A NUMERICAL STUDY OF HOMOGENEOUS TURBULENCE AND PASSIVE SCALAR TRANSPORT IN ROTATING SHEAR FLOW

G. Brethouwer
Department of Mechanics,
KTH
SE-100 44 Stockholm, Sweden
geert@mech.kth.se

Y. Matsuo
Information Technology Center,
Japan Aerospace Exploration Agency
Jindaiji-higashi, Chofu, Tokyo 182-8522, Japan

ABSTRACT

Direct numerical simulations of homogeneous turbulent shear flow subject to spanwise rotation have been carried out. A passive scalar field with an imposed mean gradient was also included in the simulations. The flow reached a state close to the equilibrium structure with a slowly varying turbulence anisotropy and nondimensional shear number SK/ε. Different rotation numbers have been used in the simulations and the rotation either accelerated the growth of kinetic energy or slowed it down. The growth was approximately exponential in a few cases at intermediate shear times. At longer shear times the kinetic energy was growing linearly in most of the cases. The rotation affected considerably the anisotropy of the flow and the velocity correlations. The scalar-velocity fluctuation correlations and the direction of the turbulent scalar flux vector were according to the simulations also strongly influenced by rotation, even as the mechanical to scalar time scale ratio.

INTRODUCTION

Turbulent shear flows subject to system rotation have been investigated by Bech & Andersson (1997), Métais *et al.* (1995) and many others. These studies have shown that rotation affects the stability of the flow and can enhance or weaken the turbulence fluctuations. A geometry which is because of its simplicity very suitable for studying the combined effect of rotation and shear is the uniformly sheared and rotating turbulent flow. Bardina *et al.* (1983) and Salhi & Cambon (1997) have studied this flow by large-eddy simulations (LES) and rapid distortion theory (RDT), respectively. Recently, Brethouwer (2005) performed direct numerical simulations (DNS) of rapidly sheared homogeneous turbulence with spanwise rotation and compared the DNS data with RDT. The Reynolds stresses but also the large- and small-scale turbulence structures were shown to be considerably affected by rotation.

It is commonly assumed that rotating homogeneous shear flows at long shear times approaches a structural equilibrium with a constant turbulence anisotropy and constant nondimensional shear number SK/ε, where S is the shear rate, K and ε the turbulent kinetic energy and its dissipation, respectively. Second-moment closure (SCM) models for instance, predict such an equilibrium structure but the predicted equilibrium structure depends criticly on the model for the pressure-strain correlations and the dissipation. For turbulence model development and validation knowledge about the equilibrium structure of rotating homogeneous shear flow is thus of utmost importance. Nonrotating homogeneous shear flow near equilibrium conditions was investigated experimentally by Tavoularis & Corrsin (1981) and Tavoularis & Karnik (1989). They observed an exponentially growing turbulent kinetic energy which can be understood if a constant value of SK/ε and P/ε, were P is the production of kinetic energy, is assumed. In that case $K \sim \exp(\alpha St)$ with

$$\alpha = \frac{\varepsilon}{SK}\Big(\frac{P}{\varepsilon} - 1\Big). \tag{1}$$

Little is known about the equilibrium structure of rotating homogeneous shear flow because the equilibrium state was not reached in the simulations of rotating homogeneous shear flow by Bardina *et al.* (1983) and Brethouwer (2005) as a result of the short shear time and the high shear rates.

The influence of rotation on scalar transport in turbulent shear flows is also of interest, but investigations on this topic are rather scarce. Scalar transport in turbulent channel flow has been investigated by Nagano & Hattori (2003) and Wu & Kasagi (2004). Brethouwer (2005) studied passive scalar transport in rotating homogeneous shear flow and found a remarkably strong influence of rotation the direction of the turbulent scalar flux vector. The conclusion of this study was that the modelling of the effect of rotation on turbulent scalar transport needs serious attention, but further studies are necessary to support the model development.

Direct numerical simulations of homogeneous shear flow subject to spanwise rotation are performed here. The simulations are set up in such a way that the flow approaches the equilibrium conditions. The aim is to obtain a more complete understanding of the influence of rotation on the equilibrium structure and, at the same time, produce data which are useful for turbulence modelling. The mixing of a passive scalar with an imposed mean gradient is also considered in this flow geometry. The aim in this case is to investigate how rotation affects the transport and dispersion of scalars in turbulent shear flows.

