

LENGTH AND TIME SCALES OF THE NEAR-SURFACE AXIAL VELOCITY IN A HIGH REYNOLDS NUMBER TURBULENT BOUNDARY LAYER

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ABSTRACT

Reynolds number effects on relevant length and time scales in the near-wall region of a canonical turbulent boundary layer are investigated. Well resolved measurements in the atmospheric surface layer are compared with existing laboratory data to give a composite Reynolds number range spanning over three orders of magnitude. In the field experiments, a vertical rake of twenty single element hot-wires was used to measure the axial velocity, u , characteristics in the lower log layer region of the atmospheric surface layer that flows over Utah's western desert. Only data acquired under conditions of near-neutral thermal stability are analyzed. The shape of the power spectra of u as a function of distance from the wall, y , and Reynolds number is investigated, with emphasis on the appropriate scaling parameters valid across different wavenumber, k , bands. In particular, distance from the wall is found to scale the region of the u spectra around $ky = 1$. The presence of a $k = -1$ slope in the spectra is also found to correlate with the Reynolds number dependence in the peak of the root mean square u profile. Reynolds number trends in the profiles of the Taylor microscales, which represent intermediate length and time scales in the boundary layer, are shown to not follow classical scaling.

INTRODUCTION

The canonical turbulent boundary layer (TBL) is of fundamental interest because it provides a basis for understanding more complicated flows. Almost all practical applications involving TBLs are characterized by high Reynolds numbers, including, for example, airfoils and submarine hulls. Due to limitations in computational and experimental resources, most numerical and laboratory model studies are conducted at much low Reynolds numbers. Results from these model studies can only be effectively extrapolated to higher, more practical Reynolds numbers, if the appropriate TBL scaling parameters and relations are known. Currently, such scaling behaviors remain largely indeterminate, for both canonical and noncanonical TBLs.

The present study aims to address this issue by probing a naturally high Reynolds number turbulent boundary, namely the atmospheric boundary layer (ABL), and comparing to laboratory data obtained at much lower Reynolds numbers. In both cases, very good spatiotemporal resolution of the measurement technique is maintained to avoid contamination of the results by spatial averaging, which tends to mask true Reynolds number effects. The two unique aspects of the present study, i.e., the very good spatial resolution over a

sufficiently large Reynolds number range (three orders of magnitude), are significant with respect to ascertaining potential scaling relations.

EXPERIMENTAL SET-UP

Hot-wire anemometry experiments were performed at the Surface Layer Turbulence and Environmental Science Test (SLTEST) facility on the salt playa of Utah's western desert under conditions of near-neutral thermal stability. The SLTEST site is particularly suitable for high Reynolds number ($R_\theta \approx 5 \times 10^6$) canonical TBL studies due to the sparsity of vegetation, the smooth flat surface ($z_o \approx 0.5$ mm), and the relatively predictable diurnal wind patterns resident in the early summer. Attributes of the site are described in further detail by Metzger (2002). In the present study, a tower of 20 simultaneously sampled hot-wires was used to interrogate the turbulent axial velocity in the near-surface region spanning $5 \leq y^+ \leq 10^4$, where y denotes the distance from the surface. Note, the superscript $+$ indicates inner normalization by kinematic viscosity, ν , and friction velocity, $u_\tau (\equiv \sqrt{\tau_w/\rho})$, where τ_w and ρ denote the wall shear stress and air density, respectively. Because of inherent temporal variations in the atmosphere, simultaneous data are needed to clarify trends in the statistical profiles. Therefore, the tower of simultaneously sampled velocity measurements represents an essential feature of the present study over previous studies.

The present hot-wire data are accompanied by coincident meteorological data, including three-dimensional velocity from sonic anemometers located at 2, 3, and 5 m above the surface, direct surface shear stress from a floating-element drag plate, mean velocity profiles up to 150 m from a minisodar, net surface radiation, and surface temperature gradient measurements. In order to deduce Reynolds number effects on the structure of the TBL, the present atmospheric results are compared with wind tunnel data acquired in the range $2500 \leq R_\theta \leq 5 \times 10^3$. In all cases, the inner normalized wire length is less than 10, yielding very good spatial resolution over three orders of magnitude in Reynolds number.

