EXPLICIT ALGEBRAIC MODELLING FOR EDDY VISCOSITY MODELS

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ABSTRACT

Explicit Algebraic Reynolds Stress Models (EARSM) provide a better description of the Reynolds stress tensor than the classical eddy viscosity models. Therefore, they act as a SST limiter and improve separation prediction. They also allow to easily capture rotation and curvature effects.

However, the classical derivation of EARSM models relies upon the notion of turbulence time scale, which is available in two-equation models but not in models solving only one transport equation for the eddy viscosity.

A way to extend EARSM modelling to one equation models is proposed. For the sake of simplicity only two-dimensional flows are addressed. Applications to simple flows such as homogeneous shear plus rotation, and boundary layer flows without pressure gradient or close to separation are given to validate the model.

INTRODUCTION

Algebraic Reynolds stress models have been proposed by Rodi (1972,1981). The basic idea is that the turbulent flow evolves, under weakly changing mean flow conditions, in such a way that the turbulent kinetic energy level can change drastically but the anisotropy tensor

$$b_{ij} = \frac{\langle u'_i u'_j \rangle}{2k} - \frac{\delta_{ij}}{3} \tag{1}$$

hardly changes. The Reynolds stress transport equation can then be rewritten as

$$\begin{aligned} \frac{D\langle u'_i u'_j \rangle}{Dt} &= P_{ij} + \Pi_{ij} - \varepsilon_{ij} + \mathrm{Diff}_{\langle u'_i u'_j \rangle} \\ = \frac{\langle u'_i u'_j \rangle}{k}\frac{Dk}{Dt} &= \frac{\langle u'_i u'_j \rangle}{k}\left[P_k - \varepsilon + \mathrm{Diff}_k\right] \end{aligned} \tag{2}$$

where P_{ij}, Π_{ij}, ε_{ij} and $\mathrm{Diff}_{\langle u'_i u'_j \rangle}$ respectively stand for the Reynolds stress production, redistribution, viscous destruction and diffusion terms while $P_k = \frac{1}{2}P_{ii}$ and $\varepsilon = \frac{1}{2}\varepsilon_{ii}$. Assuming that the anisotropy also weakly changes spatially, so that the diffusion term in the Reynolds stress transport equation is proportional to the diffusion term in the turbulent kinetic energy equation, the problem reduces to the constancy of the anisotropy tensor following fluid particles, which leads to:

$$P_{ij} + \Pi_{ij} - \varepsilon_{ij} = \frac{\langle u'_i u'_j \rangle}{k}\left[P_k - \varepsilon\right] \tag{3}$$

The above equation can be solved iteratively whatever the turbulence model.

Pope (1975) pointed out that the anisotropy tensor can be expanded as a sum of ten independent tensors, formed with the non-dimensionalized mean rate of strain and vorticity tensors

$$\begin{aligned} S^*_{ij} &= \frac{k}{\varepsilon}S_{ij} \qquad S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \\ \Omega^*_{ij} &= \frac{k}{\varepsilon}\Omega_{ij} \qquad \Omega_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \end{aligned} \tag{4}$$

the coefficients of the expansion being dependent only upon the invariants of these tensors. For two dimensional flows, only three tensors have to be considered

$$S^*_{ij} \qquad S^*_{ik}\Omega^*_{kj} - \Omega^*_{ik}S^*_{kj} \qquad S^*_{ik}S^*_{kj} - \frac{\delta_{ij}}{3}II_S \tag{5}$$

together with two invariants

$$II_S = S^*_{ij}S^*_{ji} \qquad II_\Omega = \Omega^*_{ij}\Omega^*_{ji} \tag{6}$$

For two dimensional flows, Pope showed that the solution of equation (3) can be obtained as a function of the production to dissipation ratio $\frac{P_k}{\varepsilon}$. However, an analytical solution is available only if the redistribution and viscous destruction term models are linear with respect to the Reynolds stress tensor.

