ASYMMETRIC TEMPORAL CROSS-CORRELATION FUNCTIONS IN TURBULENT SHEAR FLOWS

Bruno Eckhardt, Arne Jachens and Jörg Schumacher
Fachbereich Physik,
Philipps-Universität Marburg,
D-35032 Marburg, Germany
bruno.eckhardt@physik.uni-marburg.de

Karsten Knobloch and Hans H. Fernholz
Hermann-Föttinger-Institut für Strömungsforschung,
Technische Universität, D-10623 Berlin, Germany

ABSTRACT

Spatio-temporal cross-correlations between the turbulent velocity components in streamwise and wall-normal directions are investigated in an effort to identify effects due to non-normal amplification or lift up. The temporal cross-correlation between the downstream and normal velocity component shows a characteristic asymmetry related to the interplay between the streamwise streaks and pairs of streamwise vortices. Positive values of the asymmetry function $Q > 0$ implies that the shear flow is dominated by the no-normal lift-up of streamwise streaks by pairs of streamwise vortices. $Q < 0$ is found for regions in the shear flow where nonlinear convective effects dominate. This systematic variation of the asymmetry can be used to separate different layers in turbulent shear flows.

INTRODUCTION

The large scale flows associated with coherent structures are very effective in transporting momentum between the wall and freestream regions and thus can contribute significantly to frictional drag in turbulent flows (Robinson 1991). Methods for their detection (Jiménez & Pinelli 1999; Adrian 1991) and mechanisms for their generation and evolution have been investigated (Holmes, Lumley & Berkooz 1996; Pope 2000). A recurrent theme in many of these investigations is the interplay between downstream vortices, accounting for the normal transport, and spanwise modulations in the downstream velocity, so-called streaks. Their presence is easily detected in many flows, including plane Poiseuille flow (Hamilton, Kim & Waleffe 1995; Schoppa & Hussain 2002), pipe Poiseuille flows (Eggels *et al.* 1994; Hof *et al.* 2004), various homogeneous shear flows (Kida & Tanaka 1994; Schumacher & Eckhardt 2001) and in high-Reynolds number wall bounded turbulence (Kim, Kline & Reynolds 1971; Blackwelder & Kovasznay 1972). Studies of wall bounded flows by Hamilton *et al.* (1995) led to the development of a self-sustaining mechanism for the formation of coherent structures (Waleffe, 1997). The downstream vortices drive streaks, as mentioned before. When the streaks become sufficiently strong they create normal vortices through a linear instability. The vortices are then rotated in downstream direction to feed the downstream streaks and the process repeats. Various aspects of this dynamics have been studied and its significance for transitional flows has been confirmed repeatedly. For fully developed turbulent flows the significance is less clear, although rapid distortion theory and a turbulence model by Nazarenko *et al.* (2000) suggest that it could be relevant there as well. It is our aim here to analyze certain cross-correlation functions for evidence of the dynamical processes underlying this self-sustaining mechanism.

The flows we study all have a non-vanishing shear in the mean velocity profile, $S(y) = d\overline{U}(y)/dy \neq 0$, giving rise to a non-vanishing Reynolds shear stress. Specifically, the cross-correlation between the streamwise turbulent velocity component u and the wall-normal or shear component v does not vanish. Since this quantity describes transversal momentum transport, such one-point correlations are at the focus of many studies aimed at closures for the Reynolds equation (Pope 2000, 2002). Two-point measurements of the streamwise and wall-normal components in a turbulent boundary layer were first studied by Blackwelder and Kovasznay (1972) at $y/\delta \approx 0.2$. The boundary layer thickness δ is defined as the distance δ to the wall where $\overline{U} = 0.99 U_\infty$ and U_∞ is the free-stream velocity. Their measurements show an asymmetry of the cross-correlations under $\Delta t \to -\Delta t$ which they interpreted as a fingerprint of the ejection of retarded fluid outward from the wall. The non-normal amplification mechanism provides an intuitive explanation for such an asymmetry.

In the the next section we will summarize the stochastic model of (Eckhardt & Pandit 2003), followed by a definition of the spatio-temporal correlation functions, results from a direct numerical simulation of a nearly homogeneous shear flow and results from triple hot-wire anemometers in a wind tunnel.

