NONLOCAL ANALYSIS OF MOMENTUM TRANSPORT IN TURBULENT CHANNEL FLOW

Fujihiro Hamba
Institute of Industrial Science,
University of Tokyo
Komaba, Meguro-ku, Tokyo 153-8505, Japan
hamba@iis.u-tokyo.ac.jp

ABSTRACT

An exact expression for the Reynolds stress was derived using the Green's function for the velocity fluctuation. The nonlocal eddy viscosity involved in the expression represents a contribution to the Reynolds stress from the mean velocity gradient at remote points in space and time. A direct numerical simulation of channel flow was carried out to validate the nonlocal expression. The transport equations for the velocity and the Green's function were calculated to evaluate the nonlocal eddy viscosity; it was shown that the nonlocal expression is accurate for both the normal and shear stresses. A local expression for the shear stress was also evaluated to show that the local approximation is not accurate enough near the wall. The nonlocal eddy viscosity for rotating channel has wider profiles than that for non-rotating channel. The analysis by the nonlocal expression was shown to be useful for a better understanding of turbulent shear flow.

INTRODUCTION

The eddy viscosity and diffusivity approximations are widely used to predict the mean velocity and scalar fields in turbulent flows, respectively. In the eddy viscosity model the deviatoric part of the Reynolds stress at a point is assumed to be proportional to the strain rate of the mean velocity at the same point. This local approximation is not always valid for actual turbulent flows. Its limitation was pointed out; a gradient transport model requires that the characteristic scale of the transport mechanism be small compared with the distance over which the mean gradient of the transported property changes appreciably (Corrsin 1974). In turbulent flows the length scale of turbulence is often as large as that of the mean field variation. One of typical examples is the scalar transport in the atmospheric boundary layer; convective eddies driven by buoyancy are as large as the boundary layer height, and the eddy diffusivity model is not always accurate. To develop nonlocal models for the scalar transport, Stull (1984) proposed the transilient turbulence theory that describes the nonlocal transport using a matrix of mixing coefficients. Berkowicz and Prahm (1980) generalized the eddy diffusivity; that is, the scalar flux is expressed as a spatial integral of the scalar gradient. In contrast to the eddy diffusivity model, few nonlocal models were examined for the eddy viscosity. As one of few examples, Nakayama and Vengadesan (1993) proposed a nonlocal eddy viscosity model in engineering problems.

In addition to the application of nonlocal models, the nonlocal expression was also investigated theoretically. Using the Green's function, Kraichnan (1987) derived exact nonlocal expressions for the Reynolds stress and the scalar flux. However, his expressions involve the Reynolds stress or the scalar flux also on the right-hand side; they need to be solved iteratively. Hamba (1995,2004) modified the Green's function to obtain an explicit exact expression for the scalar flux; the Green's function was calculated in the large eddy simulation of the atmospheric boundary layer and the direct numerical simulation (DNS) of channel flow to evaluate the nonlocal eddy diffusivity.

In this work we extend this nonlocal analysis to the Reynolds stress to investigate the nonlocal properties of the momentum transport. An exact explicit expression for the Reynolds stress is derived using the modified Green's function for the velocity fluctuation. As a basic example of turbulent shear flow we carry out a DNS of channel flow without and with system rotation. The transport equation for the Green's function is calculated to evaluate the nonlocal eddy viscosity. We examine nonlocal properties of the Reynolds stress in turbulent channel flow.

FORMULATION

In order to solve the mean velocity equation it is necessary to model the Reynolds stress. In the nonlinear eddy-viscosity model the Reynolds stress is approximated by

$$\langle u_i' u_j' \rangle - \frac{1}{3}\langle u_k' u_k' \rangle \delta_{ij} = -\nu_T \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \nu_T^{(2)} \left(\frac{\partial U_i}{\partial x_k}\frac{\partial U_j}{\partial x_k} - \frac{1}{3}\delta_{ij}\frac{\partial U_k}{\partial x_m}\frac{\partial U_k}{\partial x_m} \right) + \cdots \quad (1)$$