Gatski and Speziale (1993) extended the approach to three-dimensional flows but still had to impose the production to dissipation ratio. The explicit solution was independently arrived at by Girimaji (1996) and Wallin and Johansson (2000) who pointed out that the production to dissipation ratio can be expressed as

$$\frac{P_k}{\varepsilon} = -\langle u'_i u'_j \rangle S_{ij} = -2b_{ij}S^*_{ij} \tag{7}$$

Table 1: Model coefficients for various Reynolds stress models

Model	c_1	α	β	γ	λ	μ
GL	1.8	-0.6	0	-0.6	0.4	0.4
LRR (0.4)	1.5	-0.764	-0.364	-0.109	0.127	0.345
LRR (5/9)	1.5	-0.778	-0.533	-0.222	0	0.444
SSG	3.4	-0.438	-0.473	-0.238	0.325	0.8

This leads to a third order equation for the production to dissipation ratio for two-dimensional flows, a sixth order equation for three-dimensional flows.

EXTENSION TO ONE-EQUATION MODELS

The above summarized derivation of the Explicit Algebraic Reynolds Stress Models (EARSM) requires the knowledge of the turbulent kinetic energy and its dissipation rate or, at least, of a turbulence time scale. Such data is available with any two-equation turbulence model, not with a one equation model solving an equation for the eddy viscosity. One equation models are presently very popular and only one attempt to couple them with a non-linear representation of the Reynolds stress tensor has been reported by Spalart (2000).

For the sake of simplicity, the proposed strategy will be presented for two-dimensional flows. As dealing with non-dimensional quantities such as the anisotropy tensor and the non-dimensional mean rate of strain and vorticity tensor is prohibited, the Reynolds stress tensor is first rewritten, following Pope's analysis as:

$$\begin{aligned} -\langle u_i' u_j' \rangle &= -\frac{2}{3}k\delta_{ij} + 2\nu_t S_{ij} + A\left(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}\right) \\ &\quad + B\left(S_{ik}S_{kj} - \frac{1}{3}S_{lk}S_{kl}\delta_{ij}\right) \end{aligned} \tag{8}$$

From dimensional analysis, the coefficients of the last terms can be rewritten as $A = a\nu_t T$ and $B = b\nu_t T$ where a and b are constants and T a time scale to be determined. a is just introduced for latter convenience as it could be included in the definition of the time scale T.

A linear Reynolds stress model of the following form is used:

$$\begin{aligned} \frac{D\langle u_i' u_j' \rangle}{Dt} &= P_{ij} + \alpha\left(P_{ij} - \frac{2\delta_{ij}}{3}P_k\right) + \beta k S_{ij} \\ &\quad + \gamma\left(D_{ij} - \frac{2\delta_{ij}}{3}P_k\right) \\ &\quad - c_1\frac{\varepsilon}{k}\left(\langle u_i' u_j' \rangle - \frac{2\delta_{ij}}{3}k\right) - \frac{2\delta_{ij}}{3}\varepsilon \end{aligned} \tag{9}$$

where $P_{ij} = -\langle u_i' u_k' \rangle \frac{\partial u_j}{\partial x_k} - \langle u_k' u_j' \rangle \frac{\partial u_i}{\partial x_k}$ and $D_{ij} = -\langle u_i' u_k' \rangle \frac{\partial u_k}{\partial x_j} - \langle u_k' u_j' \rangle \frac{\partial u_k}{\partial x_i}$. Models coefficients for the models by Gibson (1978), Launder et al. (1975), with the original value of the constant c_2 (0.4) and the value $\frac{5}{9}$ later recommended by Taulbee (1972), and the linearized version of model by Speziale et al. (1991), as proposed by Gatski and Speziale (1993), are given in table 1. The diffusion term has been omitted in equation (9) as it is dropped out in the standard EARSM approach, as shown previously.

