ADVANCED EDDY VISCOSITY TURBULENCE MODELLING - APPLICATION TO INERT AND REACTING BLUFF BODY FLOWS

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ABSTRACT

The non-linear $k-\varepsilon$ model by Merci *et al.* (2002a, 2002b, 2003a) is applied to inert and reacting bluff body flows (Dally, 1998). Based on observed shortcomings in the results, the turbulence model is adjusted without deteriorating previous results. The obtained improvements for the bluff body flows are, however, rather limited. Probably, the flows, which are strongly unsteady in reality, are too complex for any two-equation turbulence model to yield satisfactory results throughout the domain. An unsteady calculation seems necessary (TNF, 2003a).

INTRODUCTION

In RANS turbulence models, one turbulent scale is determined from the dissipation rate ε (or a related quantity). Its transport equation is a very important feature of the turbulence model. A non-linear $k-\varepsilon$ model with an improved ε transport equation, valid in high and low Reynolds number flow regions, has already been applied to a wide variety of flows (Merci, 2002a, 2002b, 2003a).

In this paper, the model is applied to inert and reacting bluff body flows (Dally, 1998). A detailed analysis of these complex flows reveals some shortcomings. Therefore, a further model refinement, which does not influence previous results, has been developed.

In the next section, the original model is shortly described. The experimental test case is described next and some relevant flow features are given. On that basis, the model refinement is explained. Finally, simulation results are presented.

MODEL DESCRIPTION

Only relevant model aspects are given here. A complete description is found elsewhere (Merci, 2002b).

Constitutive law

The non-linear expression for the Reynolds stresses reads:

$$-\rho \widetilde{v''_i v''_j} = -\frac{2}{3}\rho k \delta_{ij} + 2 c_\mu f_\mu \rho k \tau_t S_{ij} + H.O.T. \ , \quad (1)$$

with S_{ij} the strain rate tensor:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{1}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k} \ . \quad (2)$$

The turbulence time scale is:

$$\tau_t = \frac{k}{\varepsilon} + \sqrt{\frac{\mu}{\rho\varepsilon}} \ . \quad (3)$$

The coefficients in the higher order terms in expression (1) and c_μ depend on the tensor invariants:

$$S = \sqrt{2 S_{ij} S_{ij}} \ , \ \Omega = \sqrt{2\Omega_{ij}\Omega_{ij}} \ , \ \eta = \tau_t \sqrt{S^2 + \Omega^2} \ , \quad (4)$$

with the vorticity tensor Ω_{ij}:

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right) \ . \quad (5)$$

By far the most important coefficient in eq. (1) is c_μ. It is made locally flow dependent and accounts for streamline curvature effects:

$$c_\mu = (A_1 + A_s \eta + 25(1 - f_{R_y})|W|)^{-1} - \frac{1}{4}\frac{c_1}{f_\mu}\tau_t^2 (S^2 - \Omega^2) \ , \quad (6)$$

with c_1 defined as:

$$\begin{cases} S \geq \Omega : c_1 = -f_W min(40 c_\mu^4; 0.15) \\ S < \Omega : c_1 = -f_W min(min(600 c_\mu^4; 0.15); \\ \qquad 4 f_\mu c_\mu / (\Omega^2 \tau_t^2 - S^2 \tau_t^2)) \end{cases} \quad (7)$$

In expression (6), the constant A_1 is $A_1 = 4$ and $A_s = \sqrt{3} cos\phi$, where $\phi = \frac{1}{3} arccos(\sqrt{6} W)$, with W from:

$$W = 2^{1.5}\frac{S_{ij} S_{jk} S_{ki}}{S^3} \ . \quad (8)$$

The final part of the first term in c_μ improves predictions for turbulent impinging jets (Merci, 2003a) and is negligible in the flows of this paper. The blending function f_{R_y} is defined as:

$$f_{R_y} = \frac{1}{2} + \frac{1}{2} sin\left(\frac{\pi}{2} min[max(\frac{R_y}{500} - 3; -1); 1]\right) \ , \quad (9)$$

with R_y a dimensionless distance from the nearest solid boundary:

$$R_y = \frac{\rho\sqrt{k} y}{\mu} \ . \quad (10)$$

The blending function f_{R_y} goes from 0 to 1 in the interval $R_y = 1000$ to 2000.