Here, $\langle \ \rangle$ denotes ensemble averaging, $U_i (\equiv \langle u_i \rangle)$ is the mean velocity, u_i' is the velocity fluctuation, and δ_{ij} is the Kronecker delta symbol; the summation convention is used for repeated indices. The first term involving ν_T on the right-hand side is the eddy viscosity representation; this approximation has been widely used. The second term involving $\nu_T^{(2)}$ is one of the nonlinear eddy-viscosity terms introduced to improve the eddy viscosity model. The above expression can be rewritten as

$$\langle u_i'u_j'\rangle - \frac{1}{3}\langle u_k'u_k'\rangle\delta_{ij} = -\nu_{Tijkm}\frac{\partial U_k}{\partial x_m} \quad (2)$$

where ν_{Tijkm} is the anisotropic eddy-viscosity tensor that can explicitly depend on the mean velocity gradient. This eddy viscosity model is local in space in the sense that the Reynolds stress at a point is expressed in terms of physical quantities at the same point. This local approximation is valid only if the turbulence length scale is much less than the length scale of the mean velocity variation. However, this condition is not always satisfied in actual turbulent flows.

For the scalar transport Hamba (1995) introduced a modified Green's function for the scalar fluctuation to derive an explicit exact expression for the scalar flux. In the present work, we apply this formalism to the Reynolds stress to derive its explicit nonlocal expression. We consider the equations for the velocity fluctuation

$$\frac{\partial u_i'}{\partial t} + \frac{\partial}{\partial x_j}(u_j u_i' - \langle u_j u_i'\rangle) + \frac{\partial p'}{\partial x_i} - \nu\frac{\partial^2 u_i'}{\partial x_j \partial x_j} = -u_j'\frac{\partial U_i}{\partial x_j} \quad (3)$$

$$\frac{\partial u_i'}{\partial x_i} = 0 \quad (4)$$

We introduce a modified Green's function $g_{Mij}(\mathbf{x},t;\mathbf{x}',t')$; its equations are given by

$$\frac{\partial g_{Mij}}{\partial t} + \frac{\partial}{\partial x_k}(u_k g_{Mij} - \langle u_k g_{Mij}\rangle) + \frac{\partial p_{Gj}}{\partial x_i} - \nu\frac{\partial^2 g_{Mij}}{\partial x_k \partial x_k} = \delta_{ij}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t') \quad (5)$$

$$\frac{\partial g_{Mij}}{\partial x_i} = 0 \quad (6)$$

where p_{Gj} is a vector that plays a similar role to the pressure and guarantees the solenoidal condition (6) for g_{Mij}. Using this Green's function a formal solution for the velocity fluctuation can be written as

$$u_i'(\mathbf{x},t) = -\int d\mathbf{x}'\int_0^t dt' g_{Mij}(\mathbf{x},t;\mathbf{x}',t')u_k'(\mathbf{x}',t')\frac{\partial}{\partial x_k'}U_j(\mathbf{x}',t') \quad (7)$$

Here, a term involving the initial value $u_i'(\mathbf{x},0)$ is omitted because it does not contribute to the Reynolds stress for sufficiently large t. This solution leads to the nonlocal expression for the Reynolds stress given by

$$\langle u_i'u_j'\rangle(\mathbf{x},t) = -\int d\mathbf{x}'\int_0^t dt'\nu_{NLijkm}(\mathbf{x},t;\mathbf{x}',t')\frac{\partial}{\partial x_m'}U_k(\mathbf{x}',t') \quad (8)$$

where

$$\nu_{NLijkm}(\mathbf{x},t;\mathbf{x}',t') = (\langle u_i'(\mathbf{x},t)g_{Mjk}(\mathbf{x},t;\mathbf{x}',t')u_m'(\mathbf{x}',t')\rangle + \langle u_j'(\mathbf{x},t)g_{Mik}(\mathbf{x},t;\mathbf{x}',t')u_m'(\mathbf{x}',t')\rangle)/2 \quad (9)$$

Therefore, the Reynolds stress can be expressed as a space and time integral of the mean velocity gradient.

The nonlocal eddy viscosity $\nu_{NLijkm}(\mathbf{x},t;\mathbf{x}',t')$ involved in (8) represents the contribution to the Reynolds stress at $(\mathbf{x},t)$ from the mean velocity gradient at $(\mathbf{x}',t')$. It is expected to have a nonzero value if the distance $|\mathbf{x}-\mathbf{x}'|$ and the time difference $t-t'$ are compatible with or less than the turbulence length and time scales, respectively. If in this region in space and time the mean velocity gradient $\partial U_k/\partial x_m$ is nearly constant, the Reynolds stress can be approximated by

$$\langle u_i'u_j'\rangle \cong -\nu_{Lijkm}\frac{\partial U_k}{\partial x_m} \quad (10)$$

where ν_{Lijkm} is the local eddy viscosity given by

$$\nu_{Lijkm}(\mathbf{x},t) = \int d\mathbf{x}'\int_0^t dt'\nu_{NLijkm}(\mathbf{x},t;\mathbf{x}',t') \quad (11)$$

Therefore, whether the local approximation is good or not depends on the relation in the length and time scales between profiles of ν_{Lijkm} and $\partial U_k/\partial x_m$.

