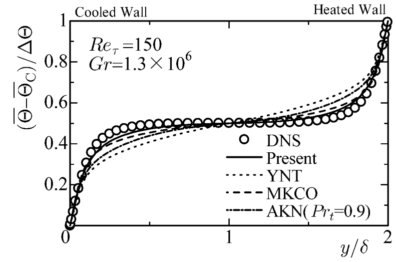

Figure 11: Distributions of mean temperature in unstable heated plane channel flow

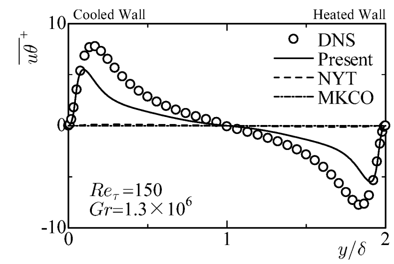

Figure 12: Distributions of streamwise heat-flux in unstable heated plane channel flow

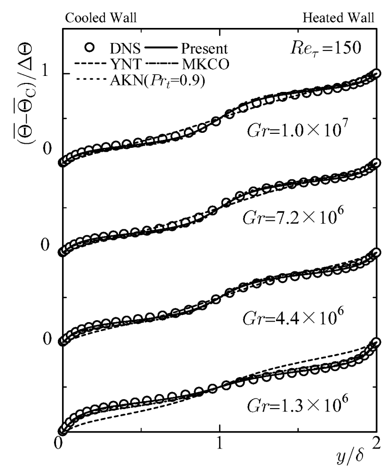

Figure 13: Distributions of mean temperature in stable heated plane channel flow

derivation of the explicit algebraic model of Reynolds stress is applied, we obtain:

$$
\begin{aligned}
b_{ij}^* = & -\frac{3}{3-2\eta^2+6\zeta^2}\left[S_{ij}^* + (S_{ik}^*\Omega_{kj}^* - \Omega_{ik}^* S_{kj}^*)\right. \\
& \left. -2\left(S_{ik}^* S_{kj}^* - \frac{1}{3} S_{mn}^* S_{nm}^* \delta_{ij}\right)\right] \\
& -\frac{1}{1+2\zeta^2}\left[f_{ij}^* + (f_{ik}^*\Omega_{kj}^* - \Omega_{ik}^* f_{kj}^*)\right. \\
& \left. +\left(f_{ik}^* S_{kj}^* + S_{ik}^* f_{kj}^* - \frac{2}{3} f_{mn}^* S_{mn}^* \delta_{ij}\right)\right] \qquad (31)
\end{aligned}
$$

where $\eta = (S_{ij}^* S_{ij}^*)^{1/2}$ and $\zeta = (\Omega_{ij}^*\Omega_{ij}^*)^{1/2}$. Note that in order to avoid an inappropriate value of D_1 and D_2 referring to the assessment results, D_1 and D_2 in Eq. (24) are modeled as $D_1 = (3-2\eta^2+6\zeta^2)/3$ and $D_2 = 1+2\zeta^2$, respectively. Here, the nondimensional forms of b_{ij}^*, S_{ij}^*, Ω_{ij}^* and f_{ij}^* are adopted as follows (Abe *et al.*, 1997; Nagano *et al.*, 1997):

$$
\begin{aligned}
& b_{ij}^* = C_D b_{ij},\ S_{ij}^* = C_D \tau S_{ij}, \\
& \Omega_{ij}^* = 2C_D\tau\Omega_{ij},\ f_{ij}^* = C_g(\tau_{mg}/k) f_{ij} \qquad (32)
\end{aligned}
$$

Consequently, the newly proposed expressions are as follows:

$$\begin{aligned}\overline{u_i u_j} =& \frac{2}{3}k\delta_{ij} - \frac{2\nu_t}{f_{R1}}S_{ij} - \frac{4C_D k f_\tau}{f_{R1}}\left(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}\right) \\ &+ \frac{4C_D k f_\tau}{f_{R1}}\left(S_{ik}S_{kj} - \frac{1}{3}S_{mn}S_{mn}\delta_{ij}\right) \\ &- \frac{2C_g\tau_{mg}}{C_D f_{R2}}f_{ij} - \frac{4C_g\tau_{mg}^2}{f_{R2}}\left(f_{ik}\Omega_{kj} - \Omega_{ik}f_{kj}\right) \\ &+ \frac{2C_g\tau_{mg}^2}{f_{R2}}\left(f_{ik}S_{kj} + S_{ik}f_{kj} - \frac{2}{3}f_{mn}S_{mn}\delta_{ij}\right) \quad (33)\end{aligned}$$

where τ_{mg} is the mixed scale of velocity and thermal fields, and model functions are given in $f_{R1} = 1 + (22/3)\left(C_D\tau_{R_o}\right)^2\Omega^2 + (2/3)\left(C_D\tau_{R_o}\right)^2\left(\Omega^2 - S^2\right)f_B$ and $f_{R2} = 1 + 8\left(C_D\tau_{R_o}\right)^2\Omega^2$, respectively, and f_τ is the function of characteristic time-scale reflecting wall-limiting behavior (Hattori and Nagano, 2003) as follows:

$$f_\tau = \tau_{R_o}^2 + \tau_{R_w}^2 \quad (34)$$

where the characteristic time-scale τ_{R_o} is given by ν_t/k, and τ_{R_w} is the wall-reflection time-scale. The model constants and functions in Eq. (33) are indicated in Table 3.

To model the transport equation for velocity field, the turbulent diffusion terms in Eqs. (6) and (7) are modeled by GGDH modeling (Nagano and Hattori, 2003):

$$T_k = \left(C_s f_{t1}\frac{\nu_t}{k}\overline{u_j u_\ell}k_{,\ell}\right)_{,j} \quad (35)$$

$$T_\varepsilon = \left(C_\varepsilon f_{t2}\frac{\nu_t}{k}\overline{u_j u_\ell}\varepsilon_{,\ell}\right)_{,j} \quad (36)$$

Also, the model constants and functions in Eqs. (6) and (7) are indicated in Table 4.

RESULTS AND DISCUSSION

The evaluations for the newly improved models are shown in Figs. 6∼13. The predictions of the conventional eddy viscosity model reflecting the buoyant effect (Yin *et al.*, 1991; YNT model, Murakami *et al.*, 1996; MKCO model) and a constant turbulent Prandtl number ($Pr_t = 0.9$) model are also included in these figures for comparison. In the case of a vertical heated plane channel flow, it can be seen that almost all models accurately predict Reynolds shear stress, mean temperature and turbulent heat-fluxes. The present model only gives an adequate prediction of the stream-wise turbulent heat-flux as shown in Fig. 9. Also, the proposed model can accurately predict the cases of unstable or stable heated plane channel flows as indicated in Figs. 11∼13. Thus, it is concluded that the present model is available for accurate prediction of wall-shear flows with buoyancy.

CONCLUSIONS

DNS-based evaluations of the modeled expressions for Reynolds stress and turbulent heat-flux are conducted in wall-shear flows with buoyancy. In particular, it is found that the streamwise turbulent heat-flux, $\overline{u\theta}$, is underpredicted in all cases. In the case of a vertical plane channel, the streamwise turbulent heat-flux, $\overline{u\theta}$, appears clearly in both the transport equations of turbulence energy and its dissipation rate. Also, the turbulent heat-flux is included in the modeled expression of Reynolds stress for the buoyancy-affected flow. Therefore, the streamwise turbulent heat-flux should also be exactly predicted by the model. We propose a new nonlinear two-equation heat-transfer model which can satisfactorily predict wall-shear flows with buoyancy.

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