CHALLENGES WITH ATMOSPHERIC BOUNDARY LAYER EXPERIMENTS

The challenges of performing hot-wire experiments in the atmospheric surface layer are multifaceted. On a practical level, field trials are physically demanding and require special facilities for on-site hot-wire calibration, which must be performed frequently. Calibration algorithms must also ex-

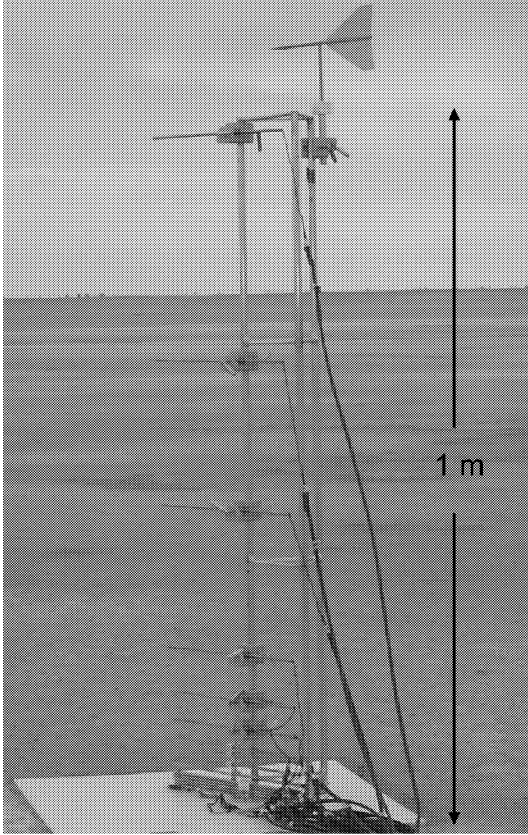


Figure 1. Photograph of the hot-wire experiment at the SLTEST site in Utah's western desert. Twenty hot-wires are spaced nearly logarithmically, spanning an overall distance of 1 m above the surface.

Explicitly incorporate temperature compensation (Metzger and Klewicki, 2003) since ambient air temperature changes dramatically over the course of the day. Since hot-wire probes are directionally sensitive, i.e., data may become contaminated by substantial crosswinds, care must be taken to constantly align the probes with the mean wind direction. In addition, field trials can be costly and risky, in the sense that the potential for instrumentation and sensor damage, due to, for example, environmental factors such as dust and high speed winds, are much greater than typical laboratory studies.

On a more technical level, the ABL differs from the canonical laboratory generated boundary layer in two important ways, aside from the Reynolds number difference: (i) it is not statistically stationary and (ii) it is thermally stratified. The former poses challenges in determining the appropriate averaging time to use in calculating the vertical momentum flux, for example, which is typically used in atmospheric studies to estimate the local friction velocity, u_τ . The second difference results from the diurnal heating and cooling of the Earth's surface, which leads to the inherent thermal stratification of the atmosphere. During the transitions between heating and cooling (sunrise and sunset), the atmospheric boundary layer passes through neutral stability whereby boundary layer turbulence is driven predominantly via mechanical shear, with buoyancy effects being negligible. Therefore, in order to make direct comparisons between ABL data and wind tunnel measurements, only field data acquired during conditions of near

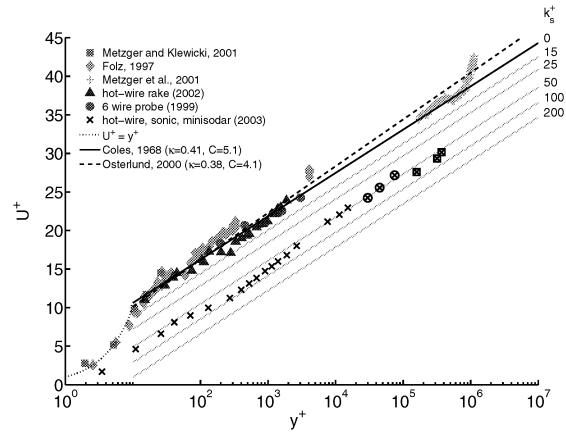


Figure 2. Inner normalized mean axial velocity profile from atmospheric data obtained at SLTEST. The set of grey lines indicates the expected log law for different surface roughness, as measured by k_s^+ .

neutral thermal stability must be considered. This poses definite challenges with respect to measuring the thermal stability and performing thorough checks on data quality to ensure that thermal effects are not influencing observed trends in the results. Work in progress by the author aims to address these issues.