A STOCHASTIC MODEL

The non-normal amplification mechanism is a linear mechanism for the evolution of perturbations in a shear flow, not unlike the evolution calculated within rapid distortion theory. In the simplest version (Eckhardt & Pandit 2003) it describes the driving of streamwise streaks by streamwise vortices: the vortices will decay in the absence of any further driving, but due to the normal mixing of fluid across the shear, u will be modulated in the spanwise direction.

Consider a linear shear profile, $\mathbf{U}_0 = Sy\mathbf{e}_x$. Coordinates are chosen with x pointing streamwise, z in the spanwise direction and y pointing in the direction of the shear. The Navier-Stokes equation for the fluid velocity $\mathbf{u}$ linearized around this flow is

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{U}_0 + (\mathbf{U}_0\cdot\nabla)\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u}\,, \qquad (1)$$

where p is the kinematic pressure and ν the kinematic viscosity of the fluid. To keep the analysis as simple as possible we work with Fourier modes appropriate for periodic boundary conditions in spanwise and downstream directions, and free-slip boundary conditions on two parallel planes in the wall-normal direction. The analysis of the linear problem with Kelvin modes shows that modulations in the downstream direction give rise to a time-dependent wave vector and faster-than-exponential damping. Farrell & Ioannou (1993) also show that the most important modes for non-normal amplification do not have a downstream variation. Therefore, we consider only perturbations with wave numbers $\mathbf{k} = (0, k_y, k_z)$, where k_z is continuous and $k_y = \pi n/d$, with n an integer and d the distance between the bounding planes.

In order to highlight the essentials of non-normal amplification, we now take a velocity field consisting of two modes, namely, a *spanwise streak*

$$\mathbf{u}_s = \begin{pmatrix} \beta\sin\alpha z\cos\beta y \\ 0 \\ 0 \end{pmatrix}, \qquad (2)$$

and a *downstream vortex*

$$\mathbf{u}_\omega = \begin{pmatrix} 0 \\ \alpha\cos\alpha z\sin\beta y \\ -\beta\sin\alpha z\cos\beta y \end{pmatrix}, \qquad (3)$$

with amplitudes $s(t)$ and $\omega(t)$, i.e.,

$$\mathbf{u} = s(t)\mathbf{u}_s + \omega(t)\mathbf{u}_\omega\,. \qquad (4)$$

In the linearized equation the pressure disappears as the velocity fields are divergence-free. The term $(\mathbf{U}_0\cdot\nabla)\mathbf{u}$ drops out and $(\mathbf{u}\cdot\nabla)\mathbf{U}_0$ results in a coupling between vortex and streak:

$$\begin{pmatrix} \dot{s} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} -\nu(\alpha^2+\beta^2) & S \\ 0 & -\nu(\alpha^2+\beta^2) \end{pmatrix} \begin{pmatrix} s \\ \omega \end{pmatrix}. \qquad (5)$$

The matrix on the right hand side is not symmetric because of the coupling of both modes through the term $(\mathbf{u}\cdot\nabla)\mathbf{U}_0 = S u_y \mathbf{e}_x$. The dynamics that follows from the non-normal system (5) has exponentially decaying vortices that drive spanwise streaks: if s_0 and ω_0 denote the initial amplitudes, then

$$\begin{aligned} s(t) &= (s_0 + S\omega_0 t)e^{-\nu(\alpha^2+\beta^2)t}; \\ \omega(t) &= \omega_0 e^{-\nu(\alpha^2+\beta^2)t}. \end{aligned} \qquad (6)$$

Clearly, even if there is no streak initially (i.e., $s_0 = 0$), there will be one as time progresses as a consequence of the mixing induced by the downstream vortex. Eventually, however, both will decay. The maximal amplitude of the streak follows from the maximum of $t\exp(-\nu(\alpha^2+\beta^2)t)$, which occurs at a time

$$t_{max} = \frac{1}{\nu(\alpha^2+\beta^2)}\,. \qquad (7)$$

Since the maximal amplitude of the downstream component of the streak $\mathbf{u}_s$ is α, the maximal modulation of the downstream velocity component follows to be

$$u_{s,max} = S\frac{\alpha}{\nu(\alpha^2+\beta^2)}\omega_0\,. \qquad (8)$$

A more complete analysis these mode interactions and the choice of optimal solutions is given by Chapman (2002).

