CLOSED-LOOP FEEDBACK CONTROL OF THE TURBULENT FLOW OVER A NACA-4412 AIRFOIL

Julie M. Ausseur, Jeremy T. Pinier, Mark N. Glauser, Hiroshi Higuchi
Department of Mechanical and Aerospace Engineering
Syracuse University
Syracuse, NY 13244, USA
jausseur@syr.edu

ABSTRACT

We are developing methods to predict the dynamics of the flow field above a NACA-4412 airfoil using real-time measurements of the pressure from the surface of the airfoil only. Through Proper Orthogonal Decomposition (POD) and modified Linear Stochastic Measurement (mLSM) low-dimensional techniques (Lumley (1967), Adrian (1975), Taylor & Glauser (2004)) these pressure measurements are coupled to Particle Image Velocimetry (PIV) data of the flow to estimate the time dependent coefficients describing the flow. Based on a least-squares technique applied to the POD spatial modes, we obtain a system of ODEs that are solved to get the time-evolution of the POD coefficients, whereby an estimate of the evolution equation of the flow is obtained. This low-dimensional estimated plant will be used to explore different Proportional, Integral & Derivative (PID) control parameters and will enable us to implement a controller of the flow state above the airfoil using leading-edge zero net-mass flow actuators while pitching the airfoil.

EXPERIMENTAL SETUP

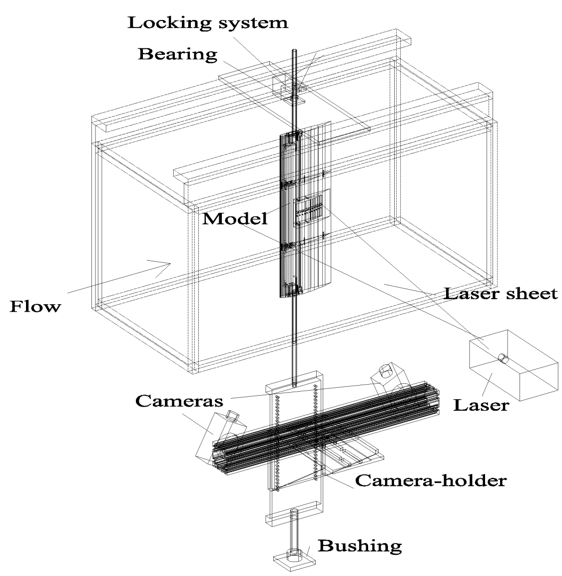

Figure 1: Overall view of the experimental setup in the test section of the Syracuse University closed-loop low-speed wind tunnel.

The subsonic wind tunnel facility at Syracuse University is a closed recirculating design with a 2 ft. by 2 ft. test section (Figure 1). The speed in the test section is continuously variable from 5 to 60 m/s. The test model NACA-4412 airfoil was designed to meet several requirements (Glauser et al. (2004_a)). It is two dimensional with a constant 8-inch chord-length and constant airfoil section geometry along the span. The model is equipped with 11 unsteady pressure sensors along the chord (Figure 2) and actuators near the leading edge.