AXIAL VELOCITY RESULTS

Statistical Profiles

Figure 2 presents the inner normalized mean axial velocity profiles from several atmospheric studies, including that of Metzger and Klewicki (2001), Metzger et al. (2001), and Folz (1997). All data exhibit a logarithmic region with a slope similar to that observed at lower R_θ . Log laws derived using parameters from Coles (1969) and Osterlund et al. (2000) are shown for comparison. The present study only considers data where the surface may be considered nearly hydraulically smooth in terms of k_s^+ , the inner normalized equivalent sand grain roughness. Two factors play into the variability of k_s^+ at the SLTEST site. For a given surface condition, higher mean wind speeds cause the friction velocity to drop, thereby increasing k_s^+ . In addition, over the course of the summer, the integrity of the desert surface degrades due to moisture depletion, causing a noticeable increase in the dimensional surface roughness by late summer. The former effect dominated in the 2003 data shown in figure 2, which includes hot-wire, sonic anemometer, and minisodar measurements. As evident by the data, the surface condition affects the mean wind profile through the entire depth of the surface layer.

The inner normalized root mean square (rms) axial velocity profiles presented in figure 3, on the other hand, reveal distinct Reynolds number trends. Compared are the data of Metzger and Klewicki (2001), Folz (1997), DeGraaff and Eaton (2000), and Klewicki and Falco (1990), along with the prediction based on the scaling relation of Marusic et al. (1997). Of particular interest is the proper scaling relation valid near the peak at $y^+ = 15$, since this corresponds, at least from low Reynolds number data, to the peak in the turbulent kinetic energy production. Renormalization using the mixed velocity

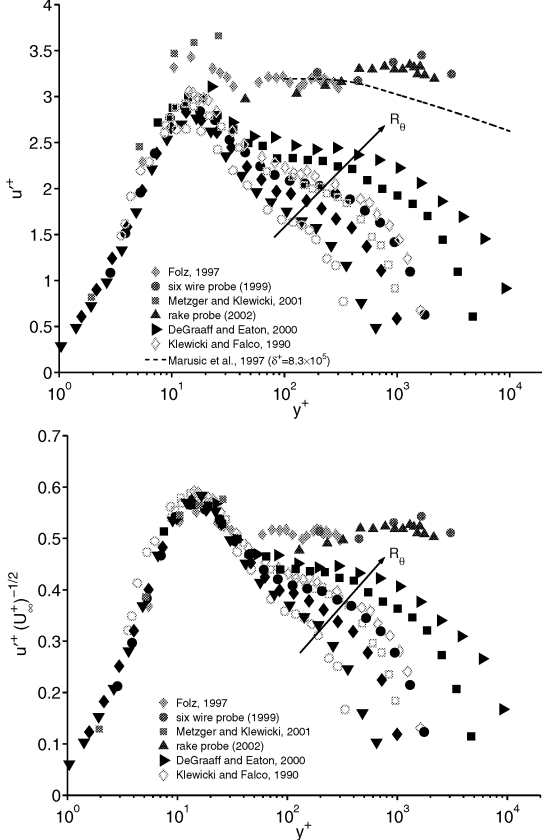


Figure 3. Root mean square axial velocity profiles as a function of Reynolds number. Inner normalization (top) and mixed scaling (bottom). The arrows indicate the direction of increasing Reynolds number. The black solid and open grey symbols are from DeGraaff and Eaton (2000) and Klewicki and Falco (1990), respectively. All colored symbols represent atmospheric data acquired at the SLTEST site.

scale of DeGraaff and Eaton (2000), i.e. $\sqrt{u_\tau U_\infty}$, appears to remove the observed Reynolds number trend near the peak at $y^+ = 15$. Interestingly, the mixed velocity scale represents the geometric mean between the smallest and largest velocity scales in the flow, and as such characterizes an intermediate velocity scale. Note, both a substantial Reynolds number range and sufficient spatial resolution are required to observe the Reynolds number behavior in figure 3. The present data are inconclusive, however, regarding a potential secondary peak near $y^+ = 550$ at $R_\theta \approx 5 \times 10^6$. Work in progress by the author aims to address this issue, as well as investigate possible scaling relations for the region of the profile between $40 < y^+ < 0.1 \delta^+$. Note, the previous work of DeGraaff and Eaton (2000) showed that outer scaling is appropriate for $y^+ > 0.1 \delta^+$.

