CLOSURE STUDY OF THE REYNOLDS NUMBER DEPENDENCY OF THE SCALAR FLUX SPECTRUM

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ABSTRACT

The EDQNM theory is used to study the behavior of the spectrum of the scalar flux in turbulent flow as the Reynolds number increases. In isotropic turbulence, at low and moderate Reynolds number, good agreement is observed with DNS and experimental results. The Reynolds number is varied up to a value of 10^7 and it is shown that at very high Reynolds numbers, the scalar flux spectrum in the inertial range behaves as predicted by the classical dimensional analysis of Lumley (1967) and scales as $K^{-7/3}$. At Reynolds numbers corresponding to laboratory experiments the closure leads to a spectrum closer to K^{-2}, showing that in the experiments the high Reynolds number asymptotic limit is not reached.

The closure is then applied to homogeneous shear flow and the spectra of cross-stream and streamwise scalar flux are investigated. The streamwise scalar flux spectrum is found to scale as $K^{-23/9}$. This result is in agreement with experiments but disagrees with classical dimensional analysis. An alternative asymptotic form is proposed.

INTRODUCTION

The effect of turbulent fluctuations on a mean scalar field is accounted for by a turbulent scalar flux, $\overline{u_i\theta}$. The scalar flux is thus a key quantity in the prediction of the mean scalar field and its understanding and modelling are of major importance for environmental and engineering applications. In order to investigate the contributions of the different turbulent length-scales to the scalar flux, we examine the spectral distribution of scalar flux over wavenumbers, i.e. the scalar flux spectrum. The aim of the present paper is to discuss the behavior of this spectrum as the Reynolds number increases.

Lumley (1964,1967) predicted that in isotropic turbulence with a mean scalar gradient $\Gamma = \partial\overline{\Theta}/\partial x_1$, the spectrum $F_{u\theta}(K)$ of the scalar flux scales as:

$$F_{u\theta}(K) \sim \Gamma\epsilon^{1/3}K^{-7/3} \tag{1}$$

with ϵ the dissipation of kinetic energy and K the wavenumber. This $K^{-7/3}$ behavior was observed in atmospheric measurements by Kader and Yaglom (1989) and Kaimal *et al.* (1972). Atmospheric experiments are however complicated by statistically non-stationary and non-homogeneous effects and recent results show a spectral slope closer to -2 (Su *et al.*, 2004).

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In more controled laboratory experiments, such as the wind-tunnel studies of Mydlarski and Warhaft (1998), the slope of the spectrum of the scalar flux is found to change with the Taylor-scale Reynolds number R_λ. As R_λ increases, the spectrum is becoming steeper and at $R_\lambda = 582$ a value close to -2 is found for the spectral exponent. In a recent work (Bos *et al.*, 2004) we showed that the K^{-2} scaling can be compatible with dimensional analysis and that it can also be found using LES.

In the present paper, we use the Eddy-Damped Quasi-Normal Markovianized (EDQNM) closure (Orszag, 1970), extended to the scalar flux spectrum by Herr, Wang and Collins (1996) (see also Ulitsky and Collins (2000)), to study the behavior of the scalar flux spectrum as R_λ varies. A range of R_λ between 30 and 10^7 is covered, so that the gap between the moderate Reynolds number experiments of Mydlarski and Warhaft (1998) and the high R_λ atmospheric measurements is bridged. Values as high as 10^7 can not even be observed in the atmosphere of our planet, but allow to check the validity of the scaling law in the high R_λ asymptotic limit.

Subsequently the EDQNM theory is applied to homogeneous shear flow with a mean scalar gradient perpendicular to the mean flow direction. The spectra of the vertical and horizontal scalar flux are examined and a non-classical inertial range behavior of the horizontal scalar flux is observed.

All through the paper the Prandtl number is supposed to be of order unity.