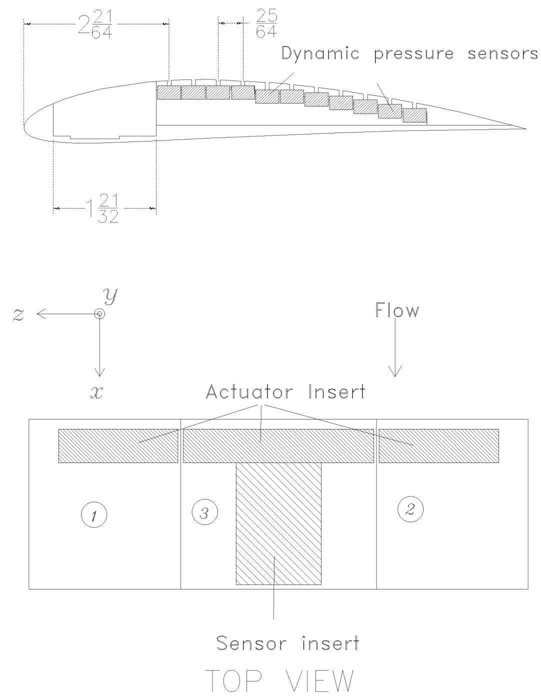

Figure 2: Side and top view of NACA 4412 with location of pressure sensors

Measurements of pressure and velocity over the airfoil are taken simultaneously at different angles of attacks. Measurements were taken at 13, 14 and 15^o angle of attack (AoA) without actuation, taking 840 statistically independent samples for each angle, and 14, 15, 16, 17 and 17.5^o with actuation taking 600 statistically independent samples for each angle. Each sample includes a PIV 2D-3C (three component) measurement of velocity using a Dantec Dynamics Stereo PIV System to capture data in the x-y (streamwise-transverse) plane above the airfoil. Concurrently, the unsteady pressure was measured at eleven locations along the chord at midspan at a sampling rate of 4 kHz. The unsteady pressure along the chord is sampled through a National Instruments SCXI signal conditioning unit along with a PXI based A/D board. The system is triggered to synchronize the pressure measurements and the PIV snapshots.

LOW-DIMENSIONAL TOOLS

Proper Orthogonal Decomposition (POD)

In 1967, Lumley proposed the POD as an unbiased technique for studying coherent structures in turbulent flows. The

POD is a logical way to build basis functions which emphasize the most energetic features of the flow (Holmes et al. (1997)). This results in the localization of a small number of the structures containing a large percentage of the systems energy. The POD is a straightforward mathematical approach based on a Karhunen-Loeve expansion. It is used to decompose the velocity field into a finite number of empirical functions intrinsic to the flow, which can be used to ascertain a subspace where a model can be constructed by projecting the governing equations on it (Holmes (1996)). These eigenfunctions, $\phi^{(n)}_{n\in N}$, are linearly independent and form a basis set chosen to maximize the mean square projection of the velocity field. POD therefore describes in an *optimal* way the energy contained in the flow. The eigenfunctions are obtained from the following integral eigenvalue problem:

$$\int R_{ij}(\vec{x},\vec{x}')\,\phi_j^{(n)}(\vec{x}')\,d\vec{x}' = \lambda^{(n)}\phi_i(\vec{x}) \quad (1)$$

where $R_{ij}(\vec{x},\vec{x}\,')$ is the ensemble averaged two-point spatial velocity correlation tensor.

We then can extract the time dependent expansion coefficients describing the flow, by projecting the velocities onto the eigenfunctions, as follows:

$$a_n(t_o) = \int_D u_i(\vec{x},t_o)\,\phi_i^{(n)}(\vec{x})\,d\vec{x} \quad (2)$$

where $u_i(\vec{x},t_o)$ is the velocity field at a given PIV-snapshot time.

The eigenfunctions of equation 1 give the optimal basis, and are termed empirical eigenfunctions since they are derived from the ensemble of the observations. The Hilbert-Schmidt theory ensures that if the random field occurs over a finite domain, an infinite number of orthonormal solutions can be used to express the original random velocity field, u_i, therefore we can then partially or totally reconstruct the original velocity field by projecting $a_n(t)$ on the eigenfunctions:

$$u_i(\vec{x},t_o) = \sum_{n=1}^{N} a_n(t_o)\phi_i^{(n)}(\vec{x}) \quad (3)$$

where N is the number of modes with which we wish to reconstruct the velocity field. If N is ∞ the velocity field is totally reconstructed. We use the time dependent information provided by equation 3 to develop the low-dimensional descriptions of the flow when N is a finite number.

Depending on the data included in the ensemble averages involved in the calculation of the two-point spatial velocity correlation tensor R_{ij}, we will refer to the POD method as either *conditional* or *global*. The conditional approach solves each flow state (angle of attack, Reynolds number, control On/Off) separately, the kernel therefore becoming of the following form:

$$R_{ij}(\vec{x},\vec{x},'t_o,\alpha) = \langle u_i(\vec{x},t_o,\alpha)\,u_j(\vec{x},'t_o,\alpha)\rangle. \quad (4)$$

while the global POD approach includes ensembles of different flow states in the average process, as follows:

$$R_{ij}(\vec{x},\vec{x},'t_o) = \langle u_i(\vec{x},t_o)\,u_j(\vec{x},'t_o)\rangle. \quad (5)$$