NUMERICAL SIMULATIONS

The flow configuration and the coordinate system are sketched in figure 1. The governing equations are the Navier-

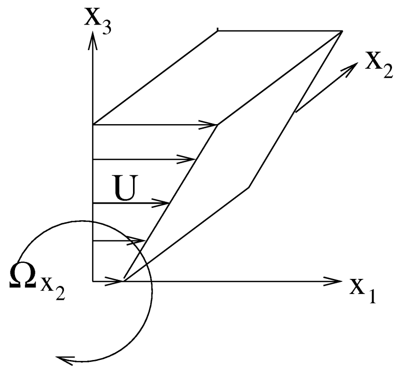

Figure 1: The coordinate system and mean velocity profile

Stokes and the continuity equations for the flow field and the advection-diffusion equation for the passive scalar. The passive scalar field has a linear and constant mean gradient in the transverse x_3-direction. The numerical approach follows Rogallo's (1981) method with a moving computational grid which is periodically remeshed. A pseudo-spectral method is applied to solve the governing equations and a fourth-order Runge-Kutta method is used for the time advancement. Aliasing errors are suppressed by a combination of phase shifting and truncation.

The size of the computational domain is $4\pi \times 3\pi \times 2\pi$ in the streamwise, spanwise and transverse direction, respectively. A grid with $1536 \times 1280 \times 1024$ points is used in all simulations. The initial velocity field was isotropic turbulence. This was obtained by performing a separate simulation of decaying turbulence without shear and rotation. When shear is imposed on the flow, the turbulence length scales growth significantly. To carry out a simulation of homogeneous shear flow whereby the turbulence can evolve freely for a sufficiently long shear time without significant influences of the periodic boundary conditions on the evolution of largest scales, the initial turbulence length scales have to be small. In the simulations the initial longitudinal integral length scale was 0.0636 or smaller. Because of the small initial length scales and the resolution requirements the initial Reynolds number was necessarily rather low. In the simulations the initial $Re_\lambda = u'\lambda/\nu$, were u' is the root-mean-square of the velocity fluctuations and λ the Taylor micro scale, was between was between 29 and 45. The Reynolds number, however, increased significantly during the simulations. The Schmidt number of the passive scalar was $Sc = 0.7$ and the initial passive scalar field was without fluctuations.

RESULTS

In this section the results of the DNS of homogeneous turbulent shear flow with spanwise rotation and a passive scalar field are presented. Several simulations with different rotation numbers $R = 2\Omega/S$, where Ω is the rotation rate of the flow domain, have been carried out.

Flow field

The time development of the turbulent kinetic energy, extracted from the DNS, is shown in figure 2. The kinetic energy is in all cases scaled with its initial value. In correspondence