In the next section we will evaluate the nonlocal eddy viscosity by carrying out a DNS of channel flow. It takes very much computing cost to calculate g_{Mij} straightforwardly because it depends on $\mathbf{x}'$ and t'. In this work, making use of the streamwise and spanwise homogeneities and the stationarity of channel flow, we evaluate the nonlocal eddy viscosity as follows. Since $U(\mathbf{x}',t')$ depends only on y' in channel flow, the integral with respect to x', z', and t' applies only to $g_{Mij}u_k'$ in (7). We define another Green's function as

$$g_{i12}(\mathbf{x},t;y') = \int dx'\int dz'\int_0^t dt' g_{Mi1}(\mathbf{x},t;\mathbf{x}',t')u_2'(\mathbf{x}',t') \quad (12)$$

satisfying

$$\frac{\partial g_{i12}}{\partial t} + \frac{\partial}{\partial x_k}(u_k g_{i12} - \langle u_k g_{i12}\rangle) + \frac{\partial p_{G12}}{\partial x_i} - \nu\frac{\partial^2 g_{i12}}{\partial x_k \partial x_k} = \delta_{i1}u_2'\delta(y-y') \quad (13)$$

$$\frac{\partial g_{i12}}{\partial x_i} = 0 \quad (14)$$

where indices $j=1$ and 2 correspond to the streamwise and wall-normal components, respectively. Using this Green's function the Reynolds stress given by Eq. (8) can be rewritten as

$$\langle u_i'u_j'\rangle(y) = -\int dy'\nu_{NLij12}(y;y')\frac{\partial U(y')}{\partial y'} (\equiv \langle u_i'u_j'\rangle_{NL}) \quad (15)$$

where

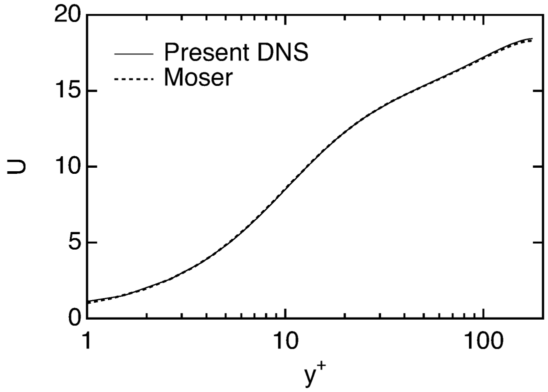

Figure 1. Mean velocity profiles.

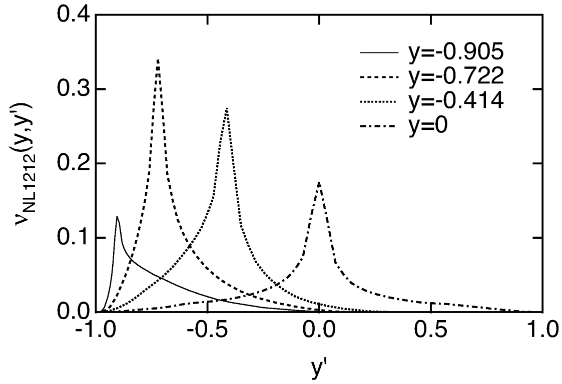

Figure 2. Profiles of nonlocal eddy viscosity ν_{NL1212} .

$$\nu_{NLij12}(y;y') = (\langle u_i'(\mathbf{x},t)g_{j12}(\mathbf{x},t;y')\rangle + \langle u_j'(\mathbf{x},t)g_{i12}(\mathbf{x},t;y')\rangle)/2 \quad (16)$$

