EFFECT OF COLLISIONS AND FEEDBACK IN AN IDEALIZED ANNULAR FLOWS

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ABSTRACT

Droplet behavior in a vertical annular flow is represented as the result of an array of point sources of spheres at the walls. The spheres mix in the turbulent flow and eventually deposit. A fully-developed condition can be reached when the rate of injection equals the rate of deposition. The deposition constant is found to be approximately equal to the root-mean-square of the wall-normal particle velocity fluctuations at a location just outside the viscous wall layer. It decreases dramatically with increasing concentrations at solid fractions as low as 4 $\times$ 10^{-4}. The main cause of this appears to be a damping of fluid turbulence rather than particle collisions. By representing the feedback effect of particles with a point force method, the deposition constant is found to decrease at solid volume fractions as low as $\alpha = 10^{-4}$. In agreement with annular flow measurements, it is calculated to vary as α^{-1} at large α. An interesting consequence of this, demonstrated in the calculations, is that a stationary state cannot be reached above a critical rate of injection from the wall sources.

INTRODUCTION

The annular pattern for gas-liquid flows is dominant in many applications. Part of the liquid flows as a film along the wall and part is entrained as drops in a high speed gas flow. A liquid exchange exists whereby the film is atomized and drops are deposited on the film. Usual practice is to assume that the rate of deposition varies linearly with the concentration of drops in the gas, and that the entrainment depends on the relative rates of atomization and deposition.

This paper addresses three unsolved problems which are of central importance to the analysis of this system: (1) At what concentration does the linear rate law for deposition break down? (2) How does one model deposition by turbulence? (3) How is the deposition rate law defined at large concentrations? The study is largely motivated by the observation that the deposition constant, k_D, decreases with concentrations and the suggestion that, at large concentrations, it varies inversely with concentration (Andreussi, 1983; Schadel et al., 1990). This result has practical consequences in that it suggests that at large liquid flows all of the liquid will be in the form of drops, except for a wall film that is too thin to atomize.

COMPUTATIONAL APPROACH

Outline of the calculations

An idealized version of an annular flow in a rectangular channel is considered. The Re_τ, defined with the friction velocity in the absence of particles, v_0^*, and the half-height of the channel, H, is 150. Cartesian coordinates x_1, x_2 and x_3 are assigned to the streamwise, wall-normal and spanwise directions. The channel walls are located at $x_2 = 0$ and $x_2 = 2H$.

Droplets are represented by solid spheres with a diameter of d_p. They are injected from $x_2 = d_p / 2$ with a velocity of (15 v_0^*, v_0^*, 0) and a rate per unit area of R_{Ab}. They are injected from $x_2 = 2\ H - d_p\ /\ 2$ with a velocity of (15 v_0^*, $-\ v_0^*$, 0) and a rate per unit area of R_{At}. The calculation is carried out until a fully-developed stationary concentration field, at which the rates of atomization and deposition are the same at both walls and the wall-normal particle mean velocities are zero at all x_2, is obtained.

Gravity is assumed to be zero, so the results can be best applied to vertical flows. Then, for fully-developed flows, R_{Ab} equals the R_{At}. With the assumption of dilute flows of particles which are much heavier than the gas, lift forces and the influences of particles on the gas turbulence are ignored. The velocity of a particle is described by the following equation:

$$\frac{dV_i}{dt} = -\frac{3\rho_f C_D}{4 d_p \rho_p} \left| \boldsymbol{V} - \boldsymbol{U} \right| (V_i - U_i) = f_{di} \qquad (1)$$

where V_i is the velocity of the particle, U_i is the gas velocity seen by the particle, ρ_p is the density of the particle, ρ_f is the density of the gas, and C_D is the drag coefficient (Mito and Hanratty, 2004). We represent the fluid turbulence both by a DNS (Mito and Hanratty, 2003) and by a stochastic method (when feedback is not considered).

A modified Langevin equation (Iliopoulos et al., 2003) is used to provide a stochastic representation of fluid velocity fluctuations seen by the particle. Jointly Gaussian random variables which correctly give all of the second moments of the fluid velocity fluctuations are chosen for the forcing functions. The mean velocities of the fluid and the turbulent statistics that appear in the model were obtained from a DNS (Mito and Hanratty, 2002). We follow the simple approach, described by Iliopoulos et al. (2003) and Mito and Hanratty (2003), that uses time constants characterizing the dispersion of fluid particles to define the time constants in the model. The time step used to solve Eq. (1) and the Langevin equation was 0.25 ν / v_0^{*2}, where ν is the kinematic viscosity.