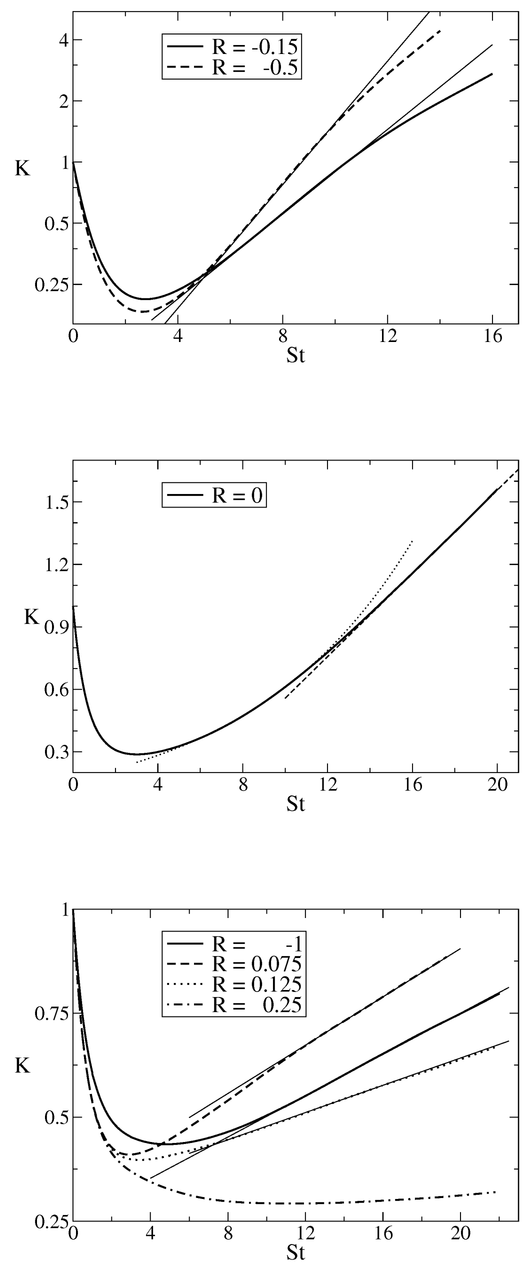

Figure 2: Development of the turbulent kinetic energy, extracted from the DNS. The straight, thin lines in the top and bottom figure are exponential or linear fits to the DNS data. The dotted and dashed lines in the middle figure are exponential and linear fits, respectively

with previous studies (Salhi & Cambon, 1997; Brethouwer, 2005) we classify $R = 0$ (no rotation) and $R = -1$ (zero absolute mean vorticity) as the neutral cases. In figure 2 we see that at $R = 0$, K has approximately an exponential growth for intermediate values of St (here St is the nondimensional shear time). The evolution can be described by $K \sim \exp(\alpha St)$, where $\alpha = 0.13$. This value agrees well with the value obtained by Tavoularis & Corrsin (1981) at about the same nondimensional shear times. At larger St values the growth of K in the DNS is approximately linear, which is rather surprising. The same result has been obtained at other resolutions and different domain sizes, so numerical artefacts seem to have a small influence. One should note, however, that for low growth rates the difference between linear and exponential growth is rather subtle.

For $-1 < R < 0$ rotation destabilises the flow with a maximum destabilisation at $R = -0.5$ according to Salhi & Cambon (1997) and Brethouwer (2005) and this result is reproduced here. A rapid growth of K is observed at $R = -0.5$ and a less rapid growth at $R = -0.15$ but still significantly faster than at $R = 0$. For intermediate St values the growth rate is approximately exponential as can be seen in figure 2, but at larger St values differences appear.

In the other neutral case, $R = -1$, the growth of K is significant whereas in the LES of Bardina *et al.* (1983) K was approximately constant. For $R > 0$ the growth rate of K is slower than at $R = 0$ as we see in figure 2 and thus rotation stabilises the flow. At $R = 0.075$, 0.125 and $R = -1$ the growth of K is approximately linear at larger St values and K is about constant at $R = 0.25$. The latter rotation number seems therefore close to the bifurcation point which separates decaying and growing turbulence.

Figure 3 shows the time development of SK/ε which is the ratio of the turbulence time scale and the time scale of the mean shear. As a result of the remeshing operation high wave number information is lost and therefore some spikes can be seen in the curves. SK/ε does not reach a really con-

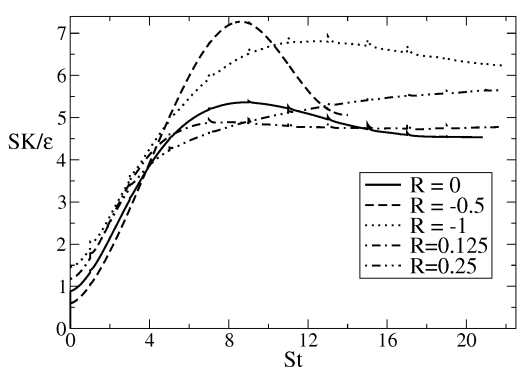

Figure 3: Development of SK/ε

stant equilibrium value at all rotation numbers. Especially for $-1 < R < 0$ when rotation destabilises the flow it is extremely difficult to approach the equilibrium structure because of the rapidly growing turbulent length scales. At $R = 0$, SK/ε approaches a value of about 4.5 which is lower than $SK/\varepsilon \simeq 6$ measured by Tavoularis & Corrsin (1981) but closer to the value observed in the log layer of channel flow which is between 3 and 4. At $R = -1$ and $R > 0$ the equilibrium value of SK/ε appears to be higher than at $R = 0$. For $-1 < R < 0$, SK/ε does not yet approach a constant value, but the value at the end of the simulations is of the same order as at $R = 0$.