EDQNM RESULTS FOR ISOTROPIC TURBULENCE WITH A MEAN SCALAR GRADIENT

The equation for the scalar flux spectrum in isotropic turbulence with a mean scalar gradient is:

$$\frac{\partial}{\partial t}F_{u\theta}(K) + (\nu+\alpha)K^2F_{u\theta}(K) = P(K) + T^{NL}_{u\theta}(K) + \Pi(K) \tag{2}$$

in which the second term on the left-hand side is the molecular destruction of scalar flux, $P(K)$ is the production of scalar flux by the velocity field, $T^{NL}_{u\theta}(K)$ is the non linear transfer term and $\Pi(K)$ stands for the pressure term. These last two terms are closed using the EDQNM theory. For the two constants that appear in the closure, values compatible with the Lagrangian History Direct Interaction Approximation (LHDIA, Kraichnan (1965)) theory are used. They differ from the ones proposed in Ulitsky and Collins (2000). The energy spectrum $E(K)$ is also predicted by the EDQNM theory. It has to be pointed out that all quantities in (2) depend only on the wavenumber K and not on the wavevector $\boldsymbol{K}$. This property stems from the fact that in the case of a fluctuating scalar field produced by the interaction of an isotropic turbulence with a mean scalar field, there is an exact relation between the 3D spectrum $\mathcal{F}_{i\theta}(\boldsymbol{K})$ and its integral over a sphere with radius K (Herr *et al.*, 1996):

$$\mathcal{F}_{u\theta}(\boldsymbol{K}) \sim (1-\mu^2)F_{u\theta}(K) \tag{3}$$

with μ the cosine of the angle between the scalar gradient axis and the wavevector.

Investigation of the different terms

In this section an analysis of the contributions to the scalar flux equation is performed. The production and molecular destruction spectra are given by:

$$\begin{aligned} P(K) &= \frac{2}{3}\Gamma E(K) \\ V(K) &= (\nu+\alpha)K^2 F_{u\theta}(K)dK \end{aligned} \tag{4}$$

The non-linear transfer is obtained by spherically integrating the triple correlation term:

$$T^{NL}_{u\theta}(K) = \int_{\Sigma_K} \Big[iK_j \Big(FT_{/\boldsymbol{r}} \Big(\overline{\theta(\boldsymbol{x})u_j(\boldsymbol{x})u_3(\boldsymbol{x}+\boldsymbol{r})} - \overline{\theta(\boldsymbol{x})u_j(\boldsymbol{x}+\boldsymbol{r})u_3(\boldsymbol{x}+\boldsymbol{r})} \Big) \Big) \Big] dA(\boldsymbol{K}) \tag{5}$$

in which $FT_{/\boldsymbol{r}}$ denotes a Fourier transform with respect to the separation vector $\boldsymbol{r}$.

It can easily be seen that this term vanishes in the one-point limit. The spectrum of this term integrated from wavenumber zero to infinity is thus zero and the transfer is conservative. The pressure term,

$$\Pi(K) = \int_{\Sigma_K} FT_{/\boldsymbol{r}} \left(\overline{\frac{1}{\rho}\theta(\boldsymbol{x})\frac{\partial}{\partial x_3}p(\boldsymbol{x}+\boldsymbol{r})} \right) dA(\boldsymbol{K}) \tag{6}$$

on the contrary has an integral which is not zero: it appears to be a destruction term. $T^{NL}_{u\theta}(K)$ and $\Pi(K)$ are both functions of the triple correlation terms. The closed expressions for these two terms are obtained with the EDQNM theory. They are not recalled here since their full length expressions can be found in the paper of Herr *et al.* (1996). In figure 1 we show the balance of the terms in equation (2) for two different Reynolds numbers. In the figure, it can be observed that $F_{u\theta}(K)$ is mainly produced at large scale by the mean gradient term and that it is destroyed at smaller scales by both pressure and molecular effects at small R_λ, and by pressure effects only at high R_λ. The conservative role played by the transfer term $T^{NL}_{u\theta}(K)$ also appears in the figure.