Four types of studies were performed: (1) Particle turbulence and deposition constants were calculated in the absence of collision or feedback. The stochastic method was used to represent the fluid turbulence seen by the particles. (2) The effects of particle collisions on particle turbulence was calculated . For most runs the fluid turbulence was represented by the stochastic model. For a few runs the DNS of the fluid in the absence of particles was used. (3) The effect of particles on the fluid turbulence (feedback) and on the particle turbulence was studied in a DNS. The effect of particle collisions was ignored. (4) For a few runs the effects of both feedback and particle collisions were studied in a DNS.

Calculation of the effects of collisions

Inter-particle collisions are observed to affect the particle turbulence even with volume fractions of $O(10^{-3})$. Their effects have been investigated in the Lagrangian framework with a deterministic method (Tanaka and Tsuji, 1991; Lun and Liu, 1997; Li et al., 2001; Yamamoto et al., 2001) and with a stochastic method (Oesterle and Petitjean, 1993; Sommerfeld, 2001). The latter needs to be modified in, as yet, an undefined way, when it is used for inhomogeneous turbulence. In this study, the inter-particle collisions are taken into account with a deterministic method. The detection method for inter-particle collisions, described by Chen et al. (1998) and Li et al. (2001), was used. Various impact patterns can be used. We consider two: (1) elastic collisions, for which the coefficient of restitution, e, equals 1, and (2) highly inelastic collisions with $e = 0.1$ and with zero relative tangential velocity after a collision (Campbell and Brennen, 1985).

Calculations are done in an infinitely wide channel when inter-particle collisions are not considered (Mito and Hanratty, 2003). A periodic boundary condition was used in the streamwise and spanwise directions for calculations in which inter-particle collisions are considered. It is noted that the dimensions of the periodicities do not appear to affect the accuracy of the simulation of the fluid phase using the stochastic method. Calculations of inter-particle collisions were done every time step.

Calculation of feedback

Significant attenuation of fluid turbulence in a vertical channel by particles has been observed at moderate mass loadings (10 % $\sim$ 40 %) in experiments by Kulick et al. (1994) and Paris and Eaton (2001). The feedback effect of particles on the fluid turbulence is studied using a DNS of the fluid turbulence. Interaction between the fluid and the particles is modeled as the sum of point forces of particles, defined at their centers, in a computational grid cell (Crowe et al., 1977):

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j^2} - \frac{\rho_p}{\rho_f}\frac{V_p}{V_{cell}}\sum_{k=1}^{N_{cell}} f_{dik} \quad (2)$$

where f_{dik} is the point force, defined in Eq. (1), for the kth particle, N_{cell} is the number of the particles in the computational cell, V_{cell} is the volume of the computational cell and V_p $(= \pi d_p^3/6)$ is the volume of a particle. All variables are made dimensionless using the friction velocity in the absence of particles, v_0^*, and the kinematic viscosity, ν.

The DNS of fully-developed turbulent fluid flow in a channel was performed in a box with dimensions of 1900 ν / v_0^* in the streamwise direction (x_1), 300 ν / v_0^* in the wall-normal direction (x_2), and 950 ν/v_0^* in the spanwise direction (x_3). The bulk flow rate was fixed in the calculations. The Reynolds number, Re_b, defined with the bulk mean velocity and the half-height of the channel was 2260, at which $Re_\tau = 150$ in the absence of particles. A pseudospectral fractional method (Lyons et al., 1991) was used for the spatiotemporal discretization. The feedback term was calculated by using a first-order Euler explicit method. The computation grid was 128 $\times$ 65 $\times$ 128. The resolutions in the streamwise and spanwise directions were $\Delta x_1^+ = 15$ and $\Delta x_3^+ = 7.4$. The resolutions in the wall-normal direction varied from $\Delta x_2^+ = 0.18$ at the wall to $\Delta x_2^+ = 7.4$ at the channel center. No slip boundary conditions were used at $x_2 = \pm H$ and periodicity was assumed in the x_1 and x_3 directions. The time step was $\Delta t^+ = 0.25$. The fluid velocity seen by particles was calculated using a mixed spectral-polynomial interpolation scheme developed by Kontomaris et al. (1992).

