NEAR-WALL TURBULENT PRESSURE DIFFUSION MODELLING AND INFLUENCE IN 3-D SECONDARY FLOWS

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ABSTRACT

The purpose of this paper is to develop a near-wall turbulent pressure diffusion model for 3-D complex flows, and to evaluate the influence of the turbulent diffusion term on the prediction of detached and secondary flows. A complete turbulent diffusion model including a near-wall turbulent pressure diffusion closure was developed based on the tensorial form of Lumley (1978) and included in a recalibrated wall-normal-free Reynolds-Stress model developped by Gerolymos and Vallet (2001). The proposed model was validated against several 1-D, 2-D and 3-D complex flows.

INTRODUCTION

The turbulent pressure diffusion term is, in general, neglected not only because of the lack of experimental or DNS data, but also because this term does not seem important in plane channel flow (Kim et al., 1987). Nevertheless, recent DNS computations over a backward-facing step have shown its importance in detached flows, especially close to the wall (Le et al., 1997).

The most widely used turbulent diffusion models are the Daly and Harlow (1970) proposal (hereafter DH), and the Hanjalić and Launder (1972) model (hereafter HL). However, these two models were initially proposed to model only the part corresponding to velocity fluctuations (divergence of the triple-velocity-correlation d^{v}_{ij}), neglecting the pressure fluctuations part. Futhermore, contrary to the (HL) model which respects the tensorial symmetry of the triple-velocity-correlation, the (DH) proposal, is not symmetric in all three indices. The exact turbulent diffusion term exhibits the same asymmetry, which means that a pressure diffusion part is certainly included in the DH model as suggested by Lumley (1979) and Launder (1984). We have recently shown (Gerolymos, Sauret, and Vallet, 2004a) that the HL model improves the prediction of boundary-layer entrainment for developing flow in a square duct (Gessner and Emery, 1981), and that the DH model improves substantially the prediction of skin-friction in the reattachment and relaxation regions in 2-D supersonic shock-compression-ramps (Settles et al., 1976b).

These results suggested the design of a model which would combine the advantages of both the DH and the HL closures. Such a model requires improvement of the closure used for the turbulent diffusion, and should include explicitly a model for pressure diffusion.

Numerous previous assessments for the triple-velocity correlation (Amano and Goel, 1986; Cormack et al., 1978; Gatski, 2004; Hanjalić, 1994; Schwarz and Bradshaw, 1994) based on *a priori* comparisons with experimental or DNS data, have shown that the HL and the Lumley (1978) models give the best overall results, in 1-D plane channel flow (Kim et al., 1987; Moser et al., 1999) and in 3-D boundary-layer (Schwarz and Bradshaw, 1993). Demuren and Sarkar (1993), in an *a posteriori* assessment, compared for fully-developed plane channel flow (Laufer, 1950), the DH, HL and MH (Mellor and Herring, 1973) models, using the Speziale et al. (1991) pressure-strain model with wall-functions, and concluded that the MH model gives the best agreement with experimental data.

Concerning the pressure-velocity correlation, the only theoretical proposal for the slow part in homogeneous flows, was etablished by Lumley (1978). The Lumley (1978) model was however used in inhomogeneous flows by several authors (Fu, 1993; Straatman, 1999; Suga, 2004). Fu (1993) successfully validated the Lumley (1978) model in 2-D plane and round jets (Ramaprian and Chandrasckhara, 1985; Taulbee et al., 1987) by using a basic Reynolds-stress closure and the DH model for the triple-velocity correlation. Straatman (1999) used the Speziale et al. (1991) and Demuren and Sarkar (1993) pressure-strain models, and compared the DH, the Lumley and a modified version of the Lumley model (coefficients recalibrated based on the analysis of zero-mean-shear turbulence), for fully-developed channel flow at R_{e_τ}=180 (Kim et al., 1987) and for flow over backward-facing step (Kim et al., 1980). The modified Lumley model performed better than the original version whose predictions were close to the DH model. Gatski (2004) reached the opposite conclusion for 1-D turbulent channel flow at R_{e_τ}=590 (Moser et al., 1999), but this was an *a priori* assessment. Nevertheless, it is difficult to generalize these results because Straatman (1999) used second moment closures with wall-functions. More recently, Suga (2004) proposed a rapid-part pressure-diffusion model and used the DH proposal to model both the triple-velocity correlation and the slow-part pressure-diffusion terms. Based on the Craft and Launder (1996) second-moment closure which is wall-normal-free, the Suga (2004) pressure-diffusion model improved the recirculating flow region behind a rectangular trailing-edge (Yao et al., 2001). There are other proposals for the pressure-diffusion term, but these models were essentially developed for the near-wall region (Craft and Launder, 1996) and some of them contain geometric normals to the wall (Launder and Tselepidakis, 1994; So and Yuan, 1999).