The factor f_W in eq. (7) is important in impingement heat transfer predictions (Merci, 2003a) and is defined as:

$$f_W = 1 - 18|W|^2 + (72/\sqrt{6})|W|^3 \ . \quad (11)$$

Finally, the damping function f_μ is defined as $f_\mu = 1 - exp(-6\ 10^{-2}\sqrt{R_y} - 2\ 10^{-4} R_y^{1.5} - 2\ 10^{-8} R_y^4)$.

Transport equations

Accurate values for k and ε in eq. (1) must be obtained from the transport equations. For the turbulent kinetic energy k, the standard transport equation is used. For ε, the following steady blended equation has been developed by Merci (2002b), based on the previous equations by Shih (1995) and Merci (2001a):

$$\begin{aligned} \frac{\partial}{\partial x_j}(\rho \varepsilon v_j) &= (1 - f_{R_y}) c_{\varepsilon 1} \frac{P_k}{\tau_t} + f_{R_y} C_1 S \rho \varepsilon \\ &- c_{\varepsilon 2} f_2 \frac{\rho \varepsilon}{\tau_t} - c_{\varepsilon 3} \frac{\mu_t}{\rho^2} \frac{\partial \rho}{\partial x_l} \frac{\partial p}{\partial x_l} \frac{1}{\tau_t} \\ &+ \frac{\partial}{\partial x_j}[(\mu + \frac{\mu_t}{\sigma_\varepsilon}) \frac{\partial \varepsilon}{\partial x_j}] + E + Y_c \ . \end{aligned} \qquad (12)$$

The model constants are $\sigma_k = 1$, $\sigma_\varepsilon = 1.2$, $c_{\varepsilon 1} = 1.44$, $c_{\varepsilon 3} = 1$ and $C_1 = max[0.43; S\tau_t/(5 + S\tau_t)]$. The production term P_k is $P_k = -\rho \widetilde{v''_i v''_j} \frac{\partial v_j}{\partial x_i}$ and the eddy viscosity is defined as:

$$\mu_t = \rho f_\mu c_\mu k \tau_t \ . \qquad (13)$$

Parameter $c_{\varepsilon 2}$ is defined as (Shih, 1995; Merci, 2001a, 2002b):

$$c_{\varepsilon 2} = max(1.83 + \frac{0.075 \tau_t \Omega_{abs}}{1 + \tau_t^2 S^2}; C_2 f_{R_y}) \ , \qquad (14)$$

with $C_2 = 1.9$. The first term guarantees a correct transformation for rotating reference frames, ensuring a correct ε behaviour for high rotation speeds. On the other hand, the value $C_2 = 1.9$ must be used in free shear flows.

The blended source term in (12) is important for the quality of the ε transport equation. In the neighbourhood of solid boundaries ($y \to 0$), $f_{R_y} \to 0$ (eq. (9)), so that the traditional source term $c_{\varepsilon 1} \frac{P_k}{\tau_t}$ is recovered. In free shear flows ($y \to \infty$), $f_{R_y} \to 1$, so that the source term becomes $C_1 S \varepsilon$. This corresponds to Shih (1995), where the ε-equation was derived from the enstrophy transport equation and the plane jet - round jet anomaly is resolved. This cannot be obtained with the classical ε-equation, with any expression for $c_{\varepsilon 2}$.

The low-Reynolds source term E has been determined from the standard $k - \omega$ model (Merci, 2001a), but is not important for the flows in this paper.

The 'Yap' correction Y_c is important for impingement heat transfer (Merci, 2003a), but is not significant in the test case considered here.