The spectral slope of the scalar flux in the inertial range

In Bos *et al.* (2004) we showed how dimensional analysis allows for an inertial range slope of the scalar flux spectrum varying from $-5/3$ to $-7/3$, depending on the behaviour of the spectral flux. The behaviour of the slope of the spectrum in the inertial range will now be investigated by EDQNM calculations. In figure 2 we show spectra for Reynolds numbers in the range $100 < R_\lambda < 10^7$.

Figure 3 illustrates the dependency of the slope on the Reynolds number. The results do not leave any doubt about

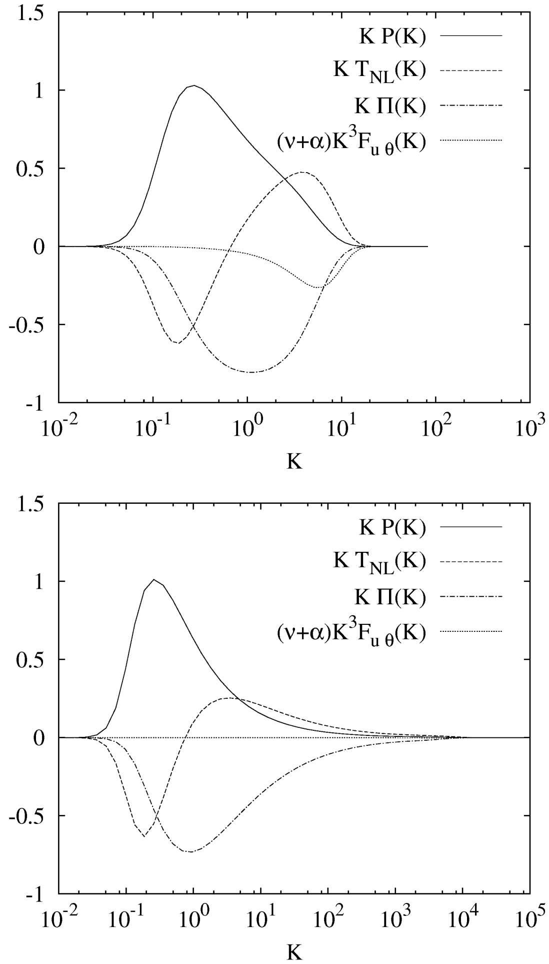

Figure 1: Spectral balance between the production, non-linear transfer, pressure and viscous destruction of scalar flux. Top: $R_\lambda = 100$, bottom: $R_\lambda = 10000$

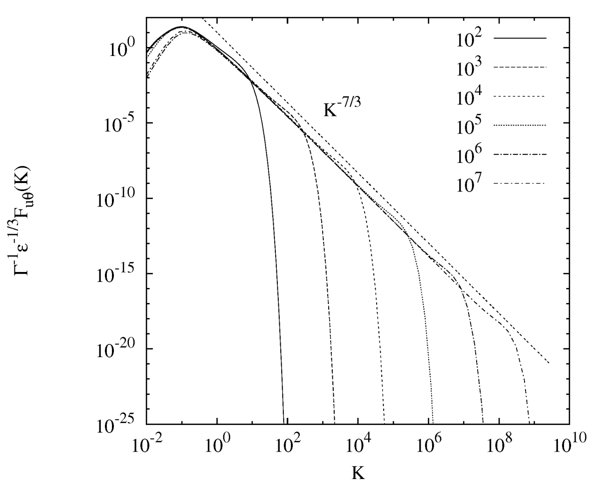

Figure 2: Spectra of the scalar flux spectra for Reynolds numbers in the range $100 < R_\lambda < 10^7$.