Introducing the expansion (8) into the above Reynolds stress equation (9), projecting it on the tensorial basis made with the four independent tensors δ_{ij}, S_{ij}, $S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}$, and $S_{ik}S_{kj} - \frac{1}{3}S_{lk}S_{kl}\delta_{ij}$ leads to the following set of equations:

$$\frac{1}{k}\frac{Dk}{Dt} = \frac{P_k}{k} - \frac{\varepsilon}{k} \tag{10}$$

$$\frac{1}{\nu_t}\frac{D\nu_t}{Dt} = \chi\frac{k}{\nu_t} - c_1\frac{\varepsilon}{k} - \left(2\mu a\Omega^2 + \frac{\lambda b}{3}S^2\right)T \tag{11}$$

$$\frac{DA}{Dt} = 2\mu\nu_t - c_1\frac{\varepsilon}{k}A \tag{12}$$

$$\frac{DB}{Dt} = -4\lambda\nu_t - c_1\frac{\varepsilon}{k}B \tag{13}$$

where $\chi = \left(\frac{2\lambda}{3} - \frac{\beta}{2}\right)$, $\lambda = 1 + \alpha + \gamma$, $\mu = 1 + \alpha - \gamma$, $S^2 = \frac{1}{2}S_{ij}S_{ij}$ and $\Omega^2 = \frac{1}{2}\Omega_{ij}\Omega_{ij}$. Equations (12, 13) can be rearranged as

$$a\left[\frac{DT}{Dt} + \frac{T}{\nu_t}\frac{D\nu_t}{Dt} + c_1\frac{\varepsilon}{k}T\right] = 2\mu \tag{14}$$

$$b\left[\frac{DT}{Dt} + \frac{T}{\nu_t}\frac{D\nu_t}{Dt} + c_1\frac{\varepsilon}{k}T\right] = -4\lambda \tag{15}$$

so that

$$b = -2a\frac{\lambda}{\mu} \tag{16}$$

Equation (12) can be rearranged as

$$\frac{DT}{Dt} = \frac{2\mu}{a} - c_1\frac{\varepsilon}{k}T - \frac{T}{\nu_t}\frac{D\nu_t}{Dt} \tag{17}$$

so that the equation set (10, 11, 17) can be interpreted as a set of relations between time scales $\frac{k}{P_k}$, $\frac{k}{\varepsilon}$, $\frac{\nu_t}{k}$, T and $T_\nu = \left(\frac{1}{\nu_t}\frac{D\nu_t}{DT}\right)^{-1}$, where only the last one is known, from the transport equation for ν_t.

Reformulating equation (8) as an equation for the anisotropy tensor

$$\begin{aligned} b_{ij} = \frac{\langle u_i' u_j' \rangle}{2k} - \frac{\delta_{ij}}{3} &= -\frac{\nu_t}{k}S_{ij} - \frac{aT\nu_t}{2k}\left(S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}\right) \\ &\quad - \frac{bT\nu_t}{2k}\left(S_{ik}S_{kj} - \frac{1}{3}S_{lk}S_{kl}\delta_{ij}\right) \end{aligned} \tag{18}$$

shows that the anisotropy remains constant, for constant mean velocity gradients, only if the time scales $\frac{\nu_t}{k}$ and T are constant. The constancy of $\frac{\nu_t}{k}$ allows to rewrite equation (10) as:

$$\frac{1}{k}\frac{Dk}{Dt} = \frac{1}{\nu_t}\frac{D\nu_t}{Dt} = \frac{P_k}{k} - \frac{\varepsilon}{k} \tag{19}$$

Equations (19, 11) together with (17) in which the l.h.s. has been set to zero now form a system of three equations for three unknown time scales. Setting for convenience $a = 2\mu$, so that $b = -4\lambda$, and noting that, for two-dimensional flows, $P_k = 4\nu_t S^2$, this system can be rewritten as:

$$\frac{1}{T_\nu} = \frac{P_k}{k} - \frac{\varepsilon}{k} = \frac{4\nu_t}{k}S^2 - \frac{\varepsilon}{k} \tag{20}$$

$$\frac{1}{T_\nu} = \chi\frac{k}{\nu_t} - c_1\frac{\varepsilon}{k} - \left(4\mu^2\Omega^2 - \frac{4\lambda^2}{3}S^2\right)T \tag{21}$$

$$\frac{1}{T_\nu} = \frac{1}{T} - c_1\frac{\varepsilon}{k} \tag{22}$$

Combining equations (20) and (22) yields:

$$T = \frac{k}{P_k + (c_1 - 1)\varepsilon} \tag{23}$$

so that, since $P_k = 4\nu_T S^2 > 0$ and $c_1 > 1$, the time scale T is always positive (while T_ν can be negative). Equation (22) can be rewritten as