Therefore in the global POD, the eigenfunctions have a greater knowledge of the multiple flow states, i.e. for different angles of attack, control On and Off. Taylor and Glauser (2002, 2004) discuss this approach at length. Boree (2003) and Fogleman et al. (2004) apply a similar approach to an engine cylinder flow. Substituting either the conditional or global kernel into equation 1 provides the desired basis functions. The velocity reconstructions displayed in this paper use a global POD approach.

modified Linear Stochastic Measurement (mLSM)

The Linear Stochastic Estimation (LSE) was proposed by Adrian in 1975. He recognized that the statistical information contained within the two-point correlation tensor, R_{ij}, could be combined with instantaneous information to form a technique for estimating the flow field. Cole et al. (1991) demonstrated this in the axisymmetric jet shear layer where they successfully estimated the velocity radially across the jet shear layer using information from only a few radial locations.

Bonnet et al. (1994) expanded on the work of Adrian (1975) and Cole (1991) to form the complementary technique which combines the POD and LSE to obtain the time resolved POD expansion coefficients from instantaneous velocity data on course hot wire grids.

Taylor and Glauser(2002, 2004) further expanded these methods and demonstrated how instantaneous time resolved wall pressure measurements could be used to construct an accurate representation of the instantaneous velocity field from wall pressure alone (i.e., the modified complementary technique or mLSE, now termed mLSM for better understanding with controls community). This approach can be applied to the POD using either the "conditional" or "global" POD eigenfunctions described above. In this study we use the mLSM method to compute the POD velocity coefficients above the airfoil using discrete pressure measurements taken on the airfoil surface. The conditional structure is the random POD coefficient, $a_n(t)$ and the conditions are the wall pressure. Since both the fluctuating pressure and the random POD coefficients are integrated quantities, the correlation between them is strong and the method makes sense from a physical standpoint. The instantaneous wall pressure is used in equation 6,

$$\tilde{a}_n(t) = \langle a_n(t)|p(t)\rangle \quad (6)$$

to obtain $\tilde{a}_n(t)$ the estimate of the random POD coefficient that describes the velocity field over $\vec{x}$ given the instantaneous surface pressures, $p_i(t)$. The estimated random coefficients for each POD mode can be described as a series expansion using the instantaneous surface pressures available at i positions on the airfoil surface:

$$\tilde{a}_n(t) = B_{n1}p_1(t) + B_{n2}p_2(t) + \cdots + B_{nq}p_q(t). \quad (7)$$

Truncating this expression to include only the linear term (plus the error associated with neglecting the higher order terms) we obtain:

$$\tilde{a}_n(t) = B_{ni}p_i(t) + O[p_i^2(t)]. \quad (8)$$

The coefficients are considered to be the conditional structures of the flow, and they effectively describe a certain percentage of the energy contained in a certain spatial POD mode. The elements of B_{ni} are chosen to minimize the mean square error, $e_{\tilde{a}_n} = \overline{[\tilde{a}_n(t) - a_n(t)]^2}$ by requiring that $\frac{\partial e_{\tilde{a}_n}}{\partial B_{ni}} = \frac{\partial \overline{[B_{ni}p_i(t)-a_n(t)]^2}}{\partial B_{ni}} = 0$. The solution to the minimization problem of equation 8 is a linear system of equations, which can be written in matrix form as:

$$\begin{bmatrix} \langle p_1^2\rangle & \langle p_1p_2\rangle & \cdots & \langle p_1p_q\rangle \\ \langle p_2p_1\rangle & \langle p_2^2\rangle & \cdots & \langle p_2p_q\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle p_qp_1\rangle & \langle p_qp_2\rangle & \cdots & \langle p_q^2\rangle \end{bmatrix} \begin{bmatrix} B_{n1} \\ B_{n2} \\ \vdots \\ B_{nq} \end{bmatrix} = \begin{bmatrix} \langle a_np_1\rangle \\ \langle a_np_2\rangle \\ \vdots \\ \langle a_np_q\rangle \end{bmatrix}$$

The elements B_{ni} are then substituted into equation 8 to estimate the random POD coefficient for each instantaneous pressure measurement. These coefficients when combined with the POD eigenfunctions provide an estimate of the instantaneous velocity field, $u_i(t)$ from application of equation 3. For flow control studies we have been using the mLSM method to provide the state of the flow from wall pressure only (Taylor and Glauser (2002, 2004) and Glauser et al. (2004_a)). This provides one method for monitoring the system state with physically realizable input from practical wall sensors.