In the present study the flow is modelled by the compressible Favre-Reynolds-averaged Navier-Stokes equations using the near-wall second-moment closure of Gerolymos and Vallet (2001) (hereafter GV–RSM), which is completely independent of wall topology, *ie* of the distance from the wall and of the normal-to-the-wall orientation. The coefficient $C_2^{\mathrm{H}}(A, Re_{\mathrm{T}})$ present in the rapid pressure-strain redistribution model was slightly recalibrated to make the model less prone to separation (hereafter GV_C_2^{H}_modified–RSM), and a complete turbulent diffusion model including a near-wall turbulent pressure-diffusion closure was developed from the Lumley (1978) model and added to the GV_C_2^{H}_modified–RSM.

NEAR-WALL TURBULENCE MODELLING

Reynolds-Stress Equations Modelling

The exact transport equations for the Favre-Reynolds-averaged Reynolds-stresses can be written symbocally

$$C_{ij} \quad = \quad d_{ij} + P_{ij} + \phi_{ij} - \bar{\rho}\varepsilon_{ij} + K_{ij} + \frac{2}{3}\phi_p \delta_{ij} \qquad (1)$$

where, the convection C_{ij} and the production P_{ij} are exact terms. Diffusion d_{ij} due to molecular d_{ij}^{ν} and turbulent d_{ij}^{T} transport, pressure-strain redistribution ϕ_{ij}, and dissipation $\bar{\rho}\varepsilon_{ij}$ terms require modelling.

$$d_{ij} \quad = \quad d_{ij}^{\nu} + d_{ij}^{\mathrm{T}} \quad ; \quad d_{ij}^{\nu} \cong \frac{\partial}{\partial x_\ell}\left[\breve{\mu}\frac{\partial \widetilde{u_i'' u_j''}}{\partial x_\ell}\right] \qquad (2)$$

In the present work, we used the wall-normal-free GV–RSM closure where direct compressibility effects K_{ij} and pressure-dilatation correlation ϕ_p terms were neglected, and redistribution ϕ_{ij} and dissipation $\bar{\rho}\varepsilon_{ij}$ terms were modelled together

$$\begin{aligned}
\phi_{ij} - \bar{\rho}\varepsilon_{ij} \quad &= \quad \left[\phi_{ij} - \bar{\rho}(\varepsilon_{ij} - \tfrac{2}{3}\delta_{ij}\varepsilon)\right] - \tfrac{2}{3}\delta_{ij}\bar{\rho}\varepsilon \\
&= \quad \phi_{ij1}^{\mathrm{H}} + \phi_{ij2}^{\mathrm{H}} + \phi_{ij1}^{\mathrm{I}} + \phi_{ij2}^{\mathrm{I}} - \tfrac{2}{3}\delta_{ij}\bar{\rho}\varepsilon \\
&\cong \quad -C_1^{\mathrm{H}}\bar{\rho}\varepsilon a_{ij} - C_2^{\mathrm{H}}\left(P_{ij} - \tfrac{1}{3}\delta_{ij}P_{\ell\ell}\right) \qquad (3)\\
&+C_1^{\mathrm{I}}\frac{\varepsilon}{k}\left[\bar{\rho}\widetilde{u_k'' u_m''}e_{\mathrm{I}_k}e_{\mathrm{I}_m}\delta_{ij} - \tfrac{3}{2}\bar{\rho}\widetilde{u_k'' u_i''}e_{\mathrm{I}_k}e_{\mathrm{I}_j} - \tfrac{3}{2}\bar{\rho}\widetilde{u_k'' u_j''}e_{\mathrm{I}_k}e_{\mathrm{I}_i}\right] \\
&+C_2^{\mathrm{I}}\left[\phi_{km2}^{\mathrm{H}}e_{\mathrm{I}_k}e_{\mathrm{I}_m}\delta_{ij} - \tfrac{3}{2}\phi_{ik2}^{\mathrm{H}}e_{\mathrm{I}_k}e_{\mathrm{I}_j} - \tfrac{3}{2}\phi_{jk2}^{\mathrm{H}}e_{\mathrm{I}_k}e_{\mathrm{I}_i}\right] - \tfrac{2}{3}\delta_{ij}\bar{\rho}\varepsilon
\end{aligned}$$