Further Support for Mixed Scaling Near the Peak

Mixed scaling of u' near the peak at $y^+ = 15$ implies

$$\frac{u'_{max}}{(u_\tau U_\infty)^{1/2}} = C, \quad (1)$$

where $C \approx 0.57$ independent of R_θ , as based on the data

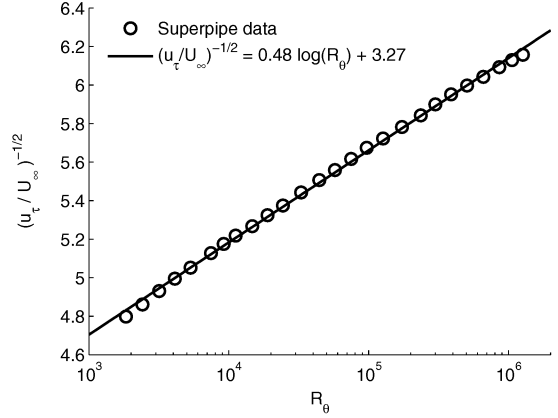


Figure 4. Reynolds number dependence of the inverted intermediate velocity scale, as based on Superpipe data derived from the work of McKeon et al. (2004).

presented in figure 3. Rearranging (1) yields

$$u'_{max} = C \left(\frac{u_\tau}{U_\infty} \right)^{-1/2}. \quad (2)$$

At the same time, a compilation of the current literature (see Metzger and Klewicki, 2001) suggests that

$$u'_{max} = A \log(R_\theta) + B, \quad (3)$$

where $A=0.22-0.29$ and $B=1.84-2.02$ are empirical constants derived from a curve fit to the available data¹. In order for both (2) and (3) to be valid, it is clear that there must be a relationship such that

$$\left(\frac{u_\tau}{U_\infty} \right)^{-1/2} = a_1 \log(R_\theta) + a_2, \quad (4)$$

where $a_1 = A/C = 0.39-0.51$ and $a_2 = B/C = 3.22-3.54$. The quantity on the left hand side of (4) is relatively easy to measure in the Superpipe, since u_τ may be determined by the pressure drop along the pipe and U_∞ is simply the center-line velocity. Figure 4 shows this quantity as a function of Reynolds number based on data from the study of McKeon et al. (2004). A curve fit through the data produces the following relation

$$\left(\frac{u_\tau}{U_\infty} \right)^{-1/2} = 0.48 \log(R_\theta) + 3.27. \quad (5)$$

The empirical coefficients in (5) fall directly within the range of a_1 and a_2 given for (4).

Although the Superpipe data in figure 4 appear to exhibit a slight curvature when plotted semilogarithmically versus R_θ , the linear relation of (5) represents a very good first order approximation. In this manner, the present analysis of Superpipe data provides strong independent support for the mixed scaling or u' near the peak. It does not, however, answer the question why U_∞ is an important velocity scale so close to the wall. Metzger and Klewicki (2001) showed that an increased contribution from low frequency motions at high Reynolds

¹An essential part of the analysis by Metzger and Klewicki (2001) is that only high resolution data, with sensor lengths less than or equal to 10 viscous units, were used in the curve fit.

number underlies the observed rise in the peak of u'^+ near $y^+ = 15$. This supposed modulation of u' by the freestream, becoming more pronounced with increasing Reynolds number, was anticipated by Bradshaw (1967). Nevertheless, a physical or mechanistic justification for this remains to be determined.

Spectra

En route to addressing the question put forward at the end of the previous section, it is helpful to investigate the spectra of u in some detail. Of particular interest is understanding the predominant time scales of the turbulent motions contributing to the overall axial velocity variance, and how these contributions vary with both distance from the surface and Reynolds number. Figure 5 compares the premultiplied spectra of u at $y^+ \approx 15$ for the two Reynolds numbers of $R_\theta = 5 \times 10^6$ (SLTEST) and $R_\theta = 2500$ (wind tunnel). Spectra are normalized such that the area under the curve is equal to the variance. Clearly, the high Reynolds number data exhibit an extended region with a -1 slope. This is consistent with the result regarding the logarithmic Reynolds number dependence of the peak in u'^+ at $y^+ \approx 15$, as described below.

