THE EFFECT OF VORTICITY AND STRAIN ON THE EVOLUTION OF MATERIAL LINES IN HOMOGENEOUS TURBULENCE

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ABSTRACT.

The Lagrangian evolution of infinitesimal material lines is investigated experimentally through three dimensional measurements of particle trajectories (3D-PTV) in quasi homogeneous isotropic turbulence with $Re_\lambda = 50$. By postprocessing 3D-PTV data obtained by Lüthi et al. (2005) we access the full set of velocity derivatives $\partial u_i/\partial x_j$ which allows for monitoring the evolution of various turbulent quantities along each particle trajectory. The main goal of the present investigation concerns the physical mechanisms that govern the Lagrangian evolution of material lines l. In particular we will focus on the effect of vorticity and strain, respectively, on the orientation and on the stretching of l.

INTRODUCTION.

In the last decade a number of numerical studies were devoted to the investigation of statistical properties of material lines and vorticity in a turbulent flow field (Drummond and Munch (1990), Girimaji and Pope (1990), Huang (1996) Kida & Goto (2002), among others). It is emphasized in Tsinober (2001) that the Lagrangian evolution of material lines differs from the evolution of vorticity (see equation (1)). It is clear that for each point there exists an infinite number of material elements, but only one single vortex line.

$$\frac{1}{2}\frac{Dl^2}{Dt} = l_i l_j s_{ij} \qquad \frac{1}{2}\frac{D\omega^2}{Dt} = \omega_i\omega_j s_{ij} + \nu\omega_i\nabla^2\omega_i \quad (1)$$

A more qualitative difference is that material lines are passively driven by the field of velocity derivatives, whereas in the case of enstrophy production there exists a complex dynamic coupling between the field of strain and the field of vorticity. We note that s^2 and ω^2 are not only coupled through the field of velocity, which they both uniquely define, but that this is a two way coupling, whereas it is one way for the material line and the field of velocity derivatives. Of particular interest in our context is also the work of Ohkitani (2002) and Tsinober & Gallanti (2003) . The preferential orientation of vortex lines and material line elements with respect to the eigenframe of strain is studied by means of DNS. The important novelty is, that also for the evolution of material lines a diffusive term is added, leading to $dl_i/dt = l_j\partial u_i/\partial x_j + \nu\nabla^2 l_i$. In this particular case, they find that l tend to align with the intermediate principal strain axis, λ_2, in a Navier Stokes Equation velocity field, whereas in a Gaussian velocity field material lines predominantly align with λ_1. The first attempt to address the above issues experimentally was made by Lüthi et al. (2005). This became possible due to the recent development of the 3D-PTV experimental technique which allowed for the estimate of the Lagrangian evolution of the full tensor of velocity derivatives with sufficient accuracy. The Lagrangian approach allows to not only reproduce statistical properties of ω and l but, more importantly, it allows to look at the Lagrangian evolution of the quantities of interest. This approach allowed to, e.g. observe - and to some degree understand - the underlying history of ω and l alignments with respect to an evolving eigenframe of strain (Guala et al., 2005). The main goal was to understand whether the mentioned preferential alignments (l, λ_1) and (ω, λ_2) are driven and maintained by vorticity and/or strain. Particular attention was thus paid to the influence of ω^2 and s^2 on the rotation Ω^2_λ, of the eigenframe of strain (see also Nomura & Post, 1998), as well as on the tilting of vorticity itself Ω^2_ω. Both Ω^2_λ and Ω^2_ω were observed to be directly related to the alignment of vorticity with respect to the eigenframe of strain, e.g. to $\cos(\omega, \lambda_i)$. The preferential alignment between ω and λ_2 and l and λ_1

was justified and found responsible for the different stretching rate of vorticity and material lines (see figure 1). The key role played by the (l, λ_i) and the (ω, λ_i) alignments is emphasized in equation (2) and in the analogue form for $\omega_i \omega_j s_{ij}/\omega^2$.

$$l_i l_j s_{ij}/l^2 = \Lambda_1 \cos^2(\lambda_1, l) + \Lambda_2 \cos^2(\lambda_2, l) + \Lambda_3 \cos^2(\lambda_3, l) \quad (2)$$