Thus, with v as an indicator of the vortices and the strength of mixing, and u as one for the response, a temporal cross correlation $C_{vu}(\Delta t) = \langle u(t+\Delta t)v(t)\rangle_t$ will be asymmetric: the vortex has a chance to influence u for $\Delta t > 0$, but not for $\Delta t < 0$. In a linear model for vortex–streak interactions with two degrees of freedom driven by stochastic forces the correlations can be calculated analytically, and the asymmetry becomes transparent (Eckhardt & Pandit 2003). Quantitatively, for this kind of flow, the maximum is delayed by about one time unit and overshoots the value at zero by about 20%.

THE CORRELATION FUNCTION

Clearly, there are many differences between the linear model with stochastic forcing and fully turbulent dynamics, where nonlinear effects are omnipresent and where the forcing is self-consistently provided by the turbulent dynamics. We, therefore, turn to data from direct numerical simulations (DNS) of nearly homogeneous shear flows. The DNS allows for an extension to two-point cross-correlations in space *and* time since they do not have to rely on Taylor's frozen flow hypothesis.

The quantity we focus on is the correlation function between the turbulent wall-normal velocity component v and the turbulent streamwise component u, displaced in the downstream direction by Δx and in time by Δt,

$$C_{vu}(\Delta t, \Delta x; y) = \frac{\langle v(x,y,z,t)u(x+\Delta x, y, z, t+\Delta t)\rangle_{x,z,t}}{\langle v(x,y,z,t)u(x,y,z,t)\rangle_{x,z,t}}\,. \qquad (9)$$

The averages are over time and also over all points in an x-z-plane at fixed height y. Time correlations at one point are given by

$$\tilde{C}_{vu}(\Delta t; y) = C_{vu}(\Delta t, 0; y) = \frac{\langle v(x,y,z,t)u(x,y,z,t+\Delta t)\rangle_{x,z,t}}{\langle v(x,y,z,t)u(x,y,z,t)\rangle_{x,z,t}}\,. \qquad (10)$$

In order to quantify the asymmetry effects we introduce the following measure for the temporal cross-correlations:

$$Q_{vu}(\Delta t) = \frac{\tilde{C}_{vu}(\Delta t) - \tilde{C}_{vu}(-\Delta t)}{\tilde{C}_{vu}(\Delta t) + \tilde{C}_{vu}(-\Delta t)}\,, \qquad (11)$$

(the dependence on height has been suppressed in these expressions). For the extended correlations due to the non-normal amplification we expect $\tilde{C}_{vu}(\Delta t) > \tilde{C}_{vu}(-\Delta t) > 0$ and $\tilde{C}_{vu}(\Delta t) < \tilde{C}_{vu}(-\Delta t) < 0$, so that $Q_{vu} > 0$ for these cases.

DIRECT NUMERICAL SIMULATIONS

The direct numerical simulations of a turbulent shear flow also refer to a flow bounded by two parallel free-slip plates, driven by a volume force that sustains a linear shear flow in the mean, $\overline{U}(y) = Sy$, except for a small boundary layer near the plates. More details on the numerical procedure and on the statistical stationarity of the flow are given in Schumacher & Eckhardt (2000) and Schumacher (2004).

In Fig. 1a we show the space-time contours of the cross-correlations for a DNS with spatial resolution of $128\times65\times128$ grid points for a box with an aspect ratio of $2\pi : 1 : 2\pi$ and a Taylor microscale Reynolds number $R_\lambda = 79$ which is