where $\vec{e}_{\mathrm{I}} = e_{\mathrm{I}_i}\vec{e}_i$ is the unit-vector pointing in the turbulent inhomogeneity direction which replaces the geometric normal-to-the-wall present in classical redistribution echo terms (Gerolymos et al., 2004a)

$$\vec{e}_{\mathrm{I}} = \frac{\mathrm{grad}\left\{\dfrac{\ell_{\mathrm{T}}[1 - \mathrm{e}^{-\frac{Re_{\mathrm{T}}}{30}}]}{1 + 2\sqrt{A_2} + 2A^{16}}\right\}}{\left\|\mathrm{grad}\left\{\dfrac{\ell_{\mathrm{T}}[1 - \mathrm{e}^{-\frac{Re_{\mathrm{T}}}{30}}]}{1 + 2\sqrt{A_2} + 2A^{16}}\right\}\right\|} \qquad (4)$$

$$A_2 = a_{ik}a_{ki} \;;\; A_3 = a_{ik}a_{kj}a_{ji} \;;\; A = \left[1 - \frac{9}{8}(A_2 - A_3)\right]$$

$$a_{ij} = \frac{\widetilde{u_i'' u_j''}}{\mathrm{k}} - \frac{2}{3}\delta_{ij} \;;\; \ell_{\mathrm{T}} = \frac{\mathrm{k}^{\frac{3}{2}}}{\varepsilon} \;;\; Re_{\mathrm{T}} = \mathrm{k}^2(\breve{\nu}\varepsilon)^{-1} \qquad (5)$$

where ℓ_{T} is the turbulence length scale, Re_{T} is the turbulent Reynolds number, A_2 and A_3 are respectively the second and the third invariant of the anisotropy tensor a_{ij}, and A is the flatness parameter introduced by Lumley (1978).

The distance-from-the-wall effects are included in the functions C_1^{I} and C_2^{I}, which are also geometrically independant, and replace the geometric distance-from-the-wall present in echo terms. We choosed the simple models proposed by Rotta (1951) for the slow homogeneous part ϕ_{ij1}^{H} and by Naot et al. (1970) for the rapid homogeneous part ϕ_{ij2}^{H}, prefering to focus on the function C_2^{H}. The particular form of $C_2^{\mathrm{H}}(A, Re_{\mathrm{T}})$ developped by Gerolymos and Vallet (2001) is directly responsible of the ability of the model to predict separation and its precise functional dependence on A controls the size of the separation zone. Furthermore this form improves the prediction of the turbulence structure (Gerolymos et al., 2004a). Nevertheless, the GV–RSM slightly underestimates 3-D separation zone. This is due to the calibration of this coefficient which was made for the compression ramp of Settles et al. (1976a). This configuration is not free of 3-D effects and contains compressibility and thermal effects, that the original GV model does not model accurately. Therefore, the function C_2^{H} was slightly reoptimized. The coefficients C_1^{I} and C_2^{I} were also slightly modified (Table 1) to give the correct near-wall turbulence structure and the correct logarithmic-law for flate-plate boundary-layer (Klebanoff, 1955). These two coefficients C_1^{I} and C_2^{I} are systematically recalibrated, for different variants of the RSM closures, to obtain the correct near-wall prediction of zero-pressure-gradient turbulent boundary-layer.

Table 1: GV_C_2^{H}_modified–RSM redistribution term coefficients*

$C_1^{\mathrm{H}}{=}1 + 2.58AA_2^{\frac{1}{4}}\left[1 - \mathrm{e}^{-(\frac{Re_{\mathrm{T}}}{150})^2}\right]$
$C_2^{\mathrm{H}}{=}\min(0.85, \min[1, 0.75 + 1.3\max[0, A - 0.55]]\times A^{[\max(0.25, 0.5 - 1.3\max[0, A-0.55])]}[1 - \max(0, 1 - \frac{Re_{\mathrm{T}}}{50})])$
$C_1^{\mathrm{I}}{=}0.83\,[1 - \frac{2}{3}(C_1 - 1)]\,\left\|\mathrm{grad}\left\{\dfrac{\ell_{\mathrm{T}}[1 - \mathrm{e}^{-\frac{Re_{\mathrm{T}}}{30}}]}{1 + 2A_2^{0.8}}\right\}\right\|$
$C_2^{\mathrm{I}}{=}\max\left[\dfrac{2}{3} - \dfrac{1}{6C_2^{\mathrm{H}}}, 0\right]\,\left\|\mathrm{grad}\left\{\dfrac{\ell_{\mathrm{T}}[1 - \mathrm{e}^{-\frac{Re_{\mathrm{T}}}{30}}]}{1 + 1.8A_2^{\max(0.6, A)}}\right\}\right\|$