An equilibrium value close to or lower than the equilibrium value at $R = 0$ seems therefore plausible.

Figure 4 presents the ratio of the production of kinetic energy P and ε. An equilibrium value of about $P/\varepsilon \simeq 1.8$ is

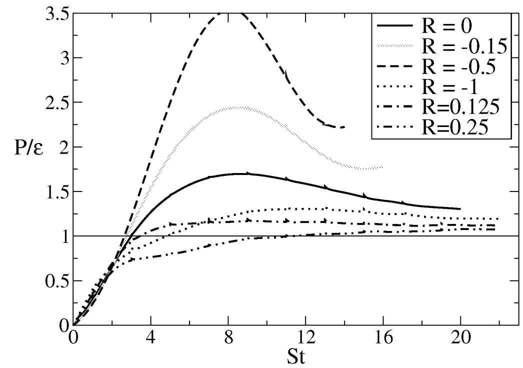

Figure 4: Development of P/ε.

generally assumed for the nonrotating case on basis of the experiments of Tavoularis & Corrsin (1981), although later experiments (Tavoularis & Karnik, 1989) do not rule out a lower value. At $R = 0$ the maximum value of P/ε is about 1.7 at $St = 9$ but then it decreases to a value of about 1.3 at $St = 20$. Tavoularis and Corrsin (1981) performed the measurements for St between 8 and 12.6. It is thus not unlikely that the flow in their experiments was not yet in an equilibrium state and that their estimation $P/\varepsilon \simeq 1.8$ for the equilibrium value is too high. The lower value of P/ε obtained in later experiments (Tavoularis & Karnik, 1989) at larger St values supports this point of view.

At $R = -0.15$ and -0.5, P/ε varies considerably, even at larger St values, and consequently a equilibrium value can not be estimated as we can see in figure 4. The curves level off at the end of the simulations and this is probably caused by turbulence structures which are becoming too large for the computational domain. At $R = -1$ and $R > 0$, P/ε approaches values somewhat lower than at $R = 0$. Taking into consideration that P/ε did not reach yet an equilibrium value at $R = -0.15$ and -0.5 we may thus conclude that the influence of rotation on the equilibrium value of P/ε is presumably quite small. Note also that if the linear growth of K, as we have observed in figure 2, proceeds and if SK/ε keeps a finite, non-zero value, we must conclude that a production-equals-dissipation equilibrium is approached. This has been predicted for nonrotating homogeneous shear flow by Bernard & Wallace (2002).

In figure 5 the time development is presented of the Reynolds stress anisotropy component b_{13}, where $b_{ij} = \overline{u_i u_j}/(2K) - \delta_{ij}/3$. At $R = 0$, b_{13} is slightly less negative at larger St values than in the experiments of Tavoularis & Corrsin (1981) but the correlation $\overline{u_1 u_3}/(u_1' u_3') = -0.45$ at $St = 20$ which is the same as in the experiments. At larger St values b_{13} reaches larger negative values at $R = -0.15$ and -0.5 and smaller negative values in the other cases compared to the case $R = 0$.

In figure 6 the time development of the anisotropy measures b_{11}, b_{22} and b_{33} at $R = 0$ and $R = -1$ are shown. The

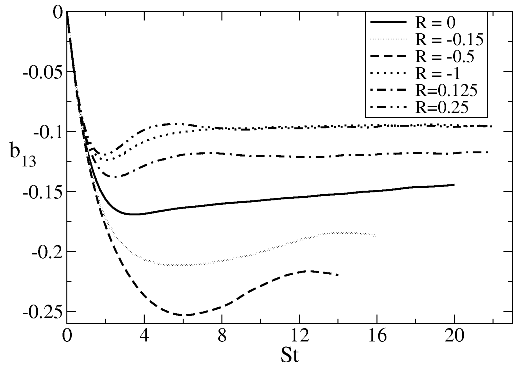

Figure 5: Development of b_{13}.