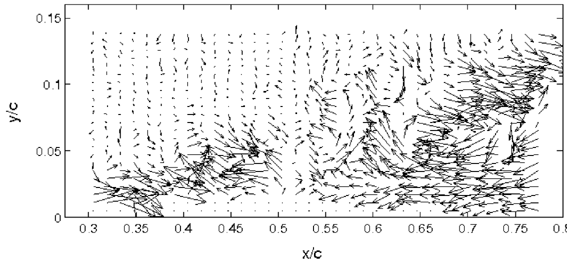

Figure 3: Instantaneous fluctuating velocity field at $\alpha = 15^o$ AoA.

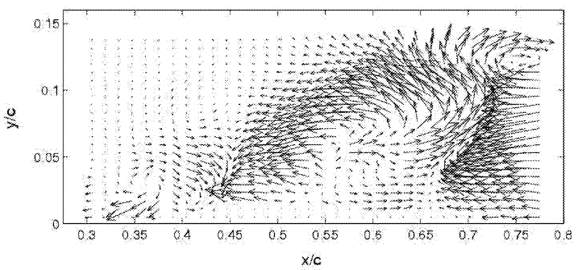

Figure 4: 50-mode mLSM estimation of the fluctuating velocity field at $\alpha = 15^o$ AoA.

Figure 3 shows an original fluctuating velocity field over the airfoil and figure 4 shows the reconstructed field estimated from pressure alone. We are able to retrieve the main features of the turbulent flow through this estimation technique. Naguib et al. have shown that a Quadratic Stochastic Measurement technique was more effective when the measurements (airfoil surface pressure here) are out of the plane of estimation. An investigation will be made to verify how a quadratic estimation improves the closed-loop control.

OPEN-LOOP CONTROL RESULTS

Zero net-mass flow actuators placed near the leading edge are used to add energy to the turbulent boundary layer thus delaying the separation of the flow (i.e. stall of the wing) as the AoA increases. The flow over the airfoil was first studied with a constant amplitude and frequency sine wave actuation to measure the effectiveness of such an actuation. The actuators were found to operate with the most efficiency at 2500 Hz because of cavity resonance phenomena. The actuation has shown effective in keeping the flow attached well over 16^o AoA, the natural stall angle of the model airfoil (Figures 5 & 6). Indeed it was able to delay separation up to 18.5^o AoA, keeping the flow in an incipient condition with an output from the speakers on the order of 1 V.

PROPORTIONAL CLOSED-LOOP FEEDBACK CONTROL RESULTS

As a validation step of our closed-loop control approach, we have been implementing a simple proportional feedback involving POD-mLSM methods. The aim is to verify if low-dimensional modelling based on POD is a potential solution for implementing a closed-loop control at high rates. The problem being to process the rich and complex information real-time. As suggested by Glauser et al. (2004_b), because the

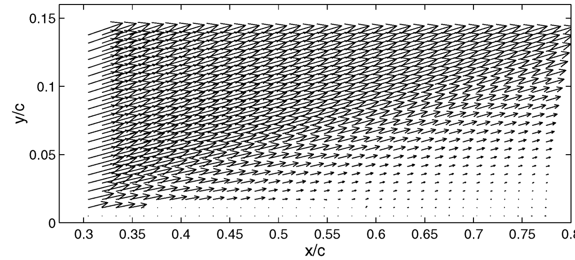

Figure 5: Mean-velocity vector map at $\alpha = 15^o$ AoA, averaged over 600 snapshots.