* the coefficient form C_1^{H} was proposed by Launder and Shima (1989)

Turbulent Diffusion Modelling

The turbulent diffusion d_{ij}^{T} is due to velocity fluctuations d_{ij}^{v} and to pressure fluctuations d_{ij}^{p}

$$\begin{aligned}
d_{ij}^{\mathrm{T}} \quad &= \quad d_{ij}^{\mathrm{v}} + d_{ij}^{\mathrm{p}} \\
&= \quad \frac{\partial}{\partial x_\ell}(-\bar{\rho}\widetilde{u_i'' u_j'' u_\ell''}) + \frac{\partial}{\partial x_\ell}(-\overline{p' u_j'}\delta_{i\ell} - \overline{p' u_i'}\delta_{j\ell}) \quad (6)
\end{aligned}$$

In the GV_C_2^{H}_modified–RSM, turbulent diffusion transport was modelled by the HL model and the pressure diffusion was neglected.

Previous *a posteriori* assessments of triple-velocity correlation models on several configurations (Gerolymos et al., 2004a; Sauret, 2004) have shown little difference beetween the HL, Lumley (1978), Cormack et al. (1978), Magnaudet (1993), and Younis et al. (1999) model, with due allowance for fine tuning of the model coefficients. Indeed, an erroneous coefficient can change the size of recirculation zone for example, and this is especially true if the second-moment closure used is able to predict separation (Gerolymos et al., 2004a). Taking into account that all of these models are in majority bilinear in the Reynolds stresses and their gradients, Gatski (2004) did not consider this conclusion surprising. Nevertheless, it should be noted that, the Younis et al. (1999) model contains mean-flow velocity gradients, but we found very little difference with the HL model in an *a priori* assessment for fully-developped plane channel flow (Kim et al., 1987), and the dissipation-rate ε gradient present in the Magnaudet (1993) proposal (and the Cormack et al. (1978) model as well) should be removed not only to avoid numerical instabilities close to the wall, in the reattachment zone (Gerolymos and Vallet, 1999), but also because it overestimates the triple-velocity correlations for 1-D plane channel flow (Hanjalić, 1994).

We have chosen for the turbulent diffusion transport the model proposed by Lumley (1978) which contains the HL triple-velocity correlation (Eq 7) and includes pressure-velocity correlation terms (Eqs 8, 9). The coefficient C_{SP} of the pressure diffusion model was modified to account for near-wall effects by using a function of the flatness parameter of Lumley (1978) and of the turbulent Reynolds number (Eq. 9). Indeed, the original value proposed by Lumley (1978) $C_{\text{SP}} = 0.2$, which means that the pressure-diffusion contribution equals to -20% of the triple-velocity correlation, was determined from mathematical developments for homogeneous flows, and in consequence this value is too high close to the wall. Furthermore, the coefficient C_{SP} cannot be zero near the wall because the pressure-diffusion term is important in this zone in detached flows (Le et al., 1997). The proposed coefficient C_{SP} value is 0.1275, except close to the wall where it is sharply damped to a value of 0.085 (Eq. 9). Then the coefficients $C_{\text{S}1}$ and $C_{\text{S}2}$ (Eq. 7) were recalibrated for improved prediction of separated flows (the original values $C_{\text{S}1} = 0.098$, $C_{\text{S}2} = 0.01265$, proposed by Schwarz-Bradshaw (1994), slightly overestimate the separation zone).

$$-\bar{\rho}\widetilde{u_i''u_j''u_\ell''} = C_{\text{S}1}\bar{\rho}\frac{k}{\varepsilon}G_{ij\ell} + C_{\text{S}2}\bar{\rho}\frac{k}{\varepsilon}\left[G_i\delta_{j\ell} + G_j\delta_{i\ell} + G_\ell\delta_{ij}\right] \quad (7)$$
$$C_{\text{S}1} = 0.0935 \quad ; \quad C_{\text{S}2} = 0.0115$$

$$G_{ij\ell} = \widetilde{u_i''u_k''}\frac{\partial\widetilde{u_j''u_\ell''}}{\partial x_k} + \widetilde{u_j''u_k''}\frac{\partial\widetilde{u_i''u_\ell''}}{\partial x_k} + \widetilde{u_k''u_\ell''}\frac{\partial\widetilde{u_i''u_j''}}{\partial x_k}$$
$$G_i = G_{imm}, G_j = G_{jmm}, G_\ell = G_{\ell mm} \quad (8)$$
$$d_{ij}^p = \frac{\partial}{\partial x_\ell}\left(C_{\text{SP}}\bar{\rho}\widetilde{u_j''u_k''u_k''}\delta_{i\ell} + C_{\text{SP}}\bar{\rho}\widetilde{u_i''u_k''u_k''}\delta_{j\ell}\right)$$
$$C_{\text{SP}} = 0.085\left[1 + \min\left(0.5, A^{\max(0.25, 2(1-6A))}\right)\right] \quad (9)$$