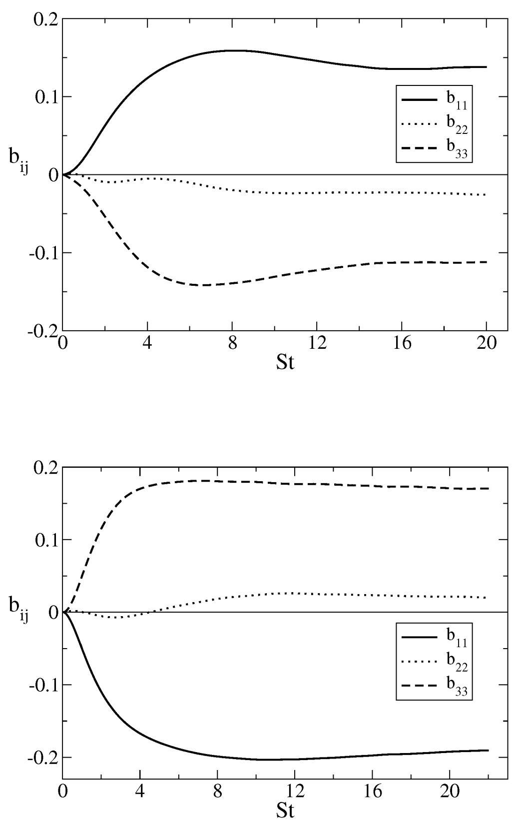

Figure 6: Development of b_{ii} at $R = 0$ (top figure) and $R = -1$ (bottom figure).

anisotropy of the turbulence at $R = 0$ in the DNS is somewhat smaller than in the experiments of Tavoularis & Corrsin (1981). Especially, b_{11} is smaller and b_{22} is larger in the DNS. Simulations with other resolutions and domain sizes gave very similar results. The turbulence is quite anisotropic at $R = -1$ with strong transverse and weak streamwise velocity fluctuations as can be seen in figure 6. The components b_{ii} are almost constant at larger St values signifying that the turbulence has approximately the equilibrium structure.

The DNS data are very useful for validation of turbulence models. As an illustration we consider here the modelling of the slow pressure-strain correlation $\Pi_{ij}^{(s)}$ which is a notorious difficult term. A much used model for $\Pi_{ij}^{(s)}$ is Rotta's model

$$\Pi_{ij}^{s} = -C_1 \varepsilon b_{ij} \tag{2}$$

where C_1 is a constant. Hallbäck *et al.* (1993) noticed that C_1 has a significant dependence on the Reynolds number and proposed a modification of the constant. In figure

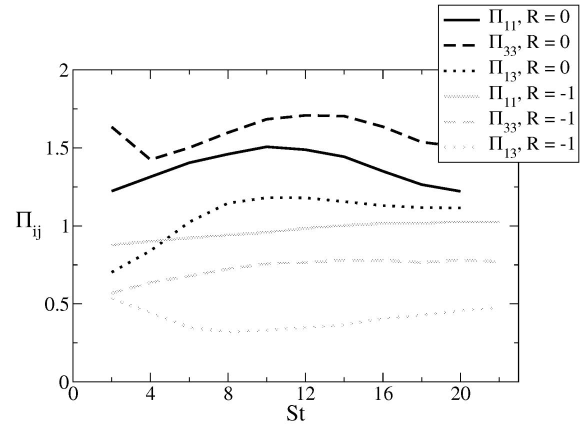

Figure 7: The slow pressure-strain correlations, extracted from DNS, scaled with the prediction of Rotta's model.

7, $-\Pi_{ij}^{(s)}/(C_1 \varepsilon b_{ij})$, i.e. the slow pressure-strain correlation, scaled with the prediction of Rotta's model, is shown. For the constant we use the Reynolds number modification proposed by Hallbäck *et al.* (1993). The data, $\Pi_{ij}^{(s)}$, ε and b_{ij} are all extracted from the DNS at $R = 0$ and -1. If Rotta's model gives a too low prediction of $\Pi_{ij}^{(s)}$ the ratio $-\Pi_{ij}^{(s)}/(C_1 \varepsilon b_{ij}) > 1$ and vise versa. Figure 7 shows that Rotta's model gives a too low prediction of $\Pi_{33}^{(s)}$ at $R = 0$ and a too high prediction of $\Pi_{13}^{(s)}$ and $\Pi_{33}^{(s)}$ at $R = -1$. In the latter case $\Pi_{22}^{(s)}$ is significant but Rotta's model gives a much too low prediction (results are omitted here) because b_{22} is very small as shown in figure 6. Sjögren & Johansson (2000) developed a non-linear model for $\Pi_{ij}^{(s)}$ through the introduction of terms quadratic or higher order in b_{ij}, but this model did not give a significant improvement.