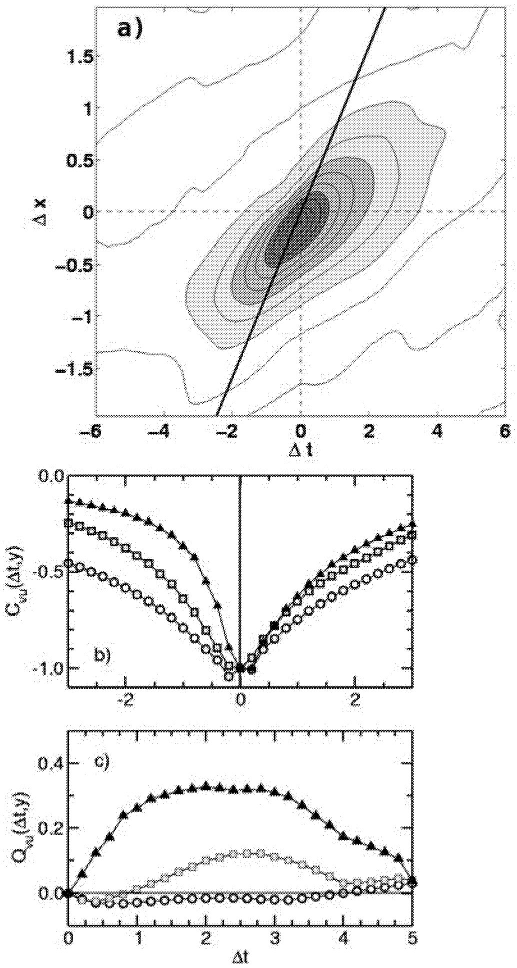

Figure 1: Space-time cross-correlation $C_{vu}(\Delta t, \Delta x; y)$ of a nearly homogeneous shear flow. Data are taken from the DNS at a height $y/L_y = 0.11$. The Reynolds number $Re = \overline{U}L_y/\nu = 1800$ where $\overline{U}$ is the mean turbulent velocity at the boundary. (a) Space-time plot of the cross-correlations. The contours increase in steps of 0.1 and the unit value is at the origin. The solid line represents the dimensionless mean turbulent velocity $\overline{U}(y/L_y = 0.11) = 0.77$. (b) Temporal cross-correlation along $\Delta x = 0$ for different heights: $\blacktriangle$: $y/L_y = 0.13$, $\blacksquare$: $y/L_y = 0.25$, $\circ$: $y/L_y = 0.50$. (c) Asymmetry coefficient for $\tilde{C}_{vu}(\Delta t; y)$ at the same heights as (b).

determined by $R_\lambda = \sqrt{15/(\nu\epsilon)}\,\langle u^2\rangle$. ϵ is the mean energy dissipation rate. The spatio-temporal correlation function shows an asymmetry with respect to time and the iso-countours are oval and not aligned with the coordinate axis. Thus, even though more spatial degrees of freedom are present, the Reynolds number is higher, and the fluctuation come from the fully developped turbulence, the non-normal amplification is reflected in the correlation function.

On closer look some differences begin to appear. In particular, the asymmetry measure (cf. Eq. (11)) in Fig. 2b shows that the correlation function for very short times has the opposite sign, $Q_{uv} < 0$. These negative values appear in almost all heights across the layer: the time interval over which they

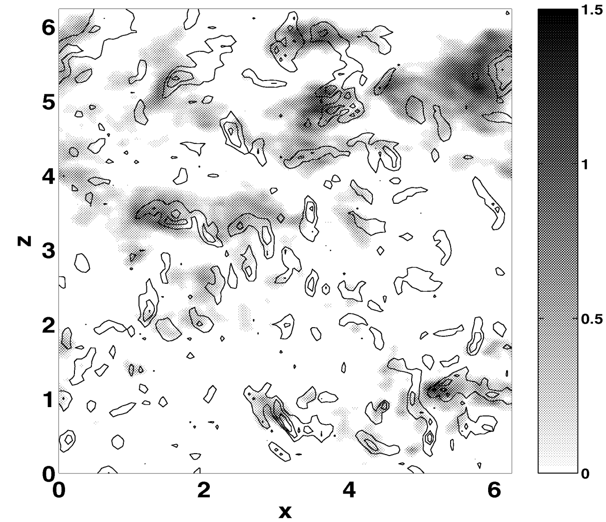

Figure 2: Snapshot of the two turbulent velocity fields entering the cross-correlation function. Data are a slice cut from a DNS at $R_\lambda = 166$. The turbulent downstream velocity u is indicated by shading for values between u between 0 and 1.5, only. The contourlines show three isolevels of the wall-normal component v at values of -0.8, -0.5 and -0.2. The maxima are shifted relative to each other by a small downstream distance that corresponds with the shift of the maximum of the space-time cross-correlation by Δx as observed in Fig. 1a.

are present is smallest close to the wall and increases as one approaches the center. Apparently, close to the center, many modes of different vertical wave length contribute and swamp the effects of the non-normal amplification. A related phenomenon will appear in boundary layer flows.