Ramifications of a -1 Slope. From the definition of the spectra,

$$u'^2 = \int_0^\infty \Phi_u df. \quad (6)$$

As apparent in figure 5, the predominant energy containing region of the spectra lies within the frequency band having a -1 slope. Therefore, integrating the spectra over this frequency band yields a good estimate of the overall variance of u , i.e.,

$$u'^2 \approx \int_{\lambda_t}^{T^{-1}} f^{-1} df, \quad (7)$$

where λ_t and T mark the approximate lower and upper bounds of the -1 slope region. Physically, λ_t and T represent the Taylor and integral time scales, respectively, which provide a measure of the characteristic intermediate and longest time scales in the flow. Performing the integration gives

$$u'^2 = \log(T^{-1}) - \log(\lambda_t^{-1}), \quad (8)$$

or

$$u'^2 = \log\left(\frac{\lambda_t}{T}\right). \quad (9)$$

The ratio of the time scales may be related to the Reynolds number yielding

$$u'^2 \propto \log(R_\theta) \quad (10)$$

The statement in (10) agrees qualitatively with the observation expressed by (3). This gives some physical justification tying the Reynolds number dependence in the peak of u'^+ to an extended region of -1 slope in the corresponding spectra that increases logarithmically with increasing R_θ .

Distance from the Wall Effect. Scaling of the spectra at y^+ values further from the surface, relative to the peak in u'^+ , were investigated to determine the extent of the -1 slope region for $y^+ > 15$. Figure 6 shows the premultiplied spectra of the axial velocity at $R_\theta \approx 5 \times 10^6$ using both inner normalization and normalization by distance from the surface. In these plots k denotes the wavenumber, i.e., $k = 2\pi f/U$ where

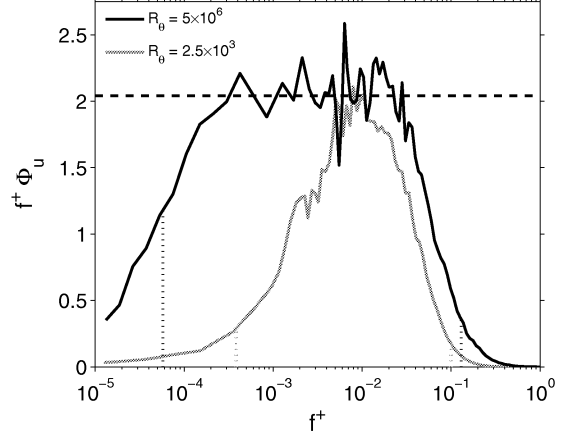


Figure 5. Comparison of premultiplied spectra at $y^+ = 15$ for high and low R_θ . The dotted lines mark the approximate locations of the inverse Taylor time scale and integral time scale for each data set. The black dashed line indicates a slope of -1, as would appear in the non-premultiplied spectra.

U represents an appropriate convection velocity dependent on y . Due to the scatter in the raw spectra at low wavenumber, some form of smoothing is necessary in order to ascertain trends in the results. Therefore, in the present study, raw premultiplied spectra were smoothed using a least squares spline approximation of fifth order. The knot sequence of the spline was adjusted to minimize the total error, defined as the sum of the squared difference between the data and the spline fit.

The results in figure 6 indicate that for $y^+ > 45$, distance from the surface scales the spectra within a wavenumber band around $ky = 1$. An inherent feature of scaling with distance from the surface is that the overall range of scaled wavenumbers contributing to the variance decreases, i.e., the scaled spectra becomes compressed, in some sense, along the x-axis as y increases. Since the present data are limited to $y^+ = 2100$, the extent to which scaling with distance from the surface applies for larger y^+ remains to be seen.

Also evident in figure 6 is that the spectra at $y^+ = 45$ exhibits a much narrower region of $k = -1$ slope, compared to that observed at $y^+ \approx 15$; while, spectra for $y^+ \geq 130$ do not possess a $k = -1$ slope at all. This is consistent with the arguments presented earlier, see (10), that a $k = -1$ slope in the spectra of u at any given y^+ corresponds to a logarithmic Reynolds number dependence in the variance of u at that same y^+ . The logarithmic Reynolds number dependence of u' at $y^+ \approx 15$, in turn, corroborates the proposed mixed scaling of u' in this same region, as described in (1)–(5). Therefore, the lack of a $k = -1$ slope in the high Reynolds number u spectra for $y^+ > 45$ is not inconsistent with the fact that mixed scaling ceases to be valid at this Reynolds number for $y^+ > 45$.