Passive scalar field

In this section the influence of rotation on the turbulent transport in homogeneous shear flow of a passive scalar with a transverse mean gradient is studied.

Figure 8 shows the development of the relative strength of the scalar fluctuations defined as

$$B = \frac{\theta'/G}{q/S} \tag{3}$$

where θ' is the root-mean-square of scalar fluctuations, G the mean scalar gradient and $q^2 = 2K$. B is order one at $R = 0$, -0.5 and 0.25, but at $R = -1$ it approaches a much larger value at larger St values. This means that the scalar fluctuations are very intensive compared to the velocity fluctuations.

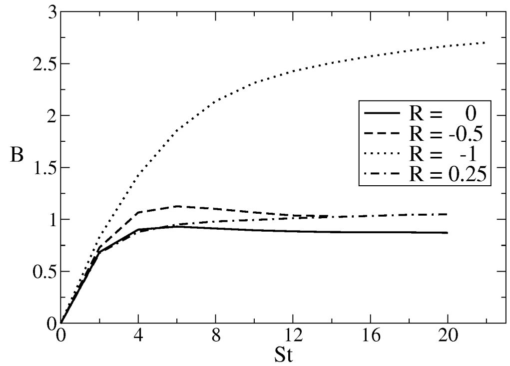

Figure 8: Development of the relative strength of the scalar fluctuations B.

The turbulent scalar flux is affected by rotation as shown by the balance equation

$$\frac{\partial \overline{u_i\theta}}{\partial t} = -\overline{u_i u_3}\, G - \overline{u_3\theta}\, S\, \delta_{i1} + 2\, \epsilon_{ij2}\, \overline{u_j\theta}\, \Omega + \Pi_{\theta i} - \varepsilon_{\theta i}, \quad (4)$$

where $\Pi_{\theta i}$ is the pressure-gradient correlation and $\varepsilon_{\theta i}$ is the viscous and diffusive destruction term. Figure 9 shows the time development of the scalar flux coefficient which is defined as

$$\zeta_i = \overline{u_i\theta}/u_i'\theta' \quad (5)$$

The scalar and velocity fluctuations in the streamwise direction have a significant correlation at $R = 0$, -0.5 and 0.25, which is typical for shear flows (Tavoularis & Corrsin 1981). However, at $R = -1$, $\zeta_1 \simeq 0$. Because the second and third term on the right-hand-side of the balance equation for $\overline{u_1\theta}$ cancel each other at $R = -1$, this result implies that the first production term $\overline{u_1 u_3}G$ is balanced by $\Pi_{\theta 1}$ and $\varepsilon_{\theta 1}$.

Strong correlations between the scalar and the transverse velocity fluctuations are observed at $R = -0.5$ and $R = -1$ and relatively weak correlations for $R > 0$ as shown in figure 9. Because of intense transverse velocity fluctuations at $R = -1$ the transverse scalar flux is very large. Hence, the production rate of scalar variance which is given by $-\overline{u_3\theta}G$ is high and this explains the high relative intensity observed in figure 8.

The scalar flux vector is thus in general not aligned with the mean gradient. It is interesting to see how the direction of the scalar flux vector is influenced by rotation. Figure 10 shows the development of the angle α_θ of the scalar flux vector with the coordinate system which is here defined as

$$\alpha_\theta = \tan^{-1}(\overline{u_3\theta}/\overline{u_1\theta}). \quad (6)$$

The scalar flux is down the mean gradient if $\alpha_\theta = -90^o$. At $R = 0$, $\alpha_\theta = -32^o$ at $St = 20$ which differs somewhat from $\alpha_\theta = -23^o$ measured by Tavoularis & Corrsin (1981) but this is probably a consequence of the lower turbulence anisotropy in the present DNS. The scalar flux vector is slightly more aligned with the flow direction for $R > 0$ and it is more aligned with the mean scalar gradient for $R < 0$. At $R = -1$ the scalar flux is in fact strictly down the mean gradient as we see in figure 10.