RESULTS

Figure 1 presents calculations, using the stochastic representation of fluid turbulence, of the variation of $k_{DB}^+ = R_{Ab}/C_B v^*$, where C_B is a bulk concentration, with the volume-averaged inertial time constant, τ_{pB}^+, for cases where no collisions and no feedback are considered. It is noted that $k_{DB}^+ = \sigma_p^+/\sqrt{2\pi}$, where σ_p^+ is the dimensionless root-mean-square of the particle velocity fluctuations in a direction normal to the wall at $x_2^+ = 40$. Good agreement is noted for a range of τ_{pB}^+ which is characteristic of annular flows.

Figure 2 presents laboratory measurements of k_{DB}^+ by Schadel et al. (1990) for upflow of air and water in a 2.54 cm pipe and by Andreussi (1983) for downflow of air and water in a 2.4 cm pipe. These are characterized by a deposition rate that varies linearly with volume fraction, α, at small α and that is roughly constant at large α, that is, a constant value of k_{DB} at small α and a α^{-1} variation at large α. The usual explanations for observed decreases in k_{DB} are that the

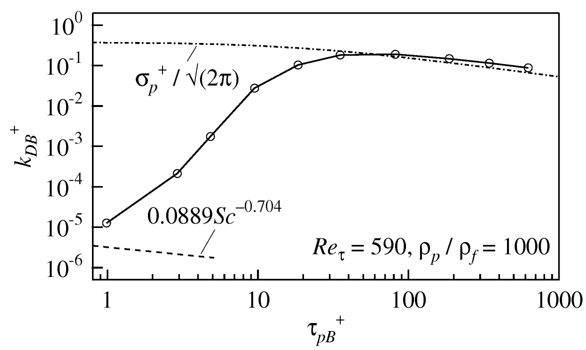

Fig. 1: Deposition constant ($Re_\tau = 590$, $\rho_p/\rho_f = 1000$, model).

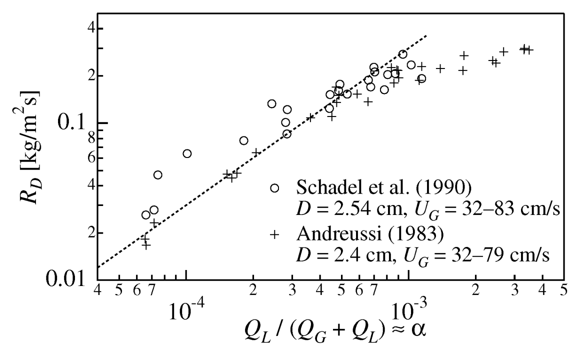

Fig. 2: Rate of deposition in annular flows.

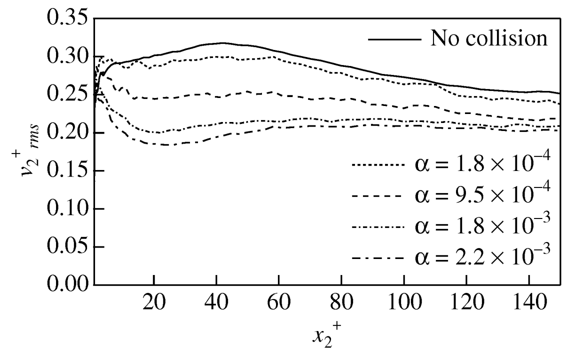

Fig. 4: Effect of inter-particle collisions on particle turbulence ($Re_\tau = 150$, $\rho_p/\rho_f = 1000$, model, inelastic collision).

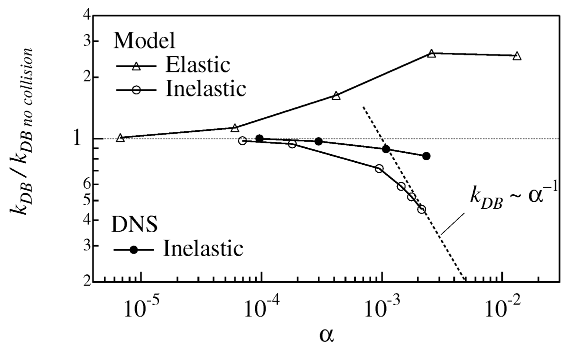

Fig. 3: Effect of inter-particle collisions on deposition constant ($Re_\tau = 150$, $\rho_p/\rho_f = 1000$).