$$\frac{1}{T} - \frac{1}{T_\nu} = c_1 \frac{\varepsilon}{k} > 0 \tag{24}$$

which then leads to:

$$\frac{T}{T_\nu} < 1 \tag{25}$$

Eliminating ε between equations (20) and (22) yields:

$$(c_1 - 1)\frac{1}{T_\nu} + \frac{1}{T} = 4c_1 \frac{\nu_t}{k} S^2 > 0$$

so that

$$\frac{T}{T_\nu} > -\frac{1}{c_1 - 1} \tag{26}$$

At last, subtracting equation (22) from (21) gives:

$$\left(\frac{2}{3}\lambda - \frac{\beta}{2}\right)\frac{k}{\nu_t} = 4\left(\mu^2\Omega^2 - \frac{\lambda^2}{3}S^2\right)T + \frac{1}{T} > 0 \tag{27}$$

which is positive as $\frac{2}{3}\lambda - \frac{\beta}{2}$ is positive whatever the considered Reynolds stress model.

Equations (20-22) can be combined to get a third order equation for T. It is more convenient to write it as an equation for $\tau = \dfrac{T}{T_\nu}$ as:

$$\begin{array}{rrcl} & 0 & = & a_3\tau^3 + a_2\tau^2 + a_1\tau + a_0 \qquad (28) \\ \text{where} & a_3 & = & 4\,(c_1 - 1)\left(3\mu^2\Omega_0^2 - \lambda^2 S_0^2\right) \\ & a_2 & = & 4\left(3\mu^2\Omega_0^2 - \lambda^2 S_0^2\right) - 2c_1 S_0^2\,(4\lambda - 3\beta) \\ & a_1 & = & 3\,(c_1 - 1) \\ & a_0 & = & 3 \end{array}$$

where $S_0^2 = T_\nu^2 S^2$ and $\Omega_0^2 = T_\nu^2\Omega^2$.

It can be noticed that a_0 and a_1 are always positive and that, if a_2 is positive, a_3 is positive. Positive a_2 corresponds to

$$\left(\frac{\Omega_0}{S_0}\right)^2 \geq \frac{\lambda^2}{3\mu^2} + \frac{c_1\,(4\lambda - 3\beta)}{6\mu^2}$$

Then, since all coefficients are positive, τ is negative and hence T_ν is negative as T is positive. As $\dfrac{k}{\nu_t}$ is constant, this means that the turbulent kinetic energy is decreasing. The dominant rotation regime given by the underlying Reynolds stress model is retrieved and had to be verified by the associated one-equation model.

Dividing by T_ν can lead to some problems. From its definition, T_ν cannot be null but can tend towards infinity. In this case, equation (28) tends towards:

$$\left[4\left(3\mu^2\Omega^2 - \lambda^2 S^2\right) - 2c_1 S^2\,(4\lambda - 3\beta)\right]T^2 + 3\,(c_1 - 1)\,T + 3 = 0$$

which as a positive root as the coefficient of T^2 is negative.

It is more convenient to write the solution of the third order equation (28) for $\dfrac{1}{\tau}$ as:

$$\mathcal{P}_1 = \frac{-1}{54a_0^3}\left(2a_1^3 - 9a_0a_1a_2 + 27a_0^2a_3\right)$$

$$\mathcal{P}_2 = \frac{1}{108a_0^4}\left(4a_0a_2^3 - a_1^2a_2^2 - 18a_0a_1a_2a_3 + 27a_0^2a_3^2 + 4a_3a_1^3\right)$$

$$\begin{array}{rcl} \dfrac{1}{\tau} & = & \dfrac{-a_1}{3a_0} + \left(\mathcal{P}_1 + \sqrt{\mathcal{P}_2}\right)^{1/3} + \left(\mathcal{P}_1 - \sqrt{\mathcal{P}_2}\right)^{1/3} \qquad (29) \\ & & \text{if} \quad (\mathcal{P}_2 \geq 0) \\ & = & \dfrac{-a_1}{3a_0} \\ & + & 2\left(\mathcal{P}_1^2 - \mathcal{P}_2\right)^{1/6}\cos\left[\dfrac{1}{3}\arccos\left(\dfrac{\mathcal{P}_1}{\sqrt{\mathcal{P}_1^2 - \mathcal{P}_2}}\right) + \dfrac{2m\pi}{3}\right] \\ & & \text{if} \quad (\mathcal{P}_2 < 0) \end{array}$$