The calibration of the coefficients C_1^{I} and C_2^{I} on flate-plate are (Table 2):

Table 2: present model

$C_1^{\text{I}}=0.83\ [1 - \frac{2}{3}(C_1 - 1)]$	$\left\|\text{grad}\left\{\dfrac{\ell_{\text{T}}[1 - \mathrm{e}^{-\frac{Re_{\text{T}}}{30}}]}{1 + 2.05A_2^{0.8}}\right\}\right\|$
$C_2^{\text{I}}= \max\left[\dfrac{2}{3} - \dfrac{1}{6C_2^{\text{H}}}, 0\right]$	$\left\|\text{grad}\left\{\dfrac{\ell_{\text{T}}[1 - \mathrm{e}^{-\frac{Re_{\text{T}}}{30}}]}{1 + 1.5A_2^{\max(0.6,A)}}\right\}\right\|$

Finally, to investigate the influence of the pressure-diffusion term, a version of present model without pressure-diffusion ($d_{ij}^p = 0$) was developed, and the coefficients C_1^{I} and C_2^{I} are given in (Table 3).

Table 3: present model without pressure-diffusion ($d_{ij}^p = 0$)

$C_1^{\text{I}}=0.83\ [1 - \frac{2}{3}(C_1 - 1)]$	$\left\|\text{grad}\left\{\dfrac{\ell_{\text{T}}[1 - \mathrm{e}^{-\frac{Re_{\text{T}}}{30}}]}{1 + 1.52A_2^{0.75}}\right\}\right\|$
$C_2^{\text{I}}= \max\left[\dfrac{2}{3} - \dfrac{1}{6C_2^{\text{H}}}, 0\right]$	$\left\|\text{grad}\left\{\dfrac{\ell_{\text{T}}[1 - \mathrm{e}^{-\frac{Re_{\text{T}}}{30}}]}{1 + 1.25A_2^{\max(0.6,A)}}\right\}\right\|$

VALIDATIONS

The present model was then validated against several flows 1) plane channel (Kim et al., 1987), 2) incident oblique shock-wave/boundary-layer interaction (Reda and Murphy, 1973), 3) 3-D transonic channel (Ott et al., 1995), and 4) high-subsonic annular cascade with large separation (Doukelis et al., 1978). Computations were carrefully checked for grid-convergence and for conformity with experimental inflow- and boundary-conditions (Gerolymos, Sauret, and Vallet, 2004b).

Plane Channel

Various d_{ij}^{T} models were compared with DNS data (Fig. 1) for fully developed plane chanel flow (Kim et al., 1987) both *a priori* (Lumley and HL models) and *a posteriori* (various wall-normal-free Reynolds stress models). Note that the GV_C_2^{H}_modified–RSM uses the HL model offering a comparison of *a priori* and *a posteriori* predictions with the same d_{ij}^v closure. All models give similar results for the simple flow. They all underestimate the level of $\overline{v'u'u'}$ and $\overline{v'u'v'}$ maxima at $y^+ \cong 50$.

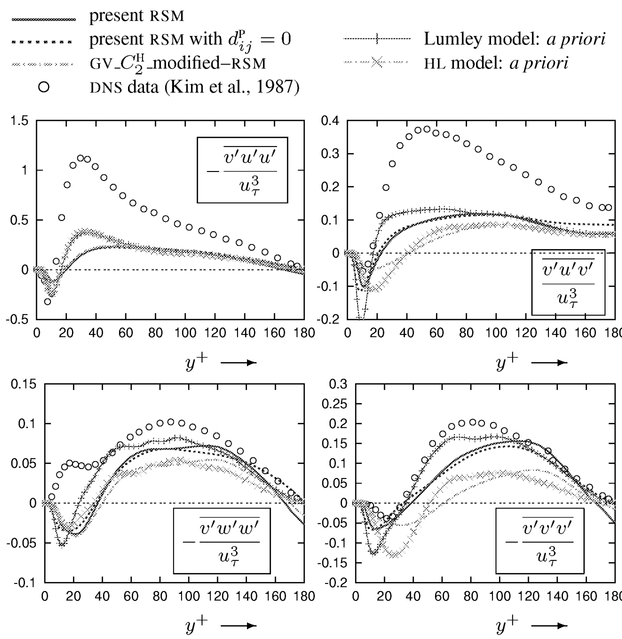

Figure 1: Comparison of computations using the present model with and without pressure-diffusion and the GV_C_2^{H}_modified–RSM, with DNS data of Kim et al. (1987) of triple-velocity correlations for plane channel flow (Re_{τ}=180, C_f=8.18 10^{-3}, Re_{B}=5600)