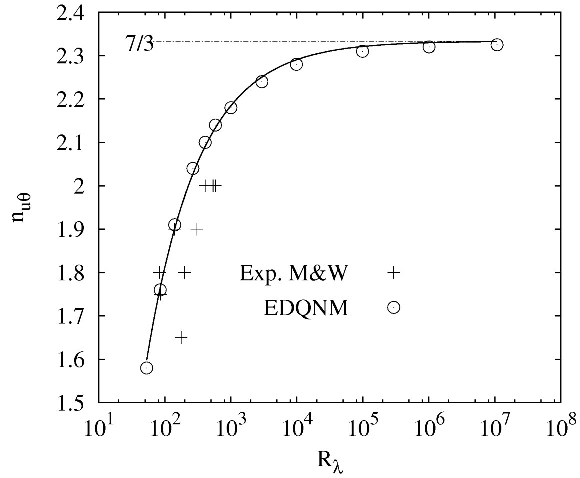

Figure 3: The slope of the inertial range of the velocity-scalar cross spectrum as a function of R_λ. Present EDQNM results compared to the experiments of Mydlarski and Warhaft (1998)

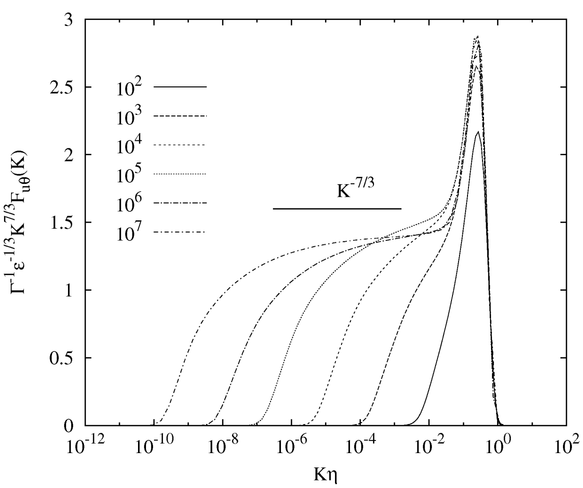

Figure 4: Scalar flux spectra for Reynolds numbers in the range $100 < R_\lambda < 10^7$

the asymptotic inertial range slope. The closure shows a tendency towards a $-7/3$ slope. It also shows that this value is only approached for very high values of the Reynolds number ($R_\lambda = 10^4$ yields a -2.27 slope). It can therefore be argued that a clear $K^{-7/3}$ behaviour will not easily be observed on earth, atmospheric experiments reaching R_λ up to 10^4. As in Mydlarski and Warhaft (1996) for the velocity spectra we try to fit a power law to the results. The empirical relation $n_{u\theta} = 7/3(1 - 2.73R_\lambda^{-0.54})$ describes the data pretty well.

It is furthermore shown that the experimental results of Mydlarski and Warhaft (1998) are in reasonable agreement with the calculations. From those results it can be concluded that a $-7/3$ slope will not be observed in DNS of decaying isotropic turbulence with a mean scalar gradient in the near future or even in wind-tunnel experiments, the Reynolds number being too low in both cases.

In figure 4 compensated spectra are plotted for Reynolds numbers in the range $100 < R_\lambda < 10^7$. The prefactor $C_{u\theta}$ in

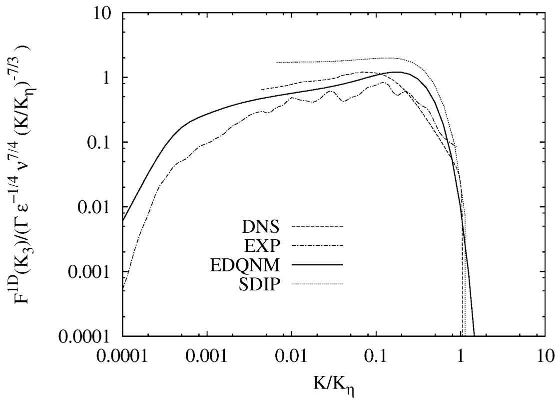

Figure 5: Comparison of one dimensional scalar flux spectra from DNS, SDIP (O'Gorman and Pullin (2005), and experiments (Mydlarski and Warhaft (1998)) with EDQNM results.

the scaling relation:

$$F_{u\theta}(K) = C_{u\theta}\Gamma\epsilon^{1/3}K^{-7/3}, \tag{7}$$

appears to be of order unity. It is found $C_{u\theta} \approx 1.5$.