For channel flow the nonlocal expressions (8) and (9) are equivalent to Eqs. (15) and (16). Since the Green's function $g_{i12}(\mathbf{x},t;y')$ depends only on y' as the source point, it is not very hard to calculate its time development. Like Eq. (10) the local approximation can be written as

$$\langle u_i'u_j'\rangle(y) \approx -\nu_{Lij12}(y)\frac{\partial U}{\partial y}(\equiv \langle u_i'u_j'\rangle_L) \quad (17)$$

where $\nu_{Lij12}(y)$ is the local eddy viscosity given by

$$\nu_{Lij12}(y) = \int dy'\nu_{NLij12}(y;y') \quad (18)$$

Comparison of Eqs. (17) and (18) with Eq. (15) shows that if $\partial U/\partial y'$ is nearly constant in the region where $\nu_{NLij12}(y;y) \neq 0$, the local approximation (17) should be good.

RESULTS AND DISCUSSION

In this section we show the result of a DNS of channel flow. Using DNS data we examine whether the nonlocal expression (15) is exact. In the DNS we numerically solve the equations for the velocity given by

$$\frac{\partial u_i}{\partial t} = -\frac{\partial}{\partial x_j}u_j u_i - \frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \delta_{i1} \quad (19)$$

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (20)$$

Equations (13) and (14) are also solved for the Green's function G_{i12} . The variables $x_1(=x)$, $x_2(=y)$, and $x_3(=z)$ denote the coordinates in the streamwise, wall-normal, and spanwise directions, respectively; corresponding velocity components are given by $u_1(=u)$, $u_2(=v)$, and $u_3(=w)$. Hereafter, all quantities are nondimensionalized by the wall-friction velocity u_τ and the channel half width L_y unless otherwise mentioned. The Reynolds number based on u_τ and $L_y/2$ is set to $Re_\tau = 180$. The size of the computational domain is $L_x \times L_y \times L_z = 6.4 \times 2 \times 3.2$. A staggered grid is adopted; it is uniform in the x and z directions and is stretched in the y direction using a hyperbolic tangent function. The number of grid points is $N_x \times N_y \times N_z = 128 \times 96 \times 128$. The periodic boundary conditions for u_i and g_{i12} are used in the x and z directions. No-slip conditions $u_i = g_{i12} = 0$ are imposed at the walls ($y = \pm 1$). We use the second-order finite-difference scheme in space and the Adams-Bashforth method for time marching. The computational time step is $\Delta t = 5 \times 10^{-4}$. The computation was run for a sufficiently long time to be statistically independent of the initial condition; then statistics such as the Reynolds stress were accumulated over a time period of 10. Since the nonlocal expression is given by a weighted integral of the mean velocity gradient, the mean velocity must be calculated accurately. Figure 1 shows the mean velocity profile as a function of $y^+(= yu_\tau/\nu)$. The result of the DNS by Moser et al. (1999) is also shown. The agreement between the two DNS is good.

First, we examine the Reynolds shear stress $\langle u'v'\rangle$. The nonlocal expression for the shear stress can be expressed as

$$\langle u'v'\rangle_{NL}(y) = -\int dy'\nu_{NL1212}(y;y')\frac{\partial U(y')}{\partial y'} \quad (21)$$

Figure 2 shows the profiles of the nonlocal eddy viscosity ν_{NL1212} as a function of y' for four locations of y. Each profile represents a contribution from the mean velocity gradient at y' to the Reynolds stress at a given point of y. The peak of each curve is located at $y' = y$; this means that the contribution from the mean velocity gradient at the same point is the largest. The profiles of ν_{NL1212} are fairly wide; the width between the two points where its value is half the maximum is about 0.2. In particular, the profile in the case of $y = -0.905$ is asymmetric and the contribution from the region of $y' > y$ is large.

Figure 3 shows the profiles of the Reynolds shear stress $\langle u'v'\rangle$. Solid line denotes the value obtained directly by averaging $u'v'$ whereas dotted line denotes $\langle u'v'\rangle_{NL}$ defined as Eq. (21). The two profiles and that of the DNS by Moser et al. (1999) agree well with each other. This agreement shows that the nonlocal expression (21) is accurate and the nonlocal eddy viscosity is appropriately evaluated from the DNS data. Dot-dashed line in Fig. 3

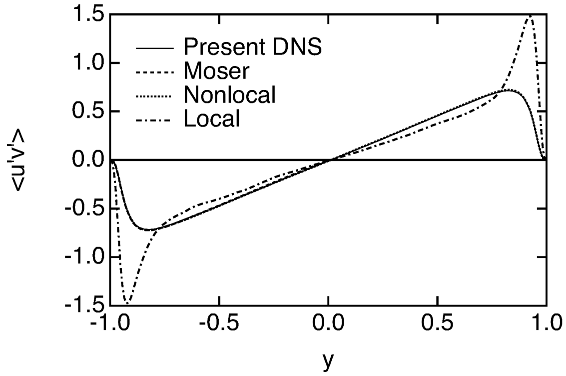

Figure 3. Profiles of turbulent shear stress $\langle u'v' \rangle$.