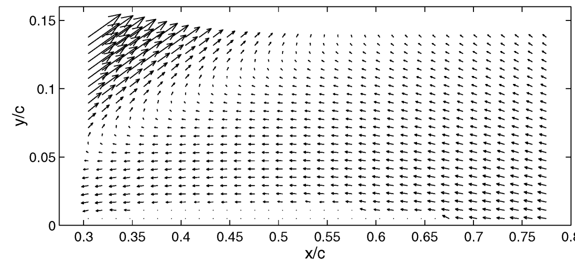

Figure 6: Mean-velocity vector map at $\alpha = 16^o$ AoA, averaged over 600 snapshots.

time series of the 1^{st} POD-mLSM estimated mode contains much of the energy along with the low frequency and high amplitude information, it was used as a real-time amplitude modulation to the 2500 Hz sine wave used in the open-loop case. The input of the actuators is therefore as follows:

$$Act_{input} = \mathrm{A}\,\varepsilon(\mathrm{t})\sin(2\pi f_0 \mathrm{t})$$

A is a fixed gain.
ε(t) is the difference between the 1^{st} POD coefficient estimated from the pressure and the amplitude of the aimed state. f_0 is the 2500 Hz high frequency carrier, given by the optimal characteristics of the speaker.

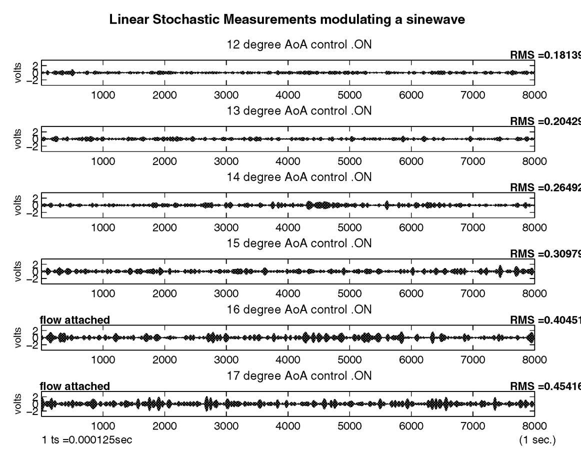

Figure 7: First POD coefficient estimated from wall pressure modulating 2000 Hz sine wave. Control ON, AoA $12 - 16^o$.

The feedback loop was performed at a rate of 10 kHz acquiring all 11 sensor channels and feeding back in real-time the actuation channel. The pressure signal sensed downstream of the actuation has to be low-pass filtered at a cutoff under the carrier frequency otherwise destabilizing the closed-loop algorithm. A dedicated real-time NI-PXI based controller was used to perform the loop at these rates. As can be seen in Figure 7 (Glauser et al. $(2004)_b$), the amplitude of the spatially estimated measurement increases with the AoA, which

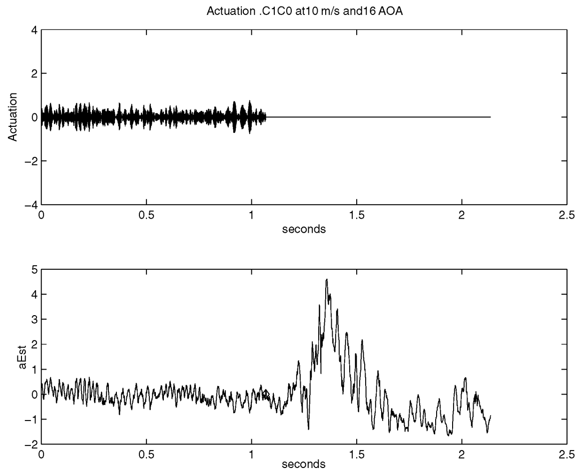

Figure 8: CLFC On then Off, actuation on top and filtered coefficient $a_1(t)$ on bottom, note hysterisis delay before separation.

is a necessary condition for this method to be stable. Indeed, as the AoA increases and the structures grow larger, the actuation amplitude will follow in real-time and increase to keep the flow attached.

Note that not only does the coefficient 'measured' from the pressure contain the amplitude information from the flow, critical for the control but it also contains the frequency changes as can be seen in Figure 8. It shows on the bottom the filtered coefficient estimated from the pressure and on top the actuation signal. As the Closed-Loop Feedback Control (CLFC) is turned off, the flow separates after a hysterisis time period on the order of a 0.5 seconds where the CLFC is OFF but the flow stays attached, phenomenon also observed by Wygnanski (2004) on a real scale application of flow control. Once the flow has completely separated we can note a lower frequency, revealing the much larger structures present in a separated flow. Proving that a simple proportional feedback method based on these low-dimensional tools is, in a practical way, feasible with surface measurements only, stable and robust is our first step toward developing a model of this system and a controller for this model.