The rotation appears to have a considerable influence on the large-scale mixing of a scalar. We now investigate if rotation influences the scalar dissipation as well. An often model

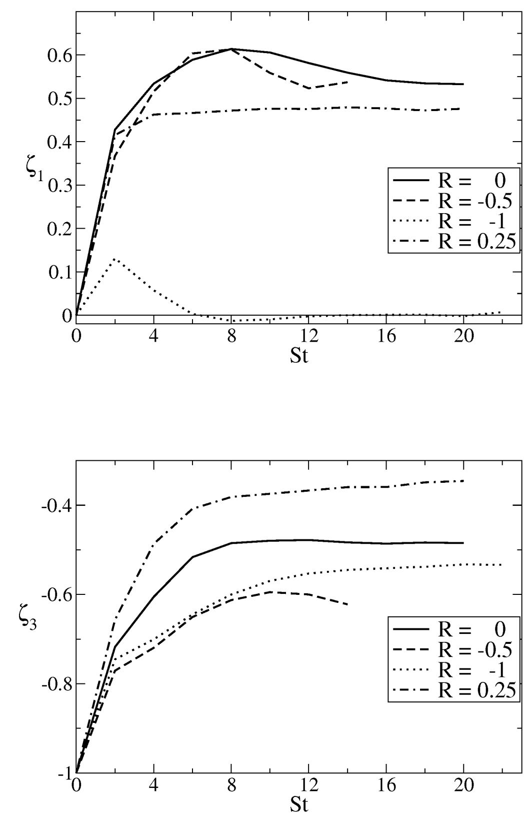

Figure 9: Development of the scalar-velocity fluctuation correlation ζ_1 (top figure) and ζ_3 (bottom figure).

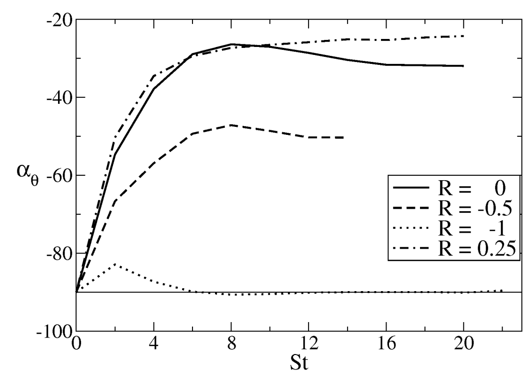

Figure 10: Development of the angle α_θ of the scalar flux vector.

for the dissipation of scalar variance is to assume a constant ratio of the mechanical to scalar time scale

$$\Psi = (2K/\varepsilon)/(\overline{\theta\theta}/\chi) \quad (7)$$

where the scalar dissipation $\chi = \kappa\overline{(\partial\theta/\partial x_j)(\partial\theta/\partial x_j)}$. Figure 11 shows this ratio, extracted from the DNS, for different rotation numbers. Ψ approaches a value of about 2 at $R = 0$

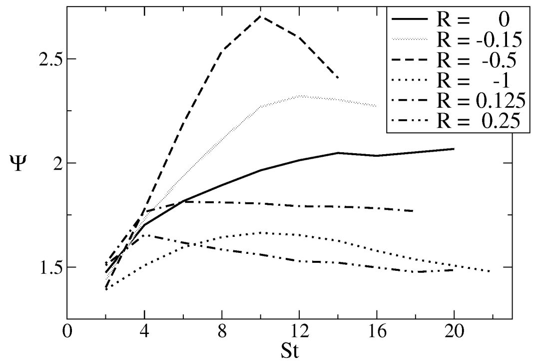

Figure 11: Development of the mechanical to scalar time scale ratio Ψ.

which is a typical value found in case of scalar mixing in shear flows. The rotation appears to have a significant influence on Ψ. For $-1 < R < 0$, Ψ is larger than at $R = 0$ while at $R = -1$ and $R > 0$ it is smaller. More analyses are necessary to find an explanation for this behaviour.