Though the influence of the the initial orientation of l (with respect to ω) affects the rate of stretching $\langle l_i l_j s_{ij}/l^2 \rangle$, we found that even for l parallel to ω, $\langle l_i l_j s_{ij}/l^2 \rangle$ is greater than $\langle \omega_i \omega_j s_{ij}/\omega^2 \rangle$. In this context, the tilting Ω_ω^2 of vorticity was investigated following equation .

$$\Omega_\omega^2 = D(\omega_i/\omega)/Dt \cdot D(\omega_i/\omega)/Dt \quad (3)$$

where

$$D(\omega_i/\omega)/Dt = \eta_i^\omega + VT \quad (4)$$

and

$$(\eta^\omega)^2 = \frac{(W^\omega)^2}{\omega^2} - \left\{ \frac{\omega_i \omega_j s_{ij}}{\omega^2} \right\}^2, \qquad W_i^\omega = \omega_j s_{ij} \quad (5)$$

VT stands for viscous terms, and the quantity $(\eta^\omega)^2$ can be interpreted as the inviscid tilting of vorticity. In Guala et al. (2005) it was shown that $\Omega_\omega^2 > \eta_\omega^2 \sim \eta_l^2$ (where $\eta_l^2 = (D(l_i/l)/Dt) \cdot (D(l_i/l)/Dt)$) is the tilting of material lines). We thus inferred that viscosity is of utmost importance in assisting vorticity in its tilting and that it is thus playing a key role in defining the mutual orientation between ω and λ_i (see also Lüthi et al., 2005).

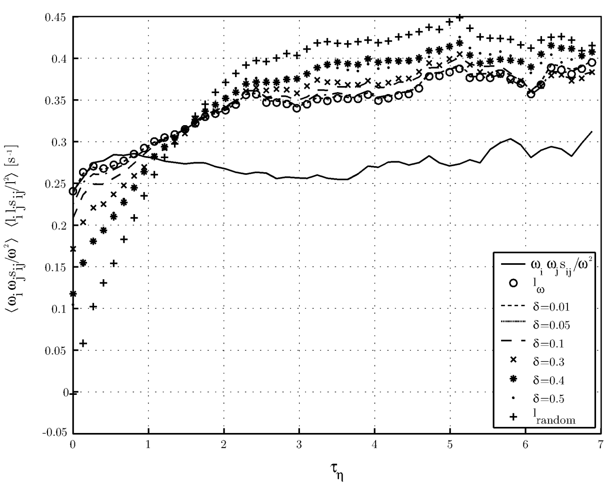

Figure 1: Comparison between the mean Lagrangian evolution of the vortex stretching production rate $\langle \omega_i \omega_j s_{ij}/\omega^2 \rangle$ with that one of infinitesimal material line $\langle l_i l_j s_{ij}/l^2 \rangle$ for different l: those initially exactly aligned with ω (l_ω) and those initially close to ω such that $cos^2(\omega, \mathbf{l}) = 1 - \delta$ for varying δ (left).

Moreover, it was found that not only the tilting of vorticity but also the rotation of the strain eigenframe, can be responsible for the mutual change of orientation between ω and λ_i (and thus also between l and λ_i) - depending on the magnitude of strain and enstrophy. In particular in high strain condition, λ_i was observed to be stable, while ω was tilting away from λ_3 and λ_1 (preventing itself from strong compression and stretching). On the other hand, in high enstrophy regions, the rotation of the eigenframe was found to overcome vorticity tilting. In particular ω was observed to be extremely inactive while laying along λ_2. Both Ω_ω^2 and Ω_λ^2 were significatively more intense compared to the tilting of material lines η_l^2. It was inferred that material lines are substantially stretched along a non persistent λ_1 direction, but it is still questionable how significant is the effect of ω on l.

It is thus clear that the evolution of material elements has to be related to the strong coupling between strain and enstrophy (see also Liberzon et al., 2005). In fact, material elements manifest with their behaviour the effect of turbulence on any other passive vectors and are thus well suitable for investigating mixing and dispersion problems. In this contribution we attempt a numerical investigation on previous experimental results focusing on the behaviour of material lines under the specific action of strain and/or enstrophy separately.

METHOD.