In the last case ($\mathcal{P}_2 < 0$), the solution is chosen by continuity and to ensure that τ satisfies

$$T > 0 \qquad 1 > \tau > -\frac{1}{c_1 - 1}$$

To sum up, the relations (29) give τ and hence the time scale T. The turbulent kinetic energy k can be deduced from equation (27) so that the Reynolds stress tensor can be computed from equation (8). If needed, the dissipation rate can also be obtained with equation (24).

MODEL VALIDATION

Homogeneous shear

For homogeneous shear $S = \Omega = \left|\dfrac{\partial U}{\partial y}\right|$ so that equation (28) reduces to

$$\begin{array}{rrcl} & 0 & = & a_3' S_0^2\tau^3 + a_2' S_0^2\tau^2 + a_1\tau + a_0 \qquad (30) \\ \text{where} & a_3' & = & 4\,(c_1 - 1)\left(3\mu^2 - \lambda^2\right) \\ & a_2' & = & 4\left(3\mu^2 - \lambda^2\right) - 2c_1 S_0^2\,(4\lambda - 3\beta) \\ & a_1 & = & 3\,(c_1 - 1) \\ & a_0 & = & 3 \end{array}$$

An analytical solution of this equation can be obtained as

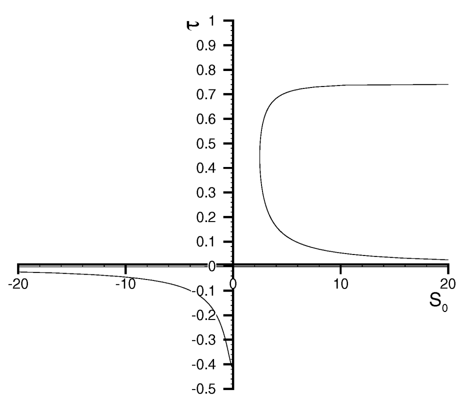

Figure 1: Homogeneous shear: time ratio τ versus non-dimensional shear rate S_0

$$S_0 = \frac{1}{\tau}\sqrt{-\frac{a_1\tau + a_0}{a_3'\tau + a_2'}} \tag{31}$$

which is shown graphically in figure 1. This figure shows that there is no solution for small positive values of S_0. This comes from the underlying hypothesis that $\frac{\nu_t}{k}$ is constant. Therefore $S_0 = S\left(\frac{1}{\nu_t}\frac{d\nu_t}{dt}\right)^{-1} = S\left(\frac{1}{k}\frac{dk}{dt}\right)^{-1} = \frac{k}{-\langle u'v'\rangle}\frac{P_k}{P_k-\varepsilon}$. When the mean shear and hence the production to dissipation ratio decreases, S_0 does not tend to zero but towards infinity. The solution is thus continuous, as shown in figure 2 on which the time scale ratio is plotted versus S_0^{-1}. Very small positive values of S_0 can only be achieved for large values of $\frac{P_k}{\varepsilon}$. According to figures 1 and 2, the time scale ratio is thus blocked to the highest value corresponding to minimum positive S_0.

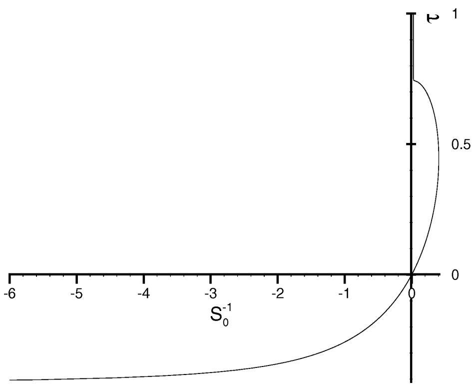

Figure 2: Homogeneous shear: time ratio τ versus inverse of the non-dimensional shear rate S_0