In figure 5 the present EDQNM results are compared with experimental data of Mydlarski and Warhaft (1998) and DNS and SDIP results of O'Gorman and Pullin (2005). The spectra are one-dimensional spectra. An exact relation between one-dimensional and spherically averaged spectra exists in the case of isotropic turbulence and scalar fluctuations created by a uniform mean scalar gradient (see O'Gorman and Pullin (2003, 2005)). It reads:

$$F^{1D}(K_1) = \frac{3}{4}\int_{K_1}^{\infty} \frac{K^2 + K_1^2}{K^3} F_{u\theta}(K)dK \tag{8}$$

SDIP stands for sparse direct inter-interaction perturbation and corresponds to a variant of the lagrangian direct interaction approximation of Kraichnan (1965). The SDIP result is given only in the asymptotically high Reynolds number limit. It yields an overestimation of the constant $C_{u\theta}$ as explained in O'Gorman and Pullin (2005). The spectrum calculated with EDQNM theory is situated in between the DNS and the experimental results.

The molecular dissipation of scalar flux

It was already noted in figure 1 that, when the Reynolds number increases, the viscous dissipation becomes small compared to the production term. We call $\Gamma\mathcal{U}^2$ the integral value of $P(K)$, $\epsilon_{u\theta}$ the integral value of $V(K)$. The dependence of the ratio, $\epsilon_{u\theta}/\Gamma\mathcal{U}^2$ has been studied in the literature. Mydlarski (2003) found a decrease proportional to $R_\lambda^{-1.2}$ and in the DNS of Overholt and Pope (1996) a $R_\lambda^{-0.77}$ scaling is observed. In figure 6 their observations are compared with the results of the EDQNM calculations. The closure is applied to a range of R_λ much wider than obtainable in DNS or wind-tunnel experiments. It can be observed that there is good agreement between the DNS of Overholt and Pope (1996) and the EDQNM calculations at low R_λ, where the $R_\lambda^{-0.77}$ scaling is found. At high R_λ, the EDQNM results scale as R_λ^{-1}. This R_λ^{-1} dependence can be analytically predicted assuming

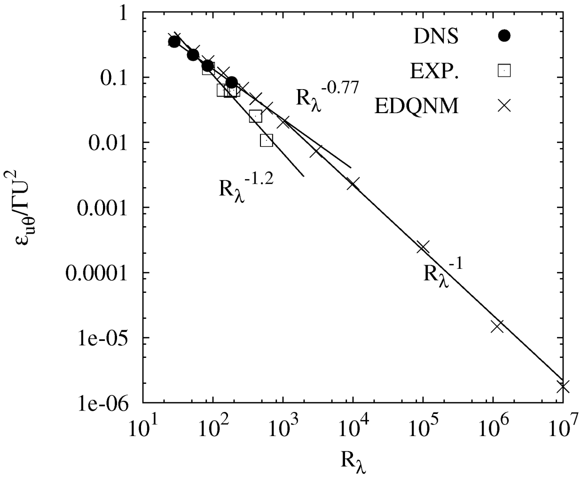

Figure 6: The ratio of molecular destruction to production of $\overline{u\theta}$ as a function of R_λ, compared to the results of the DNS of Overholt and Pope (1996) and the values of Mydlarski (2003).

Lumley's scaling (equation 7) for $F_{u\theta}(K)$. Substituting (7) in the expression for the molecular dissipation of scalar flux, one obtains:

$$\epsilon_{u\theta} = (\nu+\alpha)\int^{K_\eta} K^2\left[C_{u\theta}\Gamma\epsilon^{1/3}K^{-7/3}\right]dK \quad (9)$$

ignoring the lower bound of the integral by assuming a very high R_λ. With the expressions for the Kolmogorov scale and R_λ:

$$K_\eta = \left(\frac{2}{3C_K}\right)^{3/4}\frac{\epsilon^{1/4}}{\nu^{3/4}}, \quad R_\lambda = \sqrt{15\frac{\mathcal{U}^4}{\nu\epsilon}}$$

it is immediately found that:

$$\epsilon_{u\theta}/\Gamma\mathcal{U}^2 \sim R_\lambda^{-1}. \quad (10)$$

In the intermediate range of R_λ, the $R_\lambda^{-1.2}$ scaling found in the experiment of Mydlarski (2003) is not found with the EDQNM closure. It has to be pointed out that Mydlarski only measures one component of the dissipation.