It is known that material elements are on averaged stretched and tilted by the field of velocity derivatives. It has to be noted that s_{ij}, while it is stretching them, is also tilting them. At the same time, l's are also tilted by vorticity (see equation (6), equivalent to $(D(l_i/l)/Dt) \cdot (D(l_i/l)/Dt)$.

$$\eta_l^2 = \frac{(W^l)^2}{l^2} - \left\{ \frac{l_i l_j s_{ij}}{l^2} \right\}^2 + \frac{l_j s_{ij}(\omega \times l)}{l^2} + \frac{(\omega \times l)^2}{4l^2}. \quad (6)$$

The overall behavior of l's is then crucially depending not only on the strain intensity, but also on their orientation with respect to the eigenframe λ_i which is conditioning the stretching rate $l_i l_j s_{ij}/l^2$. Since the orientation of l is governed by both strain and vorticity, it is of utmost importance to single out how strain and vorticity are respectively acting (stretching and tilting) on material lines. This is done by performing three numerical experiments using data obtained in homogeneous turbulent flow (Luethi et al., 2005). Following Monin & Yaglom (1975) the evolution of infinitesimal material lines is governed by equation (7)

$$l(t) = B(t) \cdot l(0) \quad (7)$$

where B evolves according to:

$$\frac{d}{dt} B = h(t) \cdot B(t), \qquad h(t) = (\frac{\partial u_i}{\partial x_j})_t \qquad B(0) = I \quad (8)$$

In the present contribution we compare some statistics, e.g. the preferential orientation and the stretching rates of material elements (denoted as l_u in the following), with the correspondent statistics associated with two distinct families of material lines. Those are the ones derived by integrating along the particle trajectory not the full tensor $h(t) = (\partial u_i/\partial u_j)_t$, but the symmetric part $s_{ij}(t)$ only (it will be denoted as l_s) or the antisymmetric part $r_{ij}(t)$ only (it will be denoted as l_r).

The main point of this approach is to bypass an intrinsic difficulty. No real flows, or simulated one obeying Navier Stokes equation, can lack either enstrophy or strain. The only possible way is then to use passive vectors, as material elements, and let them evolve in a real flows as if either one of the two (ω_i or s_{ij}) is missing.

Statistics can be computed averaging quantities at the same time interval defined in a Lagrangian frame of reference, i.e. $\tau = 0$ for the first point of the trajectory and so on, regardless

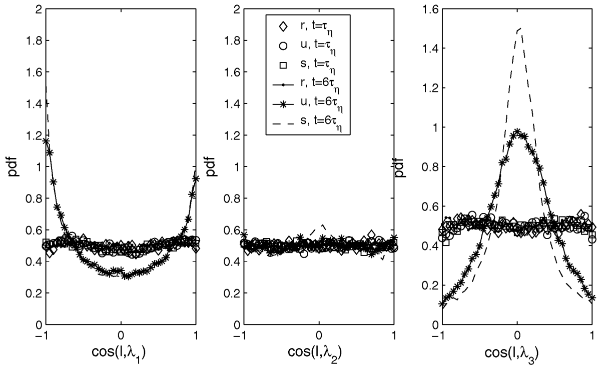

Figure 2: Pdf of the (l , λ_i) alignment for young and mature random l's governed by the full tensor $u_{i,j}$ (u), its symmetric part (s) and its antisymmetric part (r)

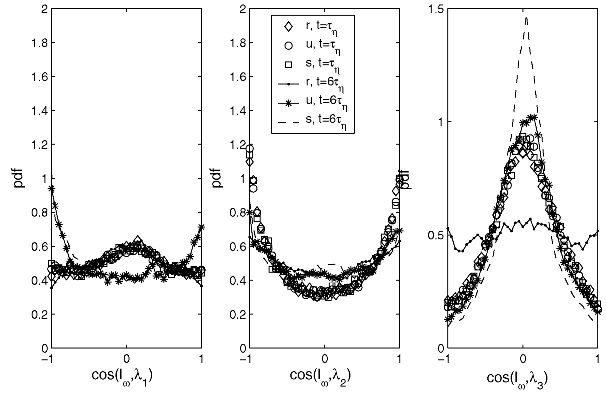

Figure 3: Pdf of the (l , λ_i) alignment for young and mature special l's governed by the full tensor $u_{i,j}$ (u), its symmetric part (s) and its antisymmetric part (r)

of the absolute time (since statistically stationary condition, in eulerian sense, are ensured).