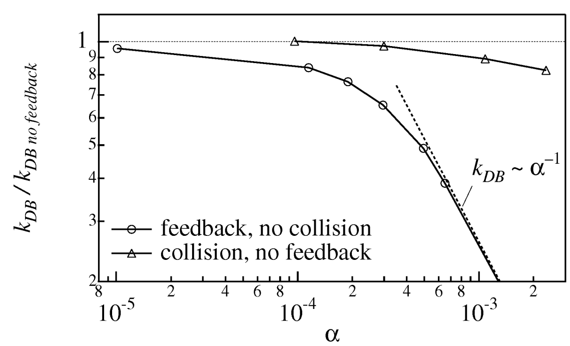

Fig. 5: Feedback effect on deposition constant.

fluid turbulence is damped or that drop coalescence is occurring. Experiments by Hay et al. (1996) showed that changes in k_{DB} could not be explained by changes in the diameters of the drops along the pipe. Furthermore, measurements of mean fluid velocity profiles seem to suggest that fluid turbulence was not decreasing. (We will show that this agreement need not indicate the absence of changes in fluid turbulence.) They, therefore, suggested that the decrease in k_{DB} was due to drop encounters. To a large extent, the exploration of this notion motivated the present study.

Calculations of the dependency of k_{DB} on α and on the collision model for particles with $\tau_p^+ = 200$ ($d_p^+ = 1.9$, ρ_p / $\rho_f = 1000$) are given in figure 3. The calculations that used the stochastic model to represent fluid turbulence show that changes in k_{DB} with increasing α are very different for different collision models and that the inelastic collision model with $e = 0.1$ can realize the laboratory measurements, shown in figure 2. The deposition parameter, k_{DB}, becomes strongly affected by drop concentration at $\alpha \cong 4 \times 10^{-4}$ and varies roughly as α^{-1} at large α. This is emphasized by the finding that it is impossible to reach a stationary state if calculations are carried out for rates of atomization larger than a critical value. Calculations that used a DNS to represent fluid turbulence and the inelastic collision model are also shown in figure 3. The dependency of k_{DB} on α is much smaller than is observed in the simulations that used a stochastic model for the fluid turbulence. This difference points to the shortcoming that the stochastic method does not represent small scale fluid turbulence correctly.

It is worthwhile to note, in figure 3, that if elastic collisions were occurring the particle turbulence, in a direction normal to the wall, and k_{DB} would be increased by collisions.

Values of the root-mean-square of the particle velocity fluctuations in a direction perpendicular to the wall corresponding to the results for the stochastic model and the inelastic collision in figure 3 are presented in figure 4. A decrease in v_{2rms}^+ with increasing α is noted. This decrease in the particle turbulence at the edge of the viscous wall layer ($x_2^+ \cong 40$) roughly compares to the decrease in k_{DB}^+. These results and those shown in figure 1 support the arguments by Hay et al. (1996) that turbulent deposition in annular flows occurs by free-flight of particles from the edge of the viscous wall layer.

Calculations of the feedback effect on k_{DB} with changes in α for particles with $\tau_p^+ = 200$ are presented in figure 5. Fluid turbulence was calculated using a DNS. Inter-particle collisions were not taken into account. The calculations show that k_{DB} becomes affected by α at $\alpha \approx 10^{-5}$ and an asymptotic behavior of $k_{DB} \sim \alpha^{-1}$. The calculations with a DNS representation of fluid turbulence that only considered inter-particle collisions are also shown in figure 5. The feedback effect appears to be much more important than inter-particle collisions. It is noted that the maximum rate of atomization, above which a statistically stationary state cannot be obtained, is much smaller for the case with feedback than for the case with only inter-particle collisions.

Since the effect of inter-particle collisions was small compared to the feedback for the system considered in this paper, results for calculations in which only feedback was taken into account are presented. Figure 6 shows the effect of α on the

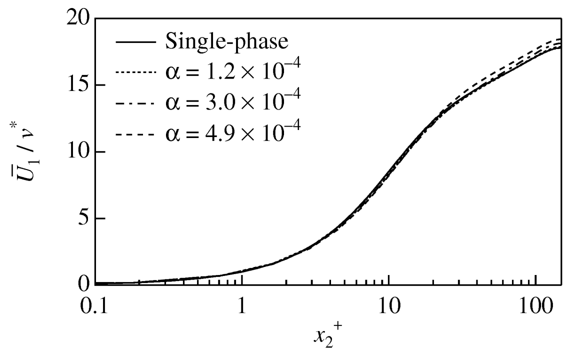

Fig. 6: Feedback effect on mean fluid velocity.