HOMOGENEOUS SHEAR FLOW WITH A MEAN SCALAR GRADIENT

We consider homogeneous shear flow, $S = \partial\overline{U_1}/\partial x_3$, with a uniform scalar gradient $\Gamma = \partial\overline{\Theta}/\partial x_3$. Due to the presence of shear the scalar flux has now two components: the vertical flux $\overline{w\theta}$, parallel to the scalar gradient, and the horizontal flux $\overline{u\theta}$, parallel to the mean flow direction. The equation for the scalar flux spectrum is:

$$\begin{aligned}\left[\frac{\partial}{\partial t}+(\nu+\alpha)K^2\right]\mathcal{F}_i + \frac{\partial\overline{U}_i}{\partial x_j}\mathcal{F}_j + \frac{\partial\overline{\Theta}}{\partial x_j}\Phi_{ij} = \\ +\frac{\partial\overline{U}_l}{\partial x_n}\frac{\partial K_l\mathcal{F}_i}{\partial K_n} + 2\frac{\partial\overline{U}_n}{\partial x_j}\frac{K_iK_n}{K^2}\mathcal{F}_j \\ -iK_n\left(T_{\theta in}-T^*_{\theta in}\right) + i\frac{K_iK_nK_j}{K^2}T_{\theta jn}\end{aligned} \quad (11)$$

in which Φ_{ij} is the spectral tensor, associated with the two-point double velocity correlations, and $T_{\theta in}$ corresponds to the two point triple correlations. $\mathcal{F}_j$ and Φ_{ij} are functions of the wavevector and time. In the presence of shear (3) does not hold anymore; nor Φ_{ij} can be expressed exactly as a function of the wavenumber only. A full EDQNM approach of the problem would then require to build and numerically integrate a wavevector dependent closed set of equations. In order to simplify the numerical task that would result of this complete approach, we integrate the equation over spherical shells with radius K to obtain the variable:

$$F_{u_i\theta}(K,t) = \iint_{\Sigma k}\mathcal{F}_i(\mathbf{K},t)dA(\mathbf{K}) \quad (12)$$

and for the spectral tensor as proposed by Cambon *et al.* (1981):

$$\varphi_{ij}(K,t) = \iint_{\Sigma k}\Phi_{ij}(\mathbf{K},t)dA(K) \quad (13)$$

The equation then becomes:

$$\begin{aligned}\left[\frac{\partial}{\partial t}+(\nu+\alpha)K^2\right]F_{u_i\theta}(K) + \frac{\partial\overline{U}_i}{\partial x_j}F_{u_j\theta}(K) \\ +\frac{\partial\Theta}{\partial x_j}\varphi_{ij}(K) = T^L_{i\theta}(K) + \Pi^L_{i\theta}(K) \\ +T^{NL}_{i\theta}(K) + \Pi^{NL}_{i\theta}(K)\end{aligned} \quad (14)$$

The model for $\varphi_{ij}(K,t)$ can be found in Touil (2002). As in Cambon *et al.* (1981), we introduce models based on isotropic tensorial functions (see also Eringen (1971) or Schiestel (1993)) to express the rapid pressure term $\Pi^L_{i\theta}$ and linear transfer $T^L_{i\theta}$. The non-linear transfer and the slow pressure term are treated with the EDQNM model as used above. Even though the closure was derived for isotropic turbulence we will use it here in the case of an anisotropic velocity field. The approach is not rigorous and has to be seen as an approximation.