Oblique Shock-Wave/Boundary-Layer Interaction

This configuration, experimentally studied by Reda and Murphy (1973), consists of an oblique shock-wave M_{SW}=2.9 and $\Delta\vartheta_{\text{SW}}$=13 deg impinging on a flat-plate turbulent boundary-layer. Comparison of skin-friction distribution for the oblique-shock-wave/boundary-layer interaction (Fig. 2) highlights the importance of pressure-velocity correlation in the reattachment region. Indeed, we note that the pressure-diffusion term is directly responsible of the good skin-friction shape prediction. This improvement is independent of the recirculation zone prediction since the present model and the GV_C_2^{H}_modified–RSM predict identical wall-pressure distributions (the $C_{\text{S}1}$ and $C_{\text{S}2}$ coefficients of the present model, were calibrated to this purpose).

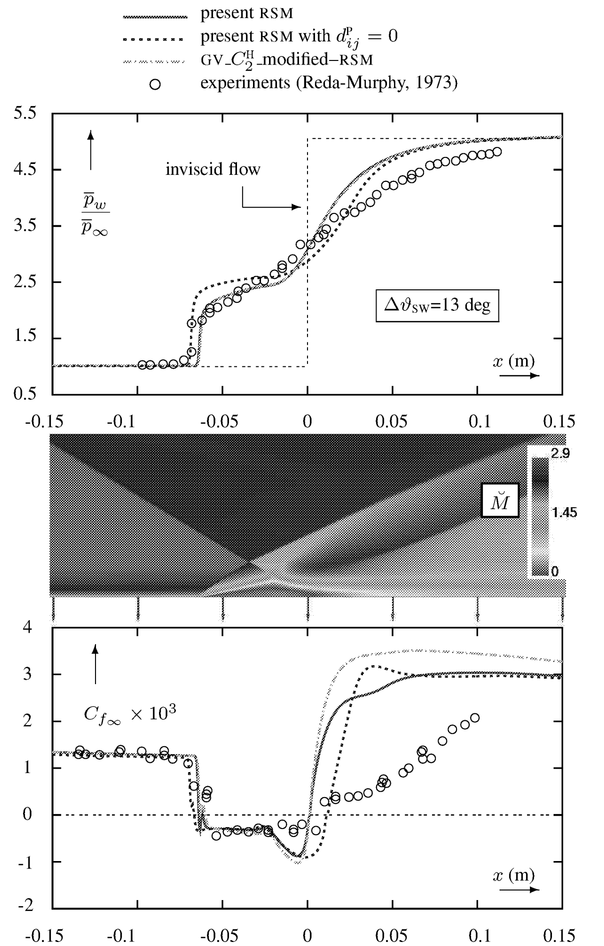

Figure 2: Comparison of grid-converged computations with measurements (Reda and Murphy, 1973) of wall-pressure and skin-friction x-wise distributions, for Reda and Murphy (1973) incident-shock-wave interaction (M_∞ =2.9, Re_{δ_0} $=0.97\times10^6$, $\Delta\vartheta_{\mathrm{SW}}$ =13 deg) using the present model with and without turbulent pressure diffusion d^{p}_{ij} and the GV_C_2^{II}_modified–RSM (iso-Machs computed with the present model RSM).

3-D Transonic Channel

The 3-D transonic channel configuration, studied experimentally by Ott et al. (1995), is an interesting test case to evaluate the capacity of a model to predict solid corner secondary flow with shock-induced recirculation. Comparison of isentropic-wall-number distributions (Fig. 3) indicate that the GV_C_2^{II}_modified Reynolds-stress model predicts a slight pressure-plateau behind the shock-wave/boundary-layer interaction (for both the outflow-static to inflow-total pressure ratios), which does not appear experimentally. The present model is in perfect agreement with experimental data, whereas the version without pressure diffusion modelling and the GV_C_2^{II}_modified–RSM predict a recirculation zone whose size varies with the shock-wave intensity (the detached zone is more pronouced with $\pi_{\text{S-T}}=0.669$). From this, it may be deduced that a simple adjustment of the C_2^{II} function, the particular form of which is able to control the size of the recirculation zone, could not give a good agreement with experimental data for both outflow-static pressures. Consequently, only explicit pressure-diffusion modelling is able to improve the corner secondary flow prediction and adjust its size in accordance with the shock-wave intensity.