RESULTS.

First of all we investigate the preferential orientation of material lines. It is shown that, consistently with the assumption of initial random orientation for the material lines, at time zero there is no preferential alignment with any of the λ_i. After 6 Kolmogorov times τ_η (mature stage of evolution), original material lines manifest the known tendency to be aligned along λ_1 (and perpendicular to λ_3). While l_r do not show any preferential orientation, l_s were found to be slightly more persistently (in a statistical sense) aligned with λ_1 (figure 2). The latter effect was found to be reduced in the case of material lines l_ω initially almost aligned with vorticity, within a cone of roughly 18°, (figure 3)

We can pose the question on whether the l_s, λ_1 alignment had to be much more probable. However we have to remind that l are stretched along a non persistent direction, which means on one hand that they are continuously tilted towards the perfect alignment but, on the other hand, that they are almost always in delay, with respect to the fast rotation of the strain eigenframe λ_i. In this context, we have to remember that the rotation of λ_i was found to be significantly conditioned on the enstrophy magnitude (Guala et al. 2005), so vorticity, though neglected in the evolution of l, is still playing an important, though indirect, role. The preferential alignment between l_s, and λ_1 affects the stretching rate $l_{si}l_{sj}s_{ij}/l_s^2$. Its Pdf is more skewed towards positive values compared to $l_{ui}l_{uj}s_{ij}/l_u^2$ (figure 5). The same effect is consistently observed in the alignment between l and the stretching vector $W^l = l_j s_{ij}$ (figure 4).

The enhanced stretching of material lines under the effect of strain, can lead to an enhanced deformation of an infinitesimal fluid volume. Following Girimaji & Pope (1990), we can think of a fluid cube with initial side length ratios of 1:1:1 (τ=0).Under the effect of strain this cube may develop into a three dimensional block with (possibly different) side length ratio a:b:c evolving in time according to the evolution of the Cauchy Green tensor $W = BB^T$, with B defined in equation (8). In particular, $a : b : c = w_1^{1/2} : w_2^{1/2} : w_3^{1/2}$, where w_i are the eigenvalue of the W tensor. It has to be noted that at time $\tau = 0$, $B = I$, thus $W = I$ and $w_i = 1$ for $i = 1, 2, 3$. For continuity, due to the conserva-

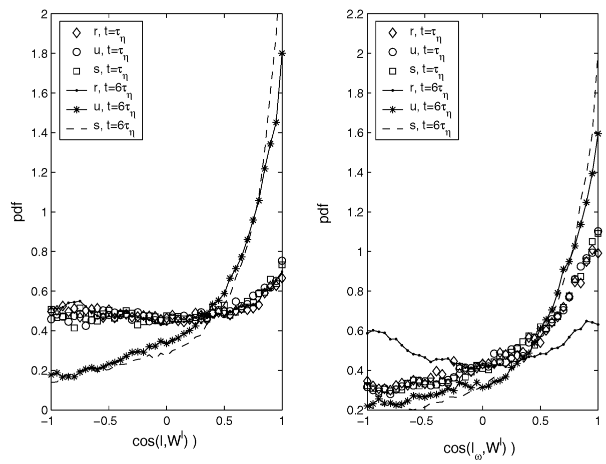

Figure 4: Pdf of the (l , W^l) alignment for young and mature random l's governed by the full tensor $u_{i,j}$ (u), its symmetric part (s) and its antisymmetric part (r)

tion of the initial volume, for any τ it has to be satisfied $w_1(\tau)\cdot w_2(\tau)\cdot w_3(\tau) = w_1(0)\cdot w_2(0)\cdot w_3(0) = 1$. This leads to:

$$\langle ln(w_1)\rangle + \langle ln(w_2)\rangle + \langle ln(w_3)\rangle = 0 \tag{9}$$

The mean lagrangian evolution of the eigenvales w_i is shown in figure 6. The volume expansion, consistently with the stretching of material lines, is also enhanced under the effect of "pure" strain. However, the shape of the s_{ij} tensor, as it is often defined the the ratio $ln(w_1)/ln(w_2)$, was not observed to be affected (not shown). It can be inferred that the enhanced stretching of the material lines does not induce any preferential deformation in the fluid volume. The statistical distribution of cigars and pancakes (Girimaji & Pope, 1990), is thus not affected.