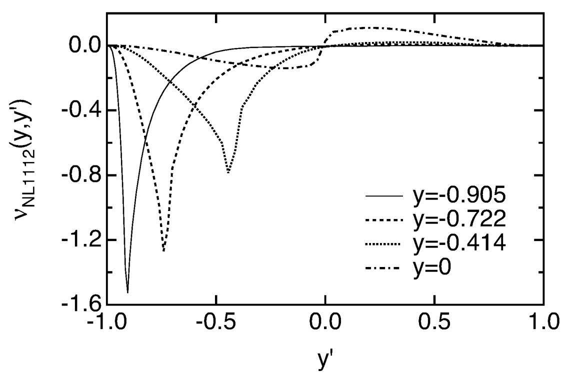

Figure 5. Profiles of nonlocal eddy viscosity ν_{NL1112}.

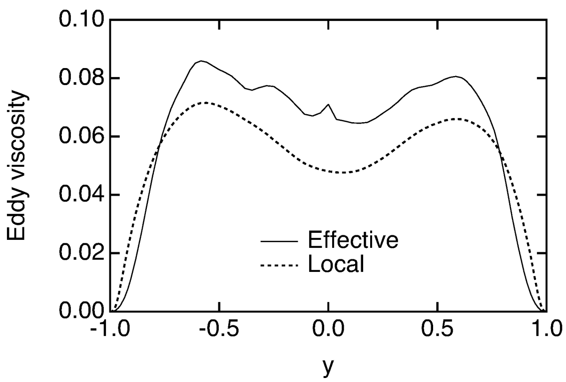

Figure 4. Profiles of effective eddy viscosity ν_{E1212} and local eddy viscosity ν_{L1212}.

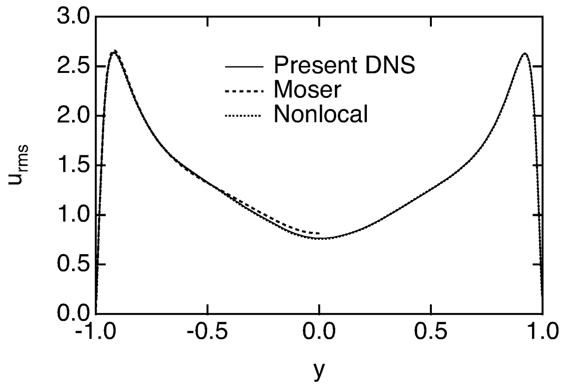

Figure 6. Profiles of rms of streamwise velocity, $\sqrt{\langle u'^2 \rangle}$.

denotes the value obtained from the local approximation

$$\langle u'v' \rangle_L(y) = -\nu_{L1212}(y)\frac{\partial U}{\partial y} \tag{22}$$

Compared with the DNS data the absolute value of $\langle u'v' \rangle_L$ is slightly small in the center region at $-0.8 < y < 0.8$ whereas it is too large near the wall at $y < -0.8$ and at $y > 0.8$. This large value is caused by the incompleteness of the local approximation; that is, the length scale of $\partial U / \partial y$ is small near the wall whereas the length scale of ν_{NL1212} is fairly large even near the wall as shown in Fig. 2. The difference between the local approximation and the exact value is clearly seen by comparing the local eddy viscosity with the effective eddy viscosity defined as

$$\nu_{E1212} = -\langle u'v' \rangle \Big/ \frac{\partial U}{\partial y} \tag{23}$$

If the local approximation is good, ν_{L1212} should be equal to ν_{E1212}. In Fig. 4 the value of ν_{L1212} is 70% of ν_{E1212} at $y = 0$ where it is about twice at $y = \pm 0.9$. The ratio between the two eddy viscosities is greater near the wall.