PREDICTION OF THE FLOW FOR CONTROLLER DEVELOPMENT

We are developing a low-dimensional dynamical model for the flow in order to develop a controller for this model. Given initial and boundary conditions of the flow, it is theoretically possible to predict the evolution of the flow by solving the full incompressible Navier-Stokes equations. Because of the complexity of the system, different works (Aubry et al. (1988), Ukeiley et al. (2001)) have been conducted in order to simplify the Navier-Stokes equations and come up with a minimal set of Ordinary Differential Equations (ODEs) that would be able to describe correctly the essential dynamical behavior of the flow. Based on this set of ODEs, which govern the time-evolution of the POD coefficients, and with an experimental rather than analytical approach, Ricaud (2001) was able to 'train' the equations with a 'learning sample' from experimental data and get an estimate of the expansion coefficients. The time-evolution ODE is of cubic form as follows:

$$\frac{da_i(t)}{dt} = \sum_{j=1}^{N} L_{ij}\, a_j + \sum_{j=1}^{N}\sum_{k=j}^{N} Q_{ijk}\, a_j\, a_k + \sum_{j=1}^{N}\sum_{k=j}^{N}\sum_{l=k}^{N} C_{ijkl}\, a_j\, a_k\, a_l \tag{9}$$

Following this idea, we built our dynamical model, function of the POD modes. Refer to Ausseur & Pinier (2005) for more detail. Our goal is to be able to correctly reproduce the estimated POD expansion coefficients of the 'learning sample' and beyond from a single initial condition for each mode. Figure 9 from Ausseur & Pinier (2005) shows the initial results in the prediction of the dynamics of the flow. Given only the initial condition $a_n(0)$ for all modes n, the integration of the ODEs through a 4^{th} order Runge-Kutta method predicted the coefficients for over 250 time-steps (0.063 sec.) before diverging due to numerical parameters not yet optimized.

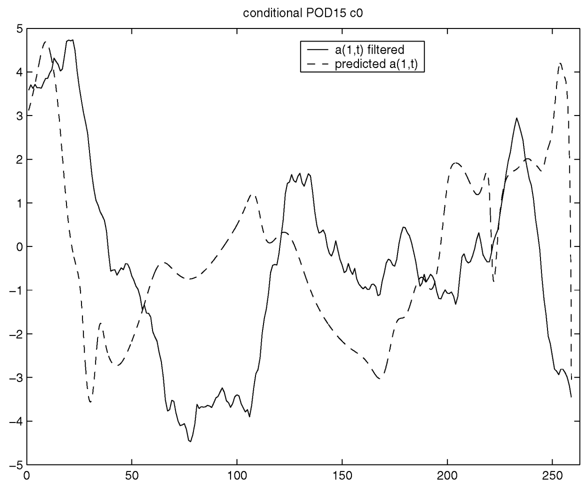

Figure 9: Comparison of measured and predicted low-order expansion coefficients $a_1(t)$ for 13^o AoA, given same initial conditions

The gained prediction capability enables us to get an estimate of a plant for the airfoil model. We are in the process of modeling the effect of the actuation to incorporate it in the modeled plant. By decomposing velocity into a mean term, a fluctuating term and an actuation term, we will be able to include the actuation term in the evolution equation and implement an elaborate closed-loop control algorithm. In a first approach, we will linearize the evolution equation. We can then simulate offline the actuation of the airfoil and develop a controller in the complex domain. The aim of the control is that the flow resembles as much as possible a completely attached flow state (e.g. at 13^o AoA) as the airfoil pitches to high angles of attack. Figure 7 shows the real-time control of the system that will be implemented. The effectiveness of our controller on the real system will highly depend on the capability of the estimated plant to describe accurately the real flow. This capability is the result of a correct integration of the low-order dynamical system, our aim being to increase significantly our prediction time.