A spatial plot of the turbulent streamwise velocity component $u(x, y_0, z, t_0)$ and the turbulent wall-normal component $v(x, y_0, z, t_0)$ for one instant in the x-z plane (see Fig. 2) reveals that the contributions to the cross-correlation function come from fragmented regions, of an extension compatible with the dimensions of coherent structures. Negative contours of v indicate streamwise vortices which are lifting up the streamwise streaks which are shown as gray-filled contours of u. Note that the maxima of u and v contours are displaced slightly in accordance with the observation that the maximum of the space-time cross-correlation is not at the origin $\Delta x = \Delta t = 0$ in Fig. 1a.

This shift can be rationalized by the observation that a streamwise vortex pair centered at height y will be advected with the corresponding mean streamwise velocity at that height, namely $\overline{U}(y)$, The pair will mix slower moving fluid into a region that streams on average faster thus diminishing locally the advection velocity U_c to values below the mean velocity $\overline{U}(y)$ for an instant. But this is the velocity which will advect the streak that is about being lifted up and thus remaining behind the vortex pair. As a result a spatial shift of the most intense cross-correlation by $-\Delta x$ follows.

Both examples of correlation functions shown in Figs. 1 and 2 and many others for different aspect ratios and Reynolds numbers show an inclination of the isocountours in the spatio-temporal cross-correlations $C_{vu}(\Delta t, \Delta x; y)$. Since the two axes being compared have dimensions of time and length, the inclination has dimension of velocity: but as the comparison with the straight lines in both figures shows, this velocity tends to be smaller than the mean velocity at that height, $\overline{U}(y)$. The speed with which these structures are advected is systematically smaller than the mean advection. We can relate this effect to an asymmetry of the spatial correlation function of the streamwise velocity in the wall-normal direction, as measured by

$$C_{uu}(\Delta x, \Delta y, y_0) = \langle u(x, y_0, z, t) u(x + \Delta x, y_0 + \Delta y, z, t) \rangle_{x,z,t} \,. \quad (12)$$

We found that this correlation function is asymmetric as well and has different correlation length with respect to y taken from the level y_0 of the measurement. The autocorrelations are obviously influenced by the presence of the free-slip walls at $y = 0$ and $y = L$. If we estimate the mean advection speed of the coherent structures from an average of the downstream speed over a domain determined by the full width at half maximum of $C_{uu}(\Delta x, \Delta y, y_0)$, we find

$$U_c = \frac{1}{\ell_2 - \ell_1} \int_{\ell_1}^{\ell_2} \overline{U}(y) \, \mathrm{d}y \,. \quad (13)$$

ℓ_2 and ℓ_1 are the widths at half of the maximum of the asymmetric $C_{uu}(\Delta x = 0, \Delta y, y_0)$. The convection velocity as defined by (13) becomes smaller as the mean velocity and coincides with the inclination of the space-time contours of the velocity cross-correlations of Fig. 1. The coherent structures thus move with the downstream speed as determined by an average over their size.

TURBULENT BOUNDARY LAYER

The third example for which we determine cross-correlation functions are measurements obtained in a high-Reynolds number boundary layer flow. Velocity data were obtained with triple hot-wire anemometers, such that all three velocity components could be extracted. The data are from a wind tunnel of the Hermann-Föttinger Institute (HFI) in Berlin at $U_\infty = 10m/s$ and the German-Dutch Windtunnel (DNW) at $U_\infty = 80m/s$. The set-up and data aquisition are described in more detail in Knobloch & Fernholz (2004). Measurements at HFI were done with a sampling rate of 20 kHz, at the DNW with 125 kHz. Typical data sets contained about a million data points at HFI and about 5 million data points at DNW. The boundary layer width was found to be $\delta = 63mm$ for HFI and $\delta = 240mm$ for DNW. Taylor microscale Reynolds numbers up to 1614 were reached in the DNW device.

In contrast to the numerical studies in the previous section, we have velocity time traces for a single spatial location only. Normally, one would invoke Taylors hypothesis to relate time delays to spatial separations and hence time correlations with spatial correlations. But as the results above show, it is possible to obtain the information about lift-up from cross-correlations of time series from a single point, if the times are short compared to the decorrelation times.