Next, we examine the normal stresses $\langle u'^2 \rangle$ and $\langle v'^2 \rangle$. The nonlocal expression for $\langle u'^2 \rangle$ is written as

$$\langle u'^2 \rangle_{NL} = -\int dy' \nu_{NL1112}(y;y')\frac{\partial U(y')}{\partial y'} \tag{24}$$

Figure 5 shows the profiles of the nonlocal eddy viscosity ν_{NL1112} as a function of y' for four locations of y. The value is negative at $-1 < y' < 0$ and positive at $0 < y' < 1$. This is because $\partial U / \partial y'$ is positive at $-1 < y' < 0$ and negative at $0 < y' < 1$ and the resulting contribution from each region to $\langle u'^2 \rangle$ in Eq. (24) should be positive. We should note that this component ν_{NL1112} is not the isotropic eddy viscosity ν_T in Eq. (1); it may correspond to $\nu_T^{(2)} \partial U_1 / \partial x_2$ in the nonlinear eddy-viscosity term in Eq. (1). The nonlinear terms are necessary to describe the anisotropy of the Reynolds stress; that is, $\langle u'^2 \rangle$ is greater than $\langle v'^2 \rangle$ and $\langle w'^2 \rangle$. A slight anisotropy $\langle u'^2 \rangle > \langle v'^2 \rangle$ remains in the channel center at $y = 0$. However, the local approximation cannot explain the anisotropy because $\partial U / \partial y = 0$ at $y = 0$. The anisotropy is caused by the nonlocal effect; as shown in $\nu_{NL1112}(y, y')$ for $y = 0$ in Fig. 5, the mean velocity gradient in the wide region of y' affects the normal stress at the center. Figure 6 shows the profiles of the root-mean-square (rms) of the streamwise velocity fluctuation, $\sqrt{\langle u'^2 \rangle}$. The profile of the nonlocal expression agrees well with the value obtained directly. This result shows that the nonlocal expression is accurate also for $\langle u'^2 \rangle$.

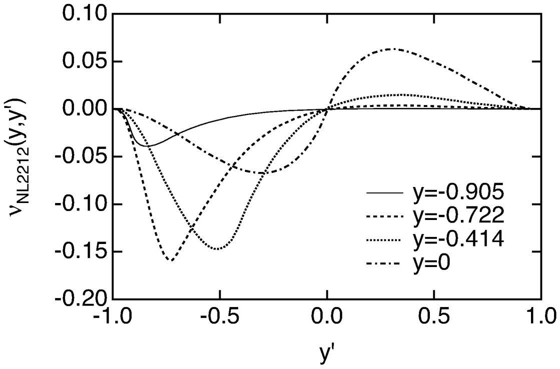

Figure 7. Profiles of nonlocal eddy viscosity ν_{NL2212} .

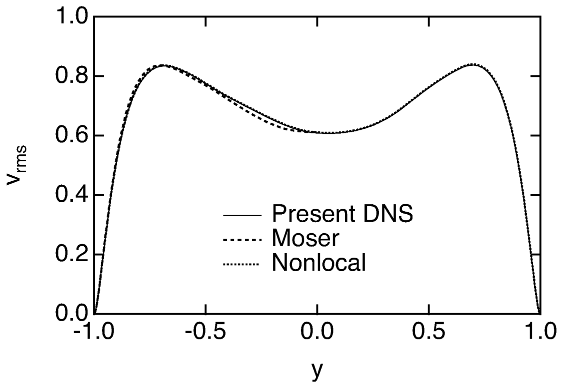

Figure 8. Profiles of rms of wall-normal velocity, $\sqrt{\langle v'^2 \rangle}$.

The nonlocal expression implies that the mean velocity gradient creates the Reynolds stress. This mechanism can be understood by considering the transport equation for the Reynolds stress. The equation for $\langle u'^2 \rangle$ is written as

$$\frac{\partial \langle u'^2 \rangle}{\partial t} = -2\langle u'v' \rangle \frac{\partial U}{\partial y} + 2\left\langle p' \frac{\partial u'}{\partial x} \right\rangle - 2\nu \left\langle \frac{\partial u'}{\partial x_k} \frac{\partial u'}{\partial x_k} \right\rangle - \frac{\partial}{\partial y}\left(\langle u'^2 v' \rangle - \nu \frac{\partial \langle u'^2 \rangle}{\partial y} \right) \quad (25)$$

The component $\langle u'^2 \rangle$ is created directly by the production term, the first term on the right hand side. If the pressure-strain term (the second term) and the diffusion term (the fourth term) vanish and the created turbulent energy is locally dissipated due to viscosity, the component may be proportional to the local mean velocity gradient involved in the production term. However, in actual turbulent flows, due to the diffusion term the turbulent energy is transferred from one point to another in space; the pressure-strain term can also have a global effect because the pressure is determined by the Poisson equation. The profiles of ν_{NL1112} in Fig. 5 reflect these nonlocal effects.