CONCLUSIONS.

The effect of strain and vorticity on the evolution of material lines and material volumes is investigated via a numerical experiment performed on data obtained by Lüthi et al. (2005) through 3DPTV in homogeneous turbulence ($Re_\lambda = 50$).

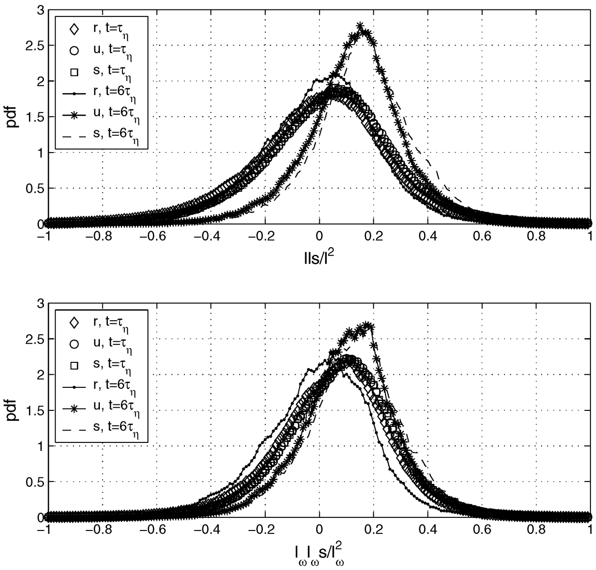

Figure 5: Pdf of $l_i l_j s_{ij}/l^2$ and $l_{\omega i} l_{\omega j} s_{ij}/l_\omega^2$ [1/s] for young and mature l's governed by the full tensor $u_{i,j}$ (u), its symmetric part (s) and its antisymmetric part (r)

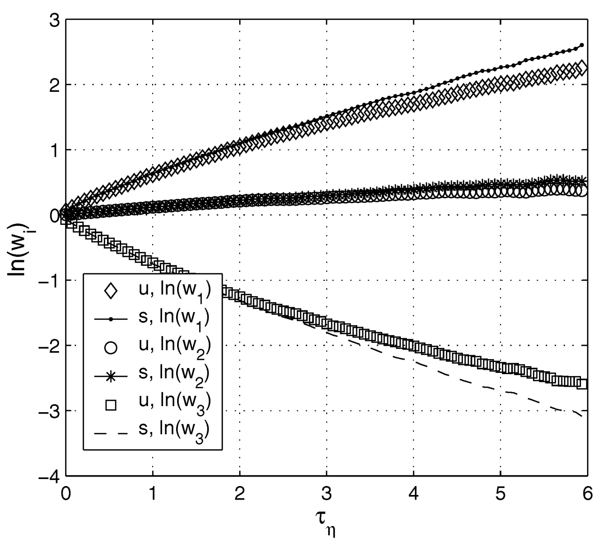

Figure 6: Lagrangian evolution of the mean eigenvalues $\langle w_i \rangle$ of $W = BB^T$ with B derived from the full tensor $u_{i,j}$ (u) or from its symmetric part (s)

Following the approach of Monin and Yaglom (1975), we estimated the Lagrangian evolution of infinitesimal material lines and volumes, in three different condition, namely under the effect of vorticity only, strain only, and (as usual for comparison) of both. The main point of the present investigation is that material lines and volumes exhibit an enhanced stretching when vorticity is "missing". From this we could infer that dispersion and mixing can be enhanced (in a real turbulent flow) in high strain and low enstrophy regions. We must however note that vorticity (or strain) is "missing" only directly in the evolution of material elements. In fact, the strong coupling between vorticity and strain is such that s_{ij}, as well as its eigenframe λ_i is always affected by vorticity. the non persistence of strain is definitely related to the effect of vorticity on the strain field. So, strictly speaking, vorticity is still indirectly affecting also the evolution of material elements.

Acknowledgements

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