Most one equation model reduce, for homogeneous shear, to:

$$\frac{d\nu_t}{dt} = c_{b1}\nu_t\left|\frac{\partial U}{\partial y}\right| = 2c_{b1}S\nu_t \qquad (32)$$

so that $\nu_t \propto \exp(2c_{b1}St)$. On the other hand, the turbulent kinetic energy equation reduces to

$$\frac{dk}{dt} = P_k - \varepsilon = P_k\left(1-\frac{\varepsilon}{P_k}\right) = 2\frac{-\langle u'v'\rangle}{k}kS\left(1-\frac{\varepsilon}{P_k}\right) \quad (33)$$

Assuming that both $\frac{-\langle u'v'\rangle}{k}$ and $\frac{P_k}{\varepsilon}$ tend towards constant levels, the turbulent kinetic energy evolves as $k \propto \exp\left(2\frac{-\langle u'v'\rangle}{k}\left(1-\frac{\varepsilon}{P_k}\right)St\right)$. Therefore, if $\frac{\nu_t}{k}$ is constant, the two power laws are the same, so that

$$c_{b1} = \frac{-\langle u'v'\rangle}{k}\left(1-\frac{\varepsilon}{P_k}\right) \qquad (34)$$

Assuming $\frac{-\langle u'v'\rangle}{k} \approx 0.3$ and $\frac{P_k}{\varepsilon} \approx 1.4 - 1.8$ yields values of c_{b1} between 0.86 and 0.133, close to the value in the Spalart (1994) model (0.1355).

From equation (32), $(T_\nu)^{-1} = \frac{1}{\nu_t}\frac{d\nu_t}{dt} = 2c_{b1}S$ so that $S_0 = ST_\nu = \frac{1}{2c_{b1}}$. The above range of c_{b1} gives values of S_0 between 3.75 and 5.8. Anisotropy levels predicted with the SSG model constants are given in table 2 and are in fair agreement with experiments.

Table 2: Anisotropy levels for sheared flows - SSG model

S_0	b_{12}	b_{11}	b_{22}	b_{33}
3.75	-0.144	0.199	-0.151	-0.047
4.8	-0.145	0.183	-0.140	-0.044
5.8	-0.146	0.169	-0.129	-0.040

Homogeneous shear plus rotation

Results for a combination of strain and rotation are plotted in figures 3 to 7. It must first be recalled that S and Ω are strain and rotation modulus so that $S_0 = ST_\nu$ and $\Omega_0 = \Omega T_\nu$ have the same sign as T_ν. Therefore, in the figures, regions where S_0 and Ω_0 have opposite signs are out of concern. The impossible domain, which corresponds to dominant rotation, is clearly visible on these figures as part of the domain ($\Omega_0 > S_0 > 0$) without iso-contour lines. The exact equation of the limiting curve is awfully complex.

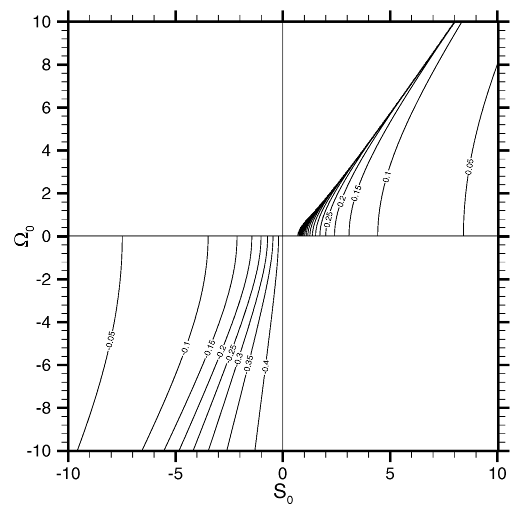

Figure 3: Homogeneous shear plus rotation: iso-time ratio ($\tau = \frac{T}{T_\nu}$) contours

Figure 3 and 4 respectively show the iso-contours of τ and $\frac{P_k}{\varepsilon}$, confirming that the region of negative values for S_0 and Ω_0 corresponds to negative values for τ, ensuring positive values for T, and to production to dissipation ratios lower than unity.

Continuity of iso-contours is better represented using $\frac{1}{S_0}$ and $\frac{1}{\Omega_0}$ as the origin now corresponds to $\frac{d\nu_t}{dt} = 0$. Figures 5 and 6 presents iso-b_{11} and iso-b_{12} contours.