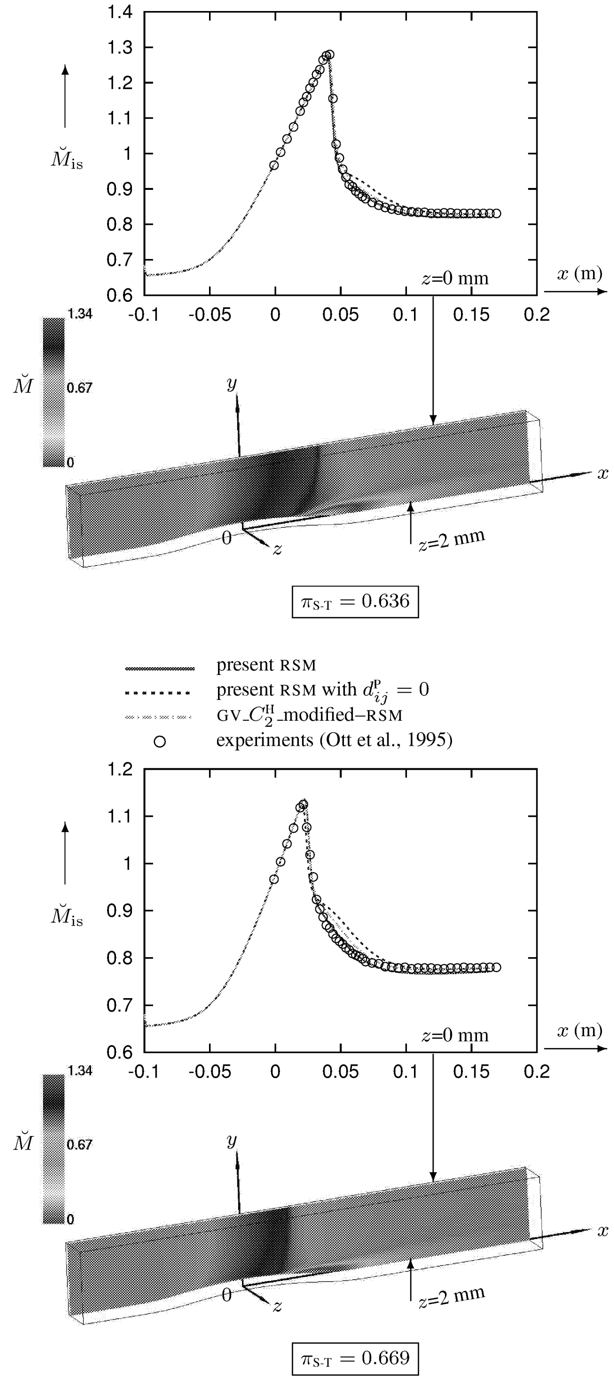

Figure 3: Comparison of computations with measurements (Ott et al., 1995) of isentropic-wall-Mach-number $\breve{M}_{\mathrm{is}}$ on the sidewall ($z=0$) at the y-symmetry plane of the transonic square nozzle configuration of Ott et al. (1995) (only $\frac{1}{4}$ of the symmetric nozzle is shown) for two outflow-static-to-inflow-total pressure ratios $\pi_{\text{S-T}}=0.636, 0.669$ ($T_{u_i}=2\%$; $\ell_{\mathrm{T}_i}=0.020$ m; $241\times121\times97$ grid), using the present model with and without turbulent pressure diffusion d^{p}_{ij} and the GV_C_2^{II}_modified–RSM; iso-Machs computed with the present model RSM at $z=2$ mm.

High-Subsonic Annular Cascade

As shown (Fig. 4) for the prediction of a large separation zone in a high-subsonic annular cascade created by an upstream scroll, the present model gives the correct prediction of the pitchwise-averaged flow-angle α_{M_o} at the outlet of the cascade. The correct prediction of this angle is directly related to the correct prediction of the large corner stall observed in this cascade (α_{M_o}= 90 deg corresponds to purely circumferential flow). If the pressure diffusion term is neglected, results are less satisfactory. Previous studies (Gerolymos et al., 2004a) have shown that turbulent diffusion models which only include triple-velocity correlation, do not have a major effect on this flow. It is important to note that an eddy-viscosity model (here, the $k-\varepsilon$ model of Launder and Sharma (1974)) completely fails in predicting this complex swirling flow.