The isotropic tensorial functions that represent the linear transfer and rapid pressure in our approximation introduce one unknown constant A. To calibrate our model, A was varied and the results of (14) were compared to the experimental results of Tavoularis and Corrsin (1981) and the DNS results of Rogers *et al.* (1986). The value of A that yielded the best agreement for the turbulent Prandtl number and the horizontal to vertical scalar flux ratio, $\overline{u\theta}/\overline{w\theta}$, was retained.

In figure 7 the slopes of the vertical and horizontal scalar flux spectra, $n_{w\theta}$ and $n_{u\theta}$ respectively, are plotted as functions of the Reynolds number. These slopes were evaluated at two different times, corresponding to $St = 0.5$ and $St = 12$. Although a large difference exists between the values found at low R_λ, it is observed that the influence of time decreases when increasing the Reynolds number.

The exponent of the vertical scalar flux spectrum tends to $n_{w\theta} = 7/3$ as in the case of isotropic turbulence with a mean scalar gradient: the asymptotic behavior of this spectrum is not affected by shear. In fig. 8 the compensated spectra are shown. The exponent of the horizontal spectrum, $n_{u\theta}$ (fig. 7 bottom) is found to tend a value larger than 7/3. However the high Reynolds number asymptote seems to be smaller than the value $n_{u\theta} = 3$ proposed by Wyngaard and Coté (1972): $n_{u\theta} = 23/9$ appears a more plausible value. This can also be observed in figure 9, where the spectra compensated by $K^{23/9}$ are plotted. It is interesting to point out that measurements

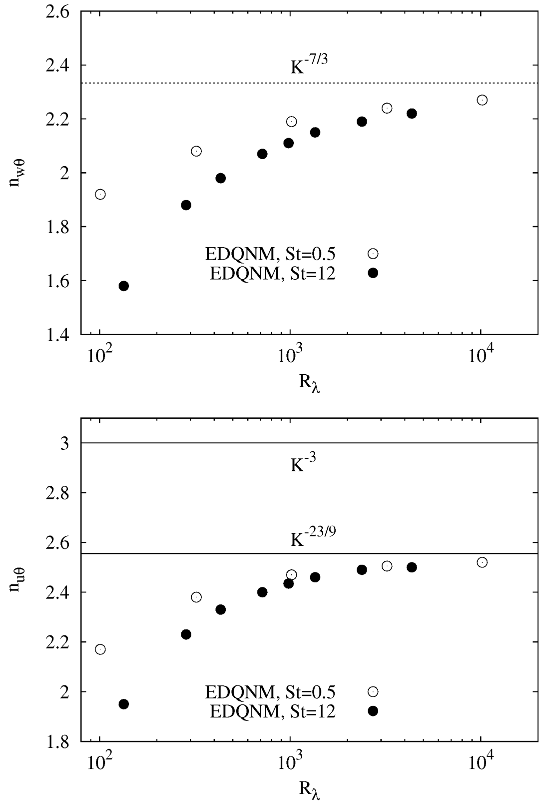

Figure 7: Inertial range slopes of the scalar flux spectra as a function of the Reynolds number. Top: vertical scalar flux. Bottom: horizontal scalar flux.

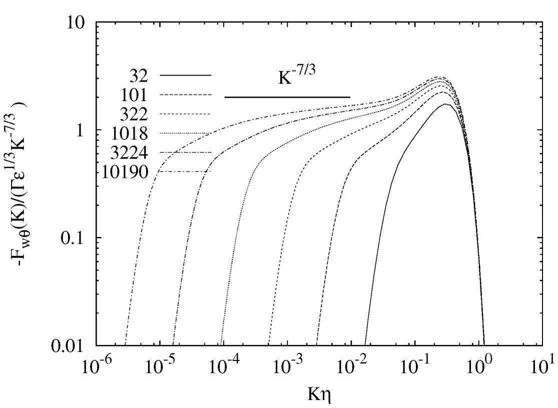

Figure 8: Compensated vertical heatflux spectra for $32 < R_\lambda < 10^4$ at $St = 0.5$.

in the atmosphere have found values values close to 2.5 (Wyngaard and Coté (1972), Caughey (1977), Kader and Yaglom (1989)) and that $23/9 \approx 2.555$.