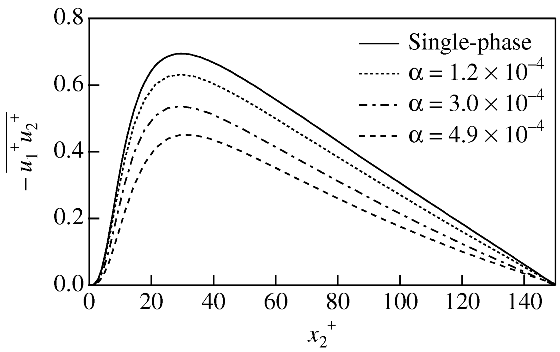

Fig. 7: Feedback effect on Reynolds shear stress.

mean fluid velocity, made dimensionless with the actual friction velocity, v^*. Changes in $\overline{U}_1/v^*$ for different α are seen to be very small as was pointed out by Kulick et al. (1994). Small increases in $\overline{U}_1/v^*$ with increasing α are observed in the outer region. These are associated with decreases in v^*, that is, drag reduction.

The fluid Reynolds shear stress is seen to decrease with increasing α (figure 7). Fluid turbulence is damped with increasing α for both the streamwise and wall-normal components (figures 8a and 8b). Since the correlation between u_1 and u_2, shown in figure 8c, does not change significantly with changes in α, changes in turbulent coherent structures due to the feedback effect are probably not significant. Thus, an explanation of the decrease in fluid turbulence seems to depend on obtaining an understanding of the effect of point forces on the Reynolds stress variation.

ACKNOWLEDGMENTS

This work is supported by DOE under grant DEFG0286ER 13556. Computer resources have been provided by the National Center for Supercomputing Applications located at the University of Illinois.

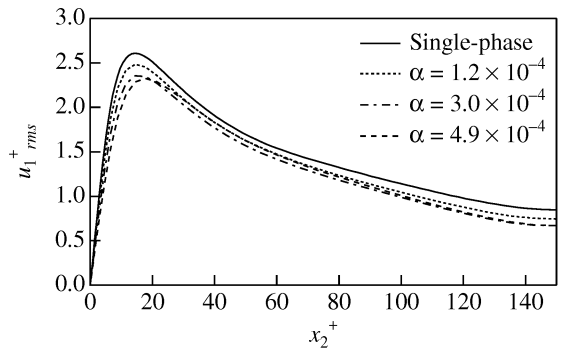

Fig. 8 (a): Feedback effect on streamwise fluid velocity fluctuation.

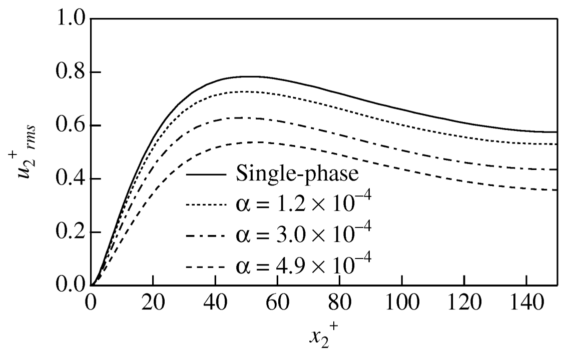

Fig. 8 (b): Feedback effect on wall-normal fluid velocity fluctuation.

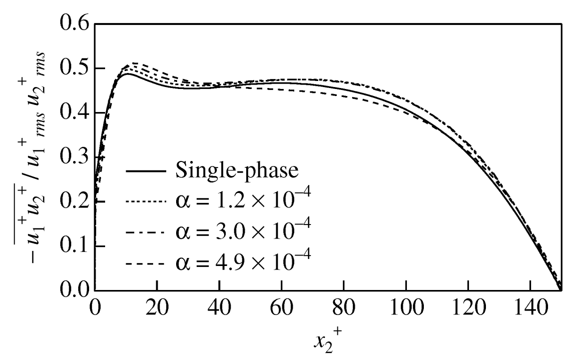

Fig. 8 (c): Feedback effect on correlation between the streamwise and wall-normal velocity fluctuations.

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