At last, the production to dissipation ratio has been plotted versus $\sigma = \frac{k}{\varepsilon}S = \frac{c_1 S_0\tau}{1-\tau}$ for various rotation to strain ratios. Although these curves have been obtained with the Speziale et al. model, they are very similar to the one published by Wallin and Johansson (2000), using another turbulence model, as shown in figure 7.

Boundary layer flows

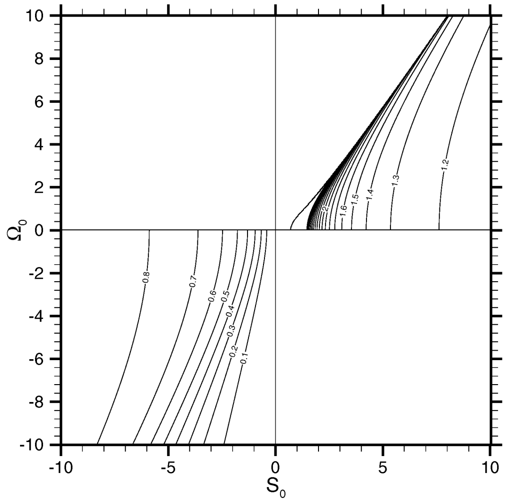

Figure 4: Homogeneous shear plus rotation: iso production to dissipation ratio ($\frac{P_k}{\varepsilon}$) contours

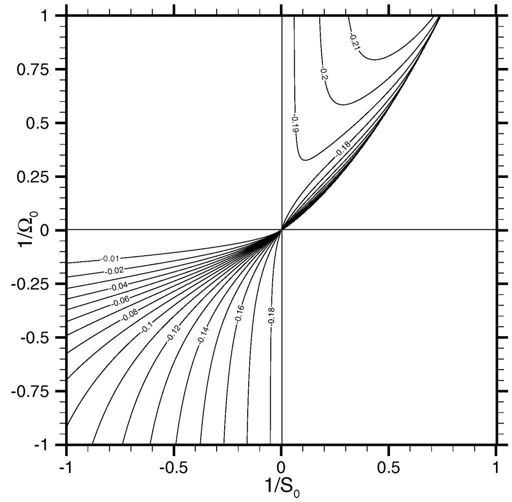

Figure 6: Homogeneous shear plus rotation: iso-b_{12} contours

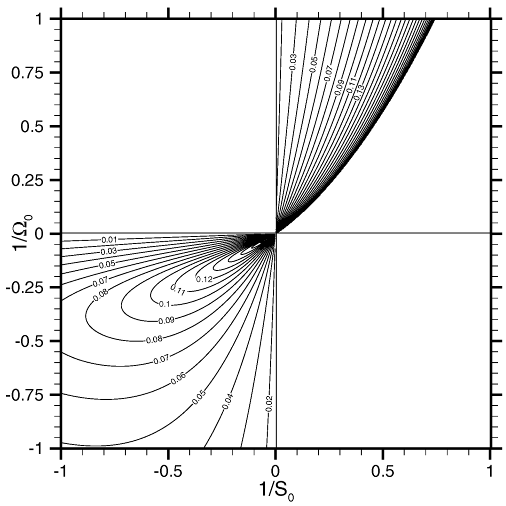

Figure 5: Homogeneous shear plus rotation: iso-b_{11} contours

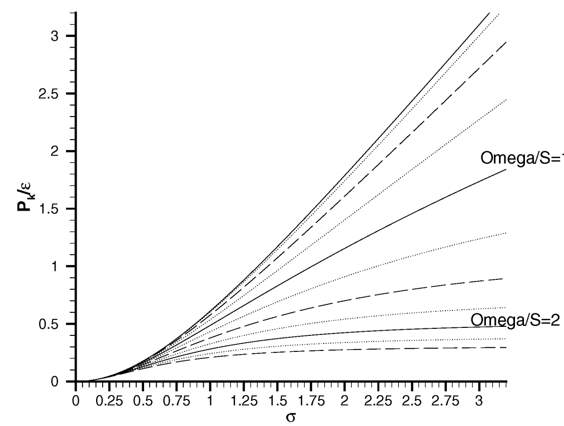

Figure 7: Homogeneous shear plus rotation: iso production to dissipation ratio contours

Two boundary layer flows have been used to validate the model. The boundary layers are first computed using a boundary layer code and the Spalart and Allmaras model. From the eddy viscosity and mean flow profiles, T_ν is computed and then all the relevant quantities, using the above model. Reynolds stress profiles are compared to experiments in figure 8 for a zero pressure gradient boundary layer. The experimental data are from Smith (1994) experiment.