The asymmetry measure for six data sets (three from HFI and three from DNW) are shown in Fig. 3. The two data sets allow to compare different distances from the wall and different Reynolds numbers. For a fixed Reynolds number the cross correlation function closest to the wall has a region with negative values of the asymmetry measure $Q_{vu}(\Delta t)$, followed by a region of positive values for intermediate times (until the correlation functions becomes so small that estimates of the asymmetry become unreliable). As one moves away from the wall, the region with negative values increases and the values become more negative. For the points furthest away from the wall no reversal to positive values is detected. The trends in the behaviour are similar to that observed in the direct numerical simulations with its free slip boundary conditions, suggesting that this effect is related to the shear and viscous boundary layers.

The asymmetry measures for the two data sets from FHI and DNW are calculated at comparable heights when measured in units of the boundary layer thickness, y/δ. They show simliar kind of behaviour, suggesting that a non-normal liftup related asymmetry in the cross-correlation function can be expected for positions $y/\delta \lesssim 0.05$. No such comparison is possible when the height is expressed in wall units. For instance, the measurement closest to the walls has height $y^+ \sim 34$ in the case of HFI data, but $y^+ \sim 709$ in the case of the DNW data. Thus while numerous studies suggest that streamwise streaks and pairs of streamwise vortices can be observed below $y^+ \lesssim 100$ (Jiménez, J. & Pinelli 1999, Pope 2000), the cross correlation function measurements suggest that some remnant of them may be active at even higher y^+, and that the relevant length scale are not wall units but boundary layer thickness. Both observations can be reconciled once the intermittent bursting activity arising from the coherent structures in the viscous sublayer is taken into account: it is known (see e.g. Blackwelder & Kovaszany, 1972) that while the streamwise vortices and streaks are present only close to the wall, the ejection of vorticity from the walls can reach further up and affect the correlation functions. For this process the relative position within the boundary layer is important, hence the scaling with δ.

SUMMARY

The comparitive study of two shear flows presented here shows that the temporal cross correlation between the downstream and normal velocities carries information about the dynamical processes responsible for this component of the Reynolds stress tensor. Closest to the wall the Reynolds stress tensor shows a clear asymmetry in time. In boundary layers this asymmetry is controlled by the height relative to the boundary layer thickness and not by that relative to the wall-normal units. This then suggests that at heights of several hundreds of wall units, where the correlation functions still has the proper asymmetry, it detects vorticity ejected from the near wall layer, rather than the vortices and streaks themselves.

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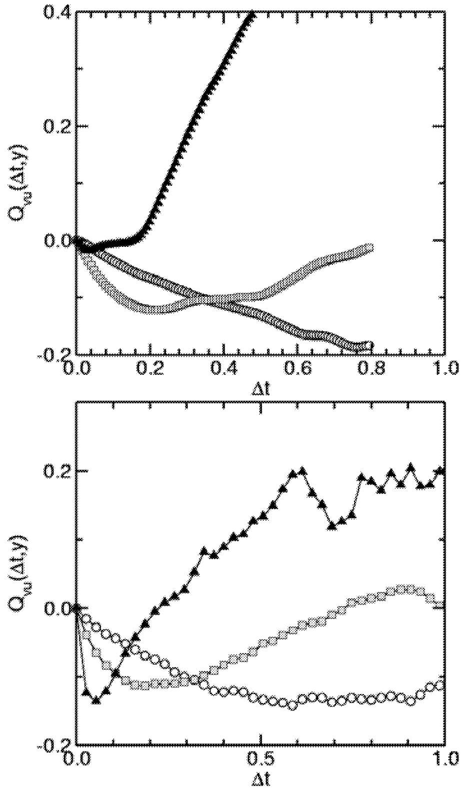

Figure 3: Asymmetry coefficient $Q_{vu}(\Delta t)$ for two sets of turbulent boundary layer data. Time is given in units of δ/U_∞ for both figures. Panel (a): HFI measurement at $Re_\delta = 41600$ for three different heights $\tilde{y} = y/\delta$ above the wall, black triangles: $\tilde{y} = 0.02$, gray filled squares: $\tilde{y} = 0.11$ and open circles: $\tilde{y} = 0.31$. Panel (b): DNW measurement at Reynolds number $Re_\delta = 1237900$. Here black triangles: $\tilde{y} = 0.02$, gray filled squares: $\tilde{y} = 0.11$ and open circles: $\tilde{y} = 0.34$.

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