CONCLUSIONS

Large-scale direct numerical simulations have been applied to study the equilibrium structure of uniformly sheared turbulence subject to spanwise rotation. At the same time the transport of a passive scalar with a transverse mean gradient has been investigated in this flow. In the nonrotating case $R = 0$ the growth of the turbulent kinetic energy is exponential at intermediate shear times, in agreement with previous experiments (Tavoularis & Corrsin, 1981). However, at longer shear times the growth seemed to be linear which was an unexpected result. For $-1 < R < 0$ the flow was destabilised by rotation resulting in a rapid exponential growth of the kinetic energy at intermediate shear times, but at longer shear times also in this case deviations from the exponential growth rate were observed. At other rotation numbers, $R = -1$ and $0 < R < 0.25$ the growth of the kinetic energy was linear at longer shear times and stabilisation of the turbulence seemed to occur around $R = 0.25$. The equilibrium values of the anisotropy measures of the turbulence were are also significantly affected by rotation. At $R = -1$ (zero absolute mean vorticity) the transverse velocity fluctuations are much more intense than the streamwise fluctuations in contrast to the situation in many shear flows. Modelling of rotating homogeneous shear flow is still a difficult topic and an example is the slow pressure-strain correlation. Predictions of Rotta's model for the slow pressure-strain correlation have been compared with the DNS and significant differences were observed.

The influence of rotation on the transport and mixing of a passive scalar in homogeneous turbulent shear flow has been studied in detail. In case of a transverse mean scalar gradient previous studies have shown that in turbulent shear flow the streamwise scalar flux is significant leading to a misalignment between the scalar flux vector and the mean gradient (Tavoularis & Corrsin, 1981). In the present DNS we observed in general also a misalignment, but the direction of the scalar flux vector was significantly affected by rotation. At $R \geq 0$ the scalar flux in the streamwise direction was larger than the scalar flux in the transverse direction, but at $R = -1$ the scalar flux was almost exactly down the mean gradient. Small-scale scalar mixing was affected by rotation as well. An illustration of this is the variation of the mechanical to scalar time scale ratio with the rotation number.

Modelling of rotating turbulent flow is of course an important topic, but the present study shows that modelling the effect of rotation on turbulent scalar transport also needs serious attention.

ACKNOWLEDGEMENTS

The first author would like to acknowledge the financial support of the Japan Society for the Promotion of Science (JSPS) and the Swedish Research Council. The computations were performed on the supercomputer of the Japan Aerospace Exploration Agency in Tokyo, Japan.

REFERENCES

Bardina, J., Ferziger, J.H., Reynolds, W.C. 1983 Improved turbulence models based on large-eddy simulation of homogeneous incompressible turbulent flow. *Tech. Rep.* TF-19. Stanford University.

Bech, K.H., Andersson, H.I. 1997 Turbulent plane Couette flow subject to strong system rotation. *J. Fluid Mech.* **347**, pp. 289–314.

Bernard, P.S., Wallace, J.M. 2002 Turbulence Flow: Analysis and Prediction, Wiley.

Brethouwer, G. 2005 The effect of rotation on rapidly sheared homogeneous turbulence and passive scalar transport. Linear theory and DNS. submitted to *J. Fluid Mech.*

Hallbäck, M., Sjögren, T, Johansson, A.V. 1993 Modelling of intercomponent transfer in Reynolds stress closures of homogeneous turbulence. *Turbulent Shear Flows IX* ed. F. Durst, N. Kasagi, B.E. Launder and J.H. Whitelaw. Springer.

Métais, O., Flores, C., Yanase, S., Riley, J.J., Lesieur, M. Rotating free-shear flows. Part 2. Numerical simulations. *J. Fluid Mech.* **293**, pp. 47–80.

Nagano, Y., Hattori, H. 2003 Direct numerical simulation and modelling of spanwise rotating channel flow with heat transfer. *J. Turbulence* **4**, 010

Salhi, A., Cambon, C. 1997 An analysis of rotating shear flow using linear theory and DNS and LES results. *J. Fluid Mech.* **347**, pp. 171–195.

Sjögren, T., Johansson, A.V. 2000 Development and calibration of algebraic nonlinear models for terms in the Reynolds stress transport equations. *Phys. Fluids* **12**, pp. 1554–1572.

Tavoularis, S., Corrsin, S. 1981 Experiments in nearly homogeneous turbulent shear flows with a uniform mean temperature gradient. Part 1. *J. Fluid Mech.* **104**, pp. 311–347.

Tavoularis, S., Karnik, U. 1989 Further experiments on the evolution of turbulent stresses and scales in uniformly sheared turbulence. *J. Fluid Mech.* **204**, pp. 457–478.

Wu, H., Kasagi, N. 2004 Turbulent heat transfer in a channel flow with arbitrary directional system rotation. *Int. J. Heat Mass Transf.* **47**, pp. 4579–4591.