$\langle u'^2 \rangle$ is slightly underpredicted and the near wall behaviour is of course not at all predicted by the model but the levels of the Reynolds stresses are fairly well reproduced.

The same comparison is plotted in figure 9 for a boundary layer close to separation. Experimental data are from Skåre and Krogstad (1994). Experimental data are fairly well reproduced, except near the wall and near the boundary layer edge where diffusion effects may be important. Considering the simplicity of the underlying turbulence model, solving just one transport equation, such an agreement on all the Reynolds stress transport is however impressive as even the turbulent kinetic energy is a result of the present model.

CONCLUSION AND PERSPECTIVES

The feasibility of the extension of Explicit Algebraic Reynolds Stress Model to turbulence models solving only one transport equation for the eddy viscosity has been demonstrated, at least for two-dimensional flows. All the Reynolds stress tensor, as well as the turbulent kinetic energy and its dissipation rate can be extracted from the mean velocity gradient, the eddy viscosity and its substantial derivative, with the help of standard equilibrium assumptions.

The present model has been validated for homogeneous flows as well as for boundary layer flows and has been shown to give fair results, in agreement with experimental data and previous EARSM models.

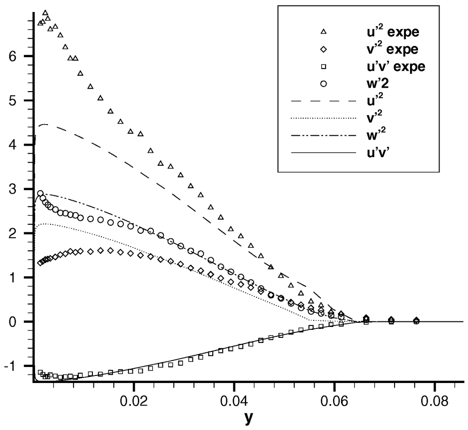

Figure 8: Reynolds stress profiles for a zero pressure gradient boundary layer: comparison of the present model with experiments

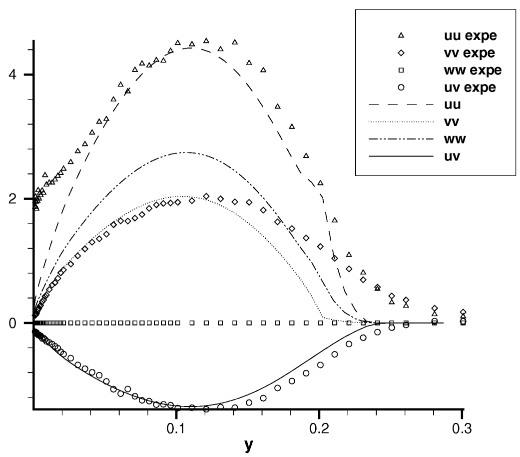

Figure 9: Reynolds stress profiles for a positive pressure gradient boundary layer: comparison of the present model with experiments

The first extension is obviously to deal with three-dimensional flows. Streamline curvature and rotation effects can easily be introduced, following Wallin and Johansson (2002). Wall damping has to be added to make the model applicable in all conditions. It has also been shown that there is some coupling with the underlying one-equation model, which deserve more attention.

A defect of the model is to assume both $\frac{\nu_t}{k}$ and T constant. A transport equation for k could be added to get rid of the first drawback but it is just getting back to an EARSM model coupled with a rather standard two-equation model. Another possible track could be to use equations (20, 22) or preferably their counterpart for three-dimensional flows, to derive a second equation which makes the model more consistent with history effects captured by the underlying Reynolds stress model.

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