TEST CASE DESCRIPTION

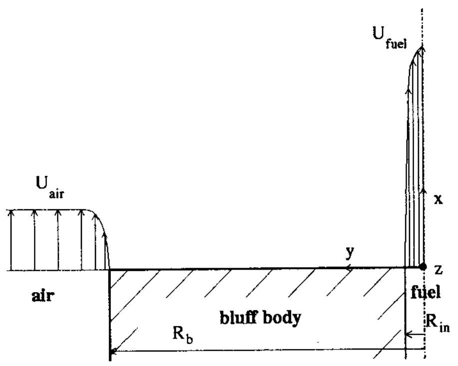

Figure 1: Test case geometry.

The experimental set-up is the Sydney bluff body burner. Both inert and reacting flows at different Reynolds numbers are studied (Dally, 1998). The non-premixed turbulent reacting flow has been a target flame in the series 'International Workshop on Measurement and Computation of Turbulent Nonpremixed Flames' (TNF, 2003a). The geometry, which is completely described by Dally (1998), is depicted in fig. 1. A central fuel jet mixes with a co-flow air stream. The central jet has a diameter $d = 3.6mm$, while the outer body diameter is $D = 50mm$. 'Fuel' has to be interpreted in a broad sense, since in the inert flow, the inner jet is air. In case of reacting flow, the flame is stabilised and attached to the burner due to the intense mixing of fuel, air and reactants in the recirculation region behind the bluff body. Table 1 summarises the flow configurations.

Table 1: Summary of flow configurations.

Flow	'Fuel'	Re_{jet}	U_{cf}
Inert	air	14400	21m/s
Reacting	CH_4/H_2 (50%/50% vol.)	15900	35m/s

The bluff body flows have a specific feature. Behind the nozzle exit, the central 'fuel' jet firstly decelerates. However, further downstream it is accelerated by the co-flow air stream. In the 'transition' region in between, the turbulence model is not satisfactory. Therefore, the described turbulence model has been adjusted, without deteriorating the results of Merci *et al.* (2002a, 2002b, 2003a).

MODEL REFINEMENT

The main problem in the mentioned 'transition' region is that the shear is small, whereas two-equation turbulence models are developed on the basis of shear flows.

For the test case under study, the overpredicted central jet deceleration at the axis is due to an excessive turbulent viscosity (and kinetic energy and shear stress: see further). In order to counteract this, model adjustments can be made in the source terms in the turbulence transport equations, and in the expression for c_μ in eq. (13).

It is observed that in the ε transport equation eq. (12), the source term $C_1 S \rho \varepsilon$ is small in the mentioned 'transition' region. Moreover, it is even smaller than $c_{\varepsilon 1} \frac{P_k}{\tau_t}$, while at the same time $S\tau_t$ is small. The ratio of the source terms is proportional to $(S\tau_t)^{-1}$, so that the first source term is expected to be dominant. This is in contrast to the observation. The reason is that the small value of $S\tau_t$ leads to a large value of c_μ (eq. (6), where the first term is dominant). As a consequence, the combination of a small source term in the ε transport equation with a large value of c_μ (implying a larger production term for k), leads to overprediction of the eddy viscosity and the turbulent shear stress.

As a first remedy, the source term $f_{R_y} C_1 S \rho \varepsilon$ in eq. (12) is replaced by:

$$f_{R_y} C_1 S \rho \varepsilon \to f_{R_y} max(C_1 S \rho \varepsilon; c_{\varepsilon 1} \frac{P_k}{\tau_t}) \ . \qquad (15)$$

This does not have any effect on the flows studied by Merci *et al.* (2002a, 2002b, 2003a), since this source term part is negligible in wall-dominated flow regions ($f_{R_y} \to 0$) and in shear stress dominated free shear flows, where normally (in particular for jets), $C_1 S \rho \varepsilon > c_{\varepsilon 1} \frac{P_k}{\tau_t}$.

Secondly, the expression of Merci *et al.* (2001a) is used for c_μ in regions where $C_1 S \rho \varepsilon < c_{\varepsilon 1} \frac{P_k}{\tau_t}$:

$$c_\mu = (8 + A_s \tau_t max(S; \Omega))^{-1} \ . \qquad (16)$$

This reduces c_μ (compared to expression (6)) in regions of low $S\tau_t$.