At lower Reynolds numbers, the u spectra typically displays a $k = -1$ slope out to much higher y^+ (Perry and Abell, 1975). One plausible reason for this discrepancy stems from the rms profile (see figure 3). At lower R_θ , mixed scaling appears to extend out to a much higher y^+ , relative to δ^+ ; as R_θ increases, the u' profiles peel away from the mixed scaling line at ever decreasing y^+ , approaching the location of the peak near $y^+ = 15$. Based on the relation established herein between mixed scaling in the u' profile and the existence of a $k = -1$ region in the spectra, it is not surprising that as

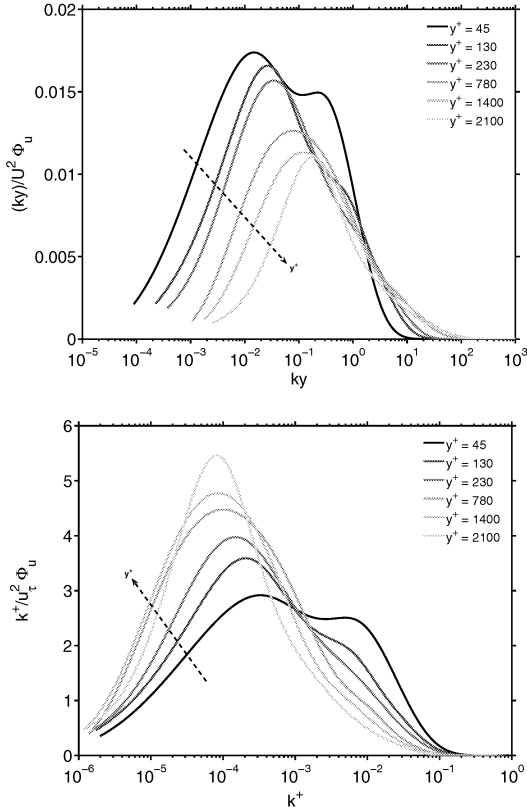


Figure 6. Spectra of the axial velocity as a function of distance from the surface at $R_\theta \approx 5 \times 10^6$. Normalization with distance from the surface (top) and inner normalization (bottom). The dashed arrow indicates the direction of increasing y^+ .

R_θ increases, the $k = -1$ region in the u spectra becomes less pronounced, and even disappears, for y^+ positions increasing beyond the peak near $y^+ = 15$.

Taylor Microscales

Previous studies indicate that the Taylor microscale plays a significant role in scaling turbulent bursting frequencies, event durations, spectra, and statistics (Klewicki and Falco, 1996; Metzger et al., 2003; Nagano et al., 1998). Because of the potential relevance toward understanding momentum and scalar transport across the TBL, deciphering the scaling properties of the Taylor time and length scales is believed to be important. Toward this end, the present study considers the variation of the Taylor microscale (both length and time) as a function of R_θ and y .

The Taylor length scale, λ , is defined as the characteristic length associated with the dissipation of turbulent kinetic energy, ϵ , and, as such, represents an intermediate scale between the inner (u_τ/ν) and outer (δ) length scales. Assuming isotropic turbulence,

$$\epsilon = 15\nu \left\langle \left(\frac{\partial u}{\partial x} \right)^2 \right\rangle \sim 15\nu \frac{\langle u^2 \rangle}{\lambda^2}, \quad (11)$$

where $\langle \cdot \rangle$ denotes an appropriate time average. Therefore, the Taylor length scale is estimated from experimental data using

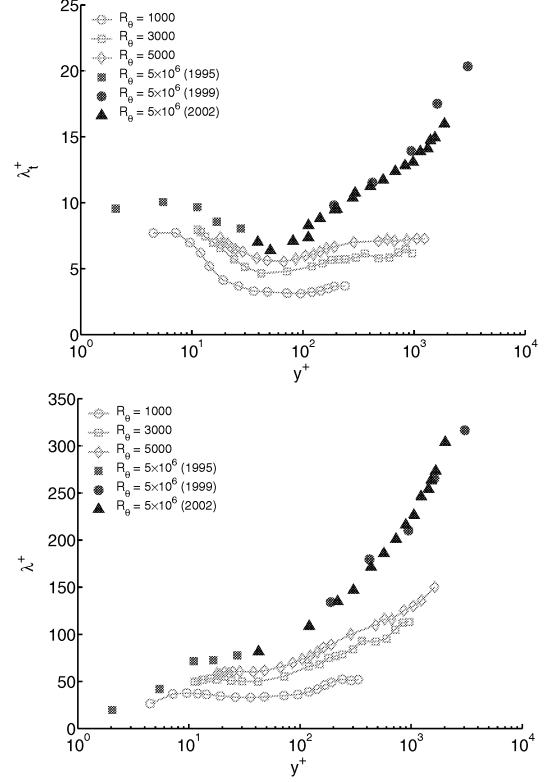


Figure 7. Inner normalized Taylor microscale as a function of distance from the surface and Reynolds number. Taylor time scale (top) and Taylor length scale (bottom). The open symbols were calculated from the data of Klewicki (1989). The solid symbols represent data acquired at the SLTEST site.