CONCLUSION

The POD and mLSM mathematical tools used have proven effective in reducing the order and complexity of the turbulent system, which is fundamental for real-time control capability. In the open-loop investigation the actuation was able to keep the flow attached or in an incipient state, well over the stall angle of the uncontrolled airfoil. The first-of-a-kind practical closed-loop feedback control of a turbulent flow based on these low-dimensional tools has shown very promising for our future implementation within an elaborate control algorithm. Close-future work includes a refinement of the integration of the evolution equations based on data to improve the prediction capability and the integration of actuation in the model evolution equation.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the support of the Air Force Office of Scientific Research through a Phase 2 STTR with Clear Science Corporation and the New York State Office of Science, Technology and Academic Research.

REFERENCES

Adrian, R. J., 1977, "On the Role of Conditional Averages in Turbulence Theory", *Turbulence in Liquids: Proceedings of the Fourth Biennial Symposium on Turbulence in Liquids (1975)*, Science Press, eds. Zakin, J. & Patterson, G., pp. 323–332.

Ausseur, J. M. & Pinier, J. T., "Towards Closed-loop Feedback Control of the flow over NACA-4412 Airfoil", 2005, *AIAA National Student Conference*, AIAA-2005-0343, Reno, NV.

Bonnet, J. P., Cole, D. R., Delville, J., Glauser, M. N. & Ukeiley, L. S., 1994,"Stochastic Estimation and Proper Orthogonal Decomposition: Complementary Techniques for Identifying Structure.", *Experiments in Fluids*, Vol. 17, pp. 307–314.

Boree, J., 2003, "Extended Proper Othogonal Decomposition: A Tool to Analyse Correlated Events in Turbulent Flows.", *Experiments in Fluids*, Vol. 35, pp. 188–192.

Cole, D. R., Glauser, M. N. & Guezennec, Y. G., 1991, "An Application of Stochastic Estimation to the Jet Mixing Layer.", *Phys. Fluids*, Vol. 4, No. 1, pp. 192–194.

Fogleman, M., Lumley, J.L., Rempfer, D. & Haworth, D., 2004, "Application of the Proper Orthogonal Decomposition to datasets of internal combustion engine flows.", *J. Turbulence*, 5, 023.

Glauser, M., Young, M., Higuchi, H., Tinney, C. & Carlson, H., 2004_a, "POD Based Experimental Flow Control on a NACA-4412 Airfoil (Invited).", *42nd AIAA Aerospace Sciences Meeting and Exhibit*, AIAA 2004-0575, Reno, NV.

Glauser, M. N., Higuchi, H., Ausseur, J. M. & Pinier, J. T., 2004_b, "Feedback Control of Separated Flows (Invited).", *2nd AIAA Flow Control Conference*, AIAA 2004-2521, Portland, OR.

Holmes, P. J., Lumley, J. L., Berkooz, G., Mattingly, J. C. & Wittenberg, R. W., 1997, "Low-Dimensional Models of Coherent Structures in Turbulence.", *Physics Reports*, Vol. 287, pp. 337–384.

Holmes, P. J., Lumley, J. L. & Berkooz, G., 1996, *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*, Cambridge University Press, Cambridge, Great Britain.

Lumley, J. L., 1967, "The Structure of Inhomogeneous Turbulent Flows.", *Atm. Turb. And Radio Wave Prop.*, Nauka, Moscow and Toulouse, France, eds. Yaglom and Tatarsky, pp. 166–178.

Naguib, A., Wark, C. & Juckenhoefel, O., 2001, "Stochastic Estimation and Flow Sources Associated with Surface Pressure Events in a Turbulent Boundary Layer.", *Physics of Fluids*, Vol. 13, No. 9, pp. 2611–2616.

Schmit, R. & Glauser, M. N., 2004, "Improvements in Low Dimensional Tools for Flow-Structure Interaction Problems: Using Global POD.", *42nd AIAA Aerospace Sciences Meeting and Exhibit*, AIAA 2004-0889.

Taylor, J. A. & Glauser, M. N., "Towards Practical Flow Sensing and Control via POD and LSE Based Low-Dimensional Tools.", *ASME Fluids Engineering Division Summer Meeting, Montreal, ASME Paper FEDSM2002-31416, 2002*, and *J. Fluids Eng.*, Vol. 126, No. 3, pp. 337–345.

Wygnanski, I, 2004, "The Variables Affecting the Control of Separation by Periodic Excitation.", *2nd AIAA Flow Control Conference*, AIAA 2004-2511, Portland, OR.