On the other hand, the nonlocal expression for the wall-normal component $\langle v'^2 \rangle$ is written as

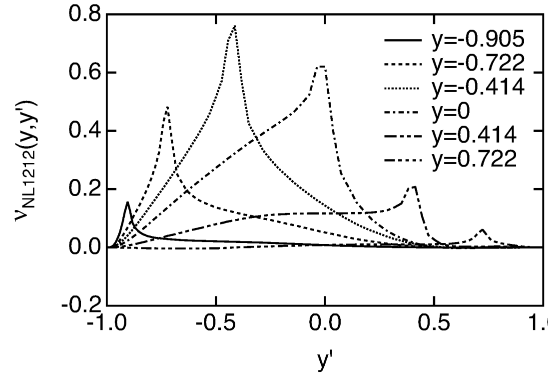

Figure 9. Profiles of nonlocal eddy viscosity ν_{NL1212} for rotating channel flow.

$$\langle v'^2 \rangle_{NL} = -\int dy' \nu_{NL2212}(y;y') \frac{\partial U(y')}{\partial y'} \quad (26)$$

Figure 7 shows the profiles of the nonlocal eddy viscosity ν_{NL2212} as a function of y' for four locations of y. The value is negative at $-1 < y' < 0$ and positive at $0 < y' < 1$ like ν_{NL1112} shown in Fig. 5. The profiles of ν_{NL2212} are wider than those of ν_{NL1112} ; the contribution to $\langle v'^2 \rangle$ is more nonlocal than that to $\langle u'^2 \rangle$. Figure 8 shows the profiles of the rms of the wall-normal velocity fluctuation, $\sqrt{\langle v'^2 \rangle}$. The profile of the nonlocal expression agrees well with the value obtained directly. The reason for the wider profiles of ν_{NL2212} can be understood by considering the transport equation for $\langle v'^2 \rangle$:

$$\frac{\partial \langle v'^2 \rangle}{\partial t} = 2\left\langle p' \frac{\partial v'}{\partial y} \right\rangle - 2\nu \left\langle \frac{\partial v'}{\partial x_k} \frac{\partial v'}{\partial x_k} \right\rangle - \frac{\partial}{\partial y}\left(\langle v'^3 \rangle + 2\langle p'v' \rangle - \nu \frac{\partial \langle v'^2 \rangle}{\partial y} \right) \quad (27)$$

There is no production term in the $\langle v'^2 \rangle$ equation. Instead the component $\langle u'^2 \rangle$ is first produced and then is transferred to $\langle v'^2 \rangle$ through the pressure-strain term. The pressure reflects the global distribution of the velocity field. Therefore, the indirect effect of the mean velocity gradient on $\langle v'^2 \rangle$ through the pressure-strain term is more nonlocal compared with the direct effect of the production term on $\langle u'^2 \rangle$.

We also carried out a DNS of channel flow at $Re_\tau = 150$ with spanwise rotation; the rotation number is $Ro_\tau (= \Omega_0 L_y / u_\tau) = 2.5$ where Ω_0 is the angular velocity of the system rotation. In this case the Coriolis force $-2\boldsymbol{\Omega}_0 \times \mathbf{u}$ is added to the velocity equation (19) and $-2\boldsymbol{\Omega}_0 \times \mathbf{u}'$ to the Green's function equation (13). The nonlocal expression for the Reynolds stress is modified as

$$\langle u_i' u_j' \rangle(y) = -\int dy' \nu_{NLij12}(y;y') \left(\frac{\partial U(y')}{\partial y'} - 2\Omega_0 \right) - \int dy' \nu_{NLij21}(y;y') 2\Omega_0 \quad (28)$$

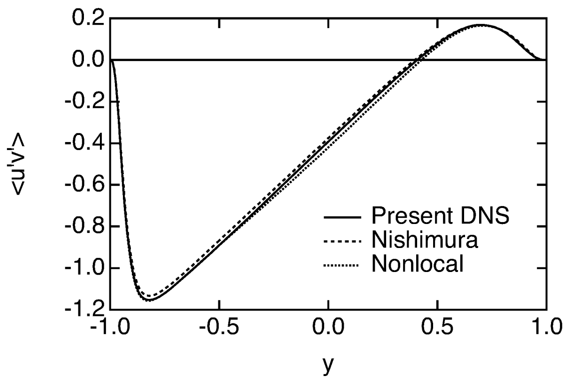

Figure 10. Profiles of turbulent shear stress $\langle u'v' \rangle$ for rotating channel flow.