CONCLUSION

The present model improves subtantially the skin-friction distribution in the reattachment and the relaxation zones without modification of the prediction of wall-pressure distribution, as did the DH model (studies presented in a previous paper (Gerolymos, Sauret, and Vallet, 2004a)), which confirm that the DH model takes into account a part of pressure-diffusion, and actually models the complete turbulent transport term. Unfortunately, the DH model is not mathematically correct and fails improving the prediction of 3-D complex flows, contrary to the Lumley model. The DH model should be used only for simple flows as suggested by Lumley (1979). The main interest of the pressure-diffusion model proposed, is the improvement of prediction of 3-D secondary flows, especially with large recirculation zones, which are encountered in many aerospace applications.

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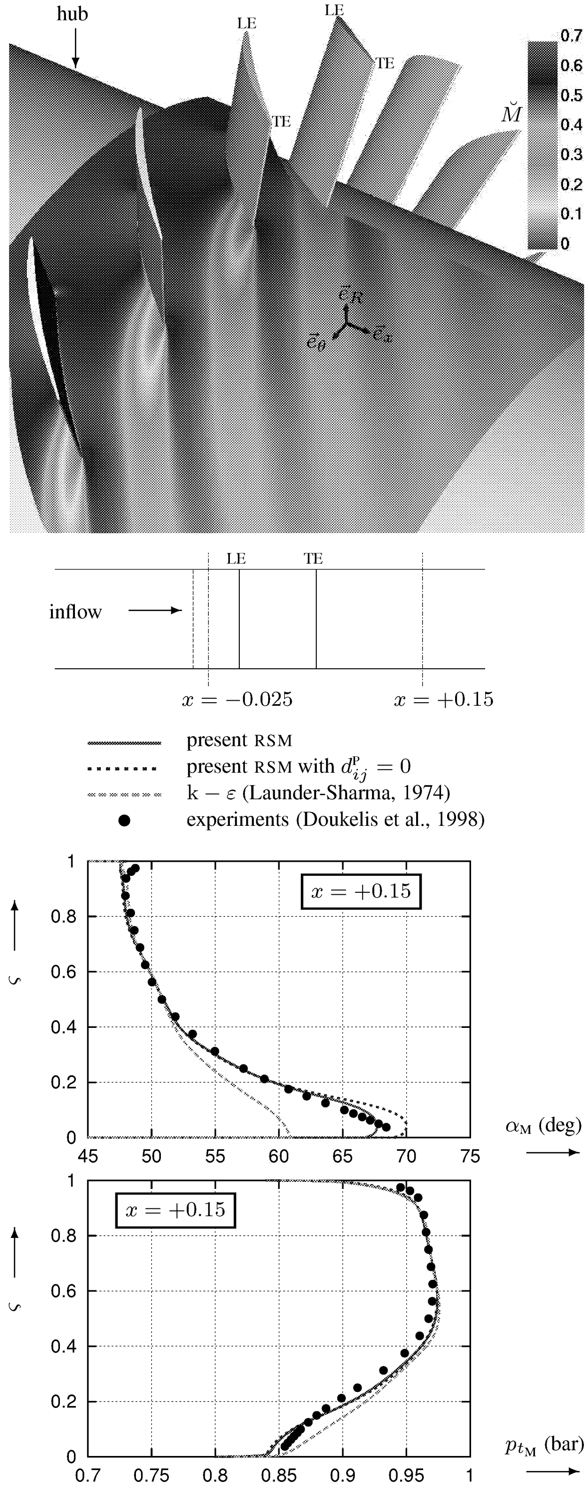

Figure 4: Partial view of the NTUA annular cascade (Doukelis et al., 1998), illustrating Mach-levels near the hub (the $xR\theta$-frame is located at the $x = +0.15$ m station), and comparison of measured (Doukelis et al., 1998) and computed spanwise (ς) distributions of pitchwise-averaged flow-angle α_{M_o} and pitchwise-averaged total-pressure $p_{t_{\mathrm{M}_o}}$ at the outlet, using the present model with and without pressure diffusion d_{ij}^{p} and the Launder and Sharma (1974) $\mathrm{k} - \varepsilon$ closure ($\dot{m} = 13.2$ kg s^{-1}; $T_{u_i} = 4\%$; $\ell_{\mathrm{T}_i} = 0.04$ m; grid_DE (Chassaing, Gerolymos, and Vallet, 2003)).