Dimensional analysis based on the quantities S, ϵ and K provides the following expression for the spectrum:

$$F_{u\theta}(K) \sim \Gamma S^{\alpha} \epsilon^{\frac{1-\alpha}{3}} K^{-\frac{7+2\alpha}{3}} \tag{15}$$

This expression is linear in Γ as it has to be to reflect the linearity of the scalar equation. Linearity in S is not mandatory since the Navier Stokes equation is not linear; if linearity in

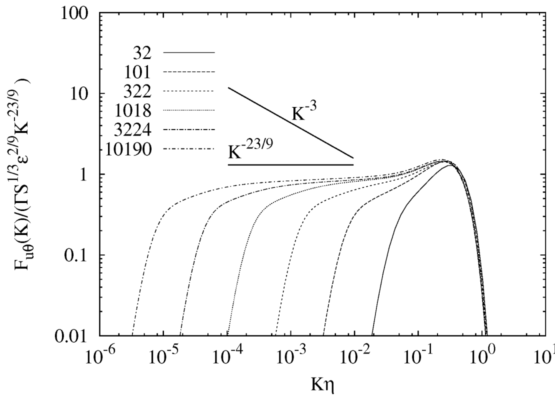

Figure 9: The $K^{-23/9}$ scaling (21) for the horizontal heatflux spectrum tested in the range $32 < R_\lambda < 10^4$ at $St = 0.5$.

S is assumed, (15) reduces to the Wyngaard and Coté (1972) formulation:

$$F_{u\theta}(K) \sim \Gamma S K^{-3}, \tag{16}$$

This formulation can also be found by assuming

$$F_{u\theta}(K) \sim \frac{\varphi_{uw}(K)}{E(K)} F_{w\theta}(K) \tag{17}$$

and using (1) to express $F_{w\theta}(K)$ and a classical expression for the $\varphi_{uw}(K)$ spectrum:

$$\varphi_{uw}(K) \sim S\epsilon(K)^{1/3} K^{-7/3}. \tag{18}$$

If one uses a tensorial extension of (1):

$$F_{i\theta}(K) \sim \frac{\partial \overline{\Theta}}{\partial x_j} \epsilon_{ij}(K)^{1/3} K^{-7/3}, \tag{19}$$

and expresses $\epsilon_{uw}(K)$ by arguments similar to the ones leading to (18):

$$\epsilon_{uw}(K) \sim S\epsilon^{2/3} K^{-2/3} \tag{20}$$

it is found:

$$F_{u\theta}(K) \sim \Gamma S^{1/3} \epsilon^{2/9} K^{-23/9}, \tag{21}$$

a scaling which is in good agreement with the present EDQNM results, figure 9, and the results of atmospherical measurements leading to $n_{u\theta} \sim 2.5$.

CONCLUSION

The EDQNM theory was used to study the Reynolds number dependency of the scalar flux spectrum in both isotropic turbulence and homogeneous shear flow. In isotropic turbulence, the asymptotic inertial range behaviour is in agreement with classical predictions and a $K^{-7/3}$ scaling is found. This scaling is however only found for very high Reynolds numbers and at Reynolds numbers corresponding to laboratory experiments, the spectral exponent is found to be closer to -2 as observed in the experiments of Mydlarski and Warhaft (1998).

The horizontal scalar flux spectrum in homogeneous shear flow is shown to behave differently from classical predictions. An asymptotic $K^{-23/9}$ behavior is observed in agreement with experimental observations. An asymptotic analytical form compatible with the observations is proposed.

This paper illustrates the role that two-point closures can still play in turbulence research. At low Reynolds number their results agree with DNS. Their low computational cost allows to perform calculations at very high Reynolds numbers where dimensional analysis at asymptotically high R_λ can be tested. At intermediate values, good agreement is observed with laboratory experiments and the gap between wind-tunnel measurements and atmospheric measurements can be bridged.

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