Figure 9 shows the profiles of the nonlocal eddy viscosity $\nu_{NL1212}(y;y')$ as a function of y'. Comparing the profiles with those for non-rotating channel in Fig. 2 we can see that the nonlocal eddy viscosity has broader profiles. This large length scale corresponds to the fact that large Taylor-Görtler-like streamwise vortices were identified in previous works of rotating channel flows.

Figure 10 shows the profiles of the Reynolds shear stress $\langle u'v' \rangle$. Its profile is asymmetric with respect to the center line. Solid line denotes the value obtained directly and dotted line denotes $\langle u'v' \rangle_{NL}$ given by (28). Dashed line is the result of the DNS by Nishimura and Kasagi (1996). The three profiles agree well with each other. The nonlocal expression was shown to be also accurate for the rotating channel flow.

CONCLUSIONS

A nonlocal expression for the Reynolds stress was derived using the Green's function for the velocity fluctuation. The nonlocal eddy viscosity involved in the expression represents a contribution to the Reynolds stress from the mean velocity gradient at remote points in space and time. A DNS of channel flow was carried out to validate the nonlocal expression; it was shown to be accurate for both the shear and normal stresses. The profile of the nonlocal eddy viscosity for the variance of the wall-normal velocity component is wider than that for the streamwise velocity component. This difference in profile comes from the fact that the wall-normal velocity variance is produced through the pressure-strain correlation where the pressure fluctuation is determined globally. A local expression for the Reynolds shear stress was also examined; the stress is overestimated near the wall. This overestimate is because the turbulence length scale near the wall is not short enough compared with the length scale of the mean velocity variation. The nonlocal eddy viscosity for rotating channel shows wider profiles than that for non-rotating channel.

The nonlocal expression is useful for a better understanding of turbulent shear flow. It can be used to investigate reasons for some defects of local turbulence models. This analysis should be applied to turbulent flows other than channel flow. Modeling the nonlocal eddy viscosity itself remains as future work.

REFERENCES

Berkowicz, R., and Prahm, L. P., 1980, "On the spectral turbulent diffusivity theory for homogeneous turbulence," *J. Fluid Mech.*, Vol. 100, pp. 433-448.

Corrsin, S., 1974, "Limitations of gradient transport models in random walks and in turbulence," *Advances in Geophysics*, Vol. 18A, pp. 25-60.

Hamba, F., 1995, "An analysis of nonlocal scalar transport in the convective boundary layer using the Green's function," *J. Atmos. Sci.*, Vol. 52, pp. 1084-1095.

Hamba, F., 2004, "Nonlocal expression for scalar flux in turbulent shear flow," *Phys. Fluids*, Vol. 16, pp. 1493-1508.

Kraichnan, R. H., 1987, "Eddy viscosity and diffusivity: exact formulas and approximations," *Complex Systems*, Vol. 1, pp. 805-820.

Moser, R. D., Kim, J., and Mansour, N. N., 1999, "Direct numerical simulation of turbulent channel flow up to $Re_\tau = 590$," *Phys. Fluids*, Vol. 11, pp. 943-945.

Nakayama, A. and Vengadesan, S., 1993, "A non-local turbulent transport model," *Proc. Ninth Symp. on Turbulent Shear Flows*, Kyoto, p. 26-4-1.

Nishimura, M., and Kasagi, N., 1996, "Direct numerical simulation of combined forced and natural turbulent convection in a rotating plane channel," *Proc. Third KSME-JSME Therm. Eng. Conf.*, Kyongju, Korea, Vol. 3, pp. 77-82.

Stull, R. B., 1984, "Transilient turbulence theory. Part I: The concept of eddy mixing across finite distances," *J. Atmos. Sci.*, Vol. 41, pp. 3351-3367.