the relation

$$\lambda^2 = \frac{\langle u^2 \rangle}{\langle (\partial u / \partial x)^2 \rangle}. \quad (12)$$

The present study utilizes Taylor's frozen turbulence hypothesis along with a Savitzky-Golay filter to evaluate the derivative in (12). The Taylor time scale, λ_t , is calculated in a more straightforward manner using an osculating parabola fit to the autocorrelation of u at zero time lag. The inner normalized results are plotted in figure 7. Importantly, the high Reynolds number atmospheric data shown were acquired over three different years with varying hot-wire probe types; yet, all of the results agree well.

Clearly, inner normalization is not the appropriate scaling measure for the Taylor microscale. Classical scaling arguments (Tennekes and Lumley, 1972, pg. 67), based on the assumption that energy dissipation equals production, predict that

$$\frac{\lambda}{\delta} \sim (R_\delta)^{-1/2}, \quad (13)$$

where $R_\delta = \tilde{u} \delta / \nu$ and $\tilde{u} = \langle u^2 \rangle^{1/2}$. The relation in (13) may be rewritten as

$$\lambda^+ \sim (\delta^+)^{1/2} (\tilde{u}^+)^{-1/2}. \quad (14)$$

Figure 8 presents the data from figure 7 rescaled using the relation in (14), without the \tilde{u}^+ factor which is much smaller than δ^+ . The results suggest that classical scaling is not quite

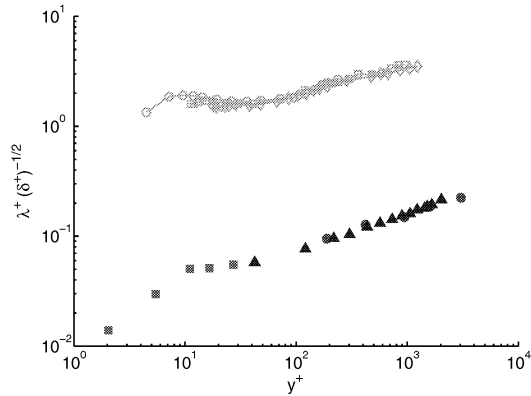


Figure 8. Classical scaling of the Taylor length scale. Symbols are the same as that in figure 7.

correct. Future works in progress aims to investigate this further, including filling the Reynolds number gap between the present wind tunnel data and atmospheric measurements.

SUMMARY

Under nearly ideal conditions, the atmospheric boundary layer at the SLTEST site comes close to mimicking the flow expected in a very large wind tunnel, and, until recently, has remained an untapped resource in the study of fundamental TBL physics. Because no other facility exists that can generate TBLs with Reynolds numbers as high as those encountered in the atmosphere, without putting severe demands on the spatial resolution of current measurement technology, it seems rational to seek answers to Reynolds number scaling relations by probing the atmosphere.

The present study compares atmospheric measurements of the axial velocity statistics, spectra, and Taylor microscale with corresponding laboratory data to address some of the open issues in the scaling properties of turbulent boundary layers. Further support of mixed scaling of the u' profile near the peak at $y^+ = 15$ was provided using independent mean velocity data from the Superpipe. Mixed scaling was also tied to the observation that the $k = -1$ region in the u spectra at $y^+ = 15$ extends further into the low wavenumber regime at high R_θ , indicating that the increase in the peak value of u' results from an increased contribution at low wavenumbers. In addition, the $k = -1$ region becomes less pronounced at high R_θ for $y^+ > 15$ and disappears altogether for $y^+ > 45$. This location corresponds roughly to the same y^+ location at which mixed scaling ceases to be valid. Finally, classical scaling arguments were shown to be inadequate in removing actual Reynolds number trends in the Taylor length scale. The key to this realization was having well resolved turbulence measurements over a sufficiently large Reynolds number range, in this case three orders of magnitude.

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