CHEMISTRY MODELLING

The pre-assumed β-PDF ('probability density function') approach is followed within the 'conserved scalar' framework. The transport equations for the mean mixture fraction and its variance are standard (Merci, 2001b). The thermochemical quantities are tabulated a priori and interpolated with the program FLAME (Peeters, 1995) during the iterative solution procedure. As chemistry model, the simplified constrained equilibrium model (Bilger, 1983), as described elswhere (Merci, 2001b), is used.

NUMERICAL METHOD

The complete set of equations is subdivided into two (for the inert flow calculations) or three (for the reacting flow) subsets. Each subset is solved in a coupled manner. The first subset consists of the continuity equation and the momentum equations, the second contains the turbulence transport equations and the third consists of the transport equations for the mean mixture fraction and its variance.

The steady-state solutions are obtained through a time marching method with a finite volume technique. The spatial discretization is an AUSM-like second order accurate scheme, in which acoustic and diffusion fluxes are discretized centrally and upwinding is used for the convective fluxes (Vierendeels, 2001). The treatment of the source terms in the turbulence transport equations is described by Merci *et al.* (2000). The higher order terms in the constitutive law (1) are treated partly implicitly (the second order terms in the turbulent normal stresses) and partly explicitly (all other terms). A complete description of the numerical method is given elsewhere (Vierendeels, 2001; Merci, 2003b, in press).

RESULTS

Results are given for the present model (with the 'refinements' as described above), the cubic model of Merci (2002a, 2002b) and the low-Reynolds standard $k-\varepsilon$ model by Yang and Shih (Yang, 1993) (referred to as 'YS').

Inert Flow

Fig. 2 shows radial profiles of mean axial velocity at different distances from the nozzle exit. The central jet firstly decelerates and then accelerates due to entrainment by the co-flow (not visible in fig. 2, since the acceleration takes place from around $x = 1.8D$). As explained (Merci, 2001b), this process is governed by the turbulent shear stress, profiles for which are presented in fig. 3. With the YS model, the radial derivative of this stress is too large near the nozzle exit ($x = 0$), so that the velocity decrease is overestimated. With the model of Merci (2002b), this effect is postponed (too large turbulent shear stress derivative near the axis at $x = 0.6D$, resulting in too low axial velocity at $x = D$), but it is not removed. With the present model, this effect is even further postponed, resulting in a better agreement for the velocity decrease near the axis (although it must be admitted that there is now a slight overprediction of the velocity around $x = D$, due to an underestimation of the turbulent shear stress derivative). It is noteworthy that the change in sign of the radial derivative of the turbulent shear stress, indicating the transition from deceleration towards acceleration, is predicted too close to the nozzle exit with the YS model, in contrast to the present model and the model of Merci (2002b). This is seen in fig. 3 ($x = 1.8D$). The consequence hereof is that the position of minimum velocity on the axis is predicted too close to the nozzle exit with the YS model. Consequently, the seemingly better agreement with the experimentally measured mean velocity on the axis at $x = 1.8D$, compared to the model of Merci (2002b), is misleading: the central jet is already accelerating again with the YS model, in contrast to what is observed experimentally, so that the underestimation of the velocity is less pronounced.

The model 'refinements' as described above, also exert an influence on the recirculation region. As explained, the level of turbulent kinetic energy is lowered by the model refinements. Consequently, the eddy viscosity (13) decreases, turbulent mixing is less intensive and the recirculation region is extended (see profile $x = D$ in fig. 2, where there is still recirculation with the present model). In fact, the recirculation region is too large with the present model.

Reacting Flow

For the reacting flow, very similar observations are made. Differences between the velocity profiles with the present model and the model of Merci (2002b) are very small here. Only around $x/D = 1$, some differences are observed (fig. 4). The more pronounced recirculation with the present model (again slightly overestimated), is again a consequence of the lower turbulent kinetic energy level.

Differences in velocity profiles with the YS model are small, but they are more pronounced in the mean mixture fraction profiles (fig. 5). Due to the higher eddy viscosity in the YS model, there is more diffusion, so that the decrease on the axis is too steep. With the present model and the model of Merci (2002b), agreement with experimental data around the axis is acceptable up to $x = D$. Further downstream, the decrease in ξ is too steep due to the underprediction of the axial velocity: convection is underestimated in comparison to the diffusion process. In the recirculation region ($x < D$), agreement with the experimental data is poor for all models for $r \in [0.2R; R]$. The relatively good agreement for the YS model is considered coincidential, since this is mainly due to excessive diffusion and turbulent mixing.

Obviously, the mean mixture fraction exerts a large influence on the mean temperature. Indeed, fig. 6 illustrates the satisfactory agreement with experimental data for the present model and the model of Merci (2002b) near the axis (except for $x = 1.8D$, as explained above), in contrast to the YS model. Due to the steeper decrease in ξ at the axis, the temperature indeed rises too rapidly (maximum temperature for $\xi = \xi_{stoich} \approx 0.05$). Within the recirculation region ($x < D$), agreement with experimental data can hardly be considered satisfactory for any of the tested turbulence models for $r \in [0.2R; R]$.

CONCLUSION

Simulation results have been presented for inert and reacting bluff body burner flows with a low-Reynolds nonlinear $k-\varepsilon$ model with a sophisticated ε-equation. The model, yielding accurate results for a variety of flows without case dependent model parameter tuning (Merci, 2002a, 2002b, 2003a), has been refined on the basis of observed shortcomings, without deteriorating previous results. However, the conclusion of the work is rather negative: despite the effort, improvements in the results are only very moderate. Probably, the studied flows, which are instationary in reality, are too complex for any two-equation RANS turbulence model in steady calculations. Unsteady RANS computations may be necessary (TNF, 2003a).

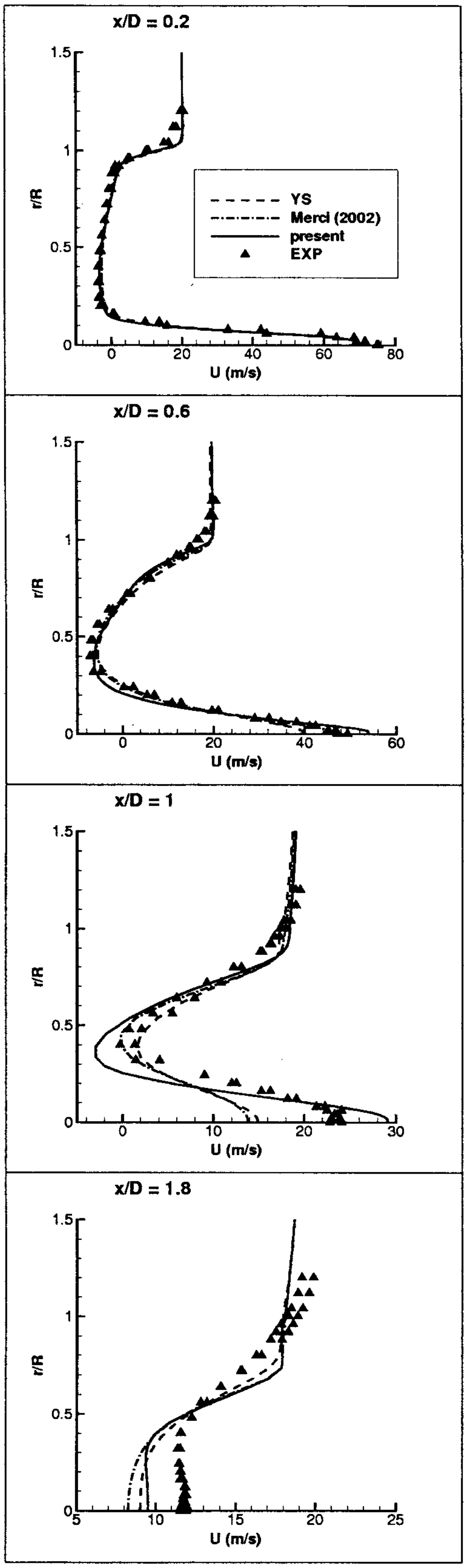

Figure 2: Mean axial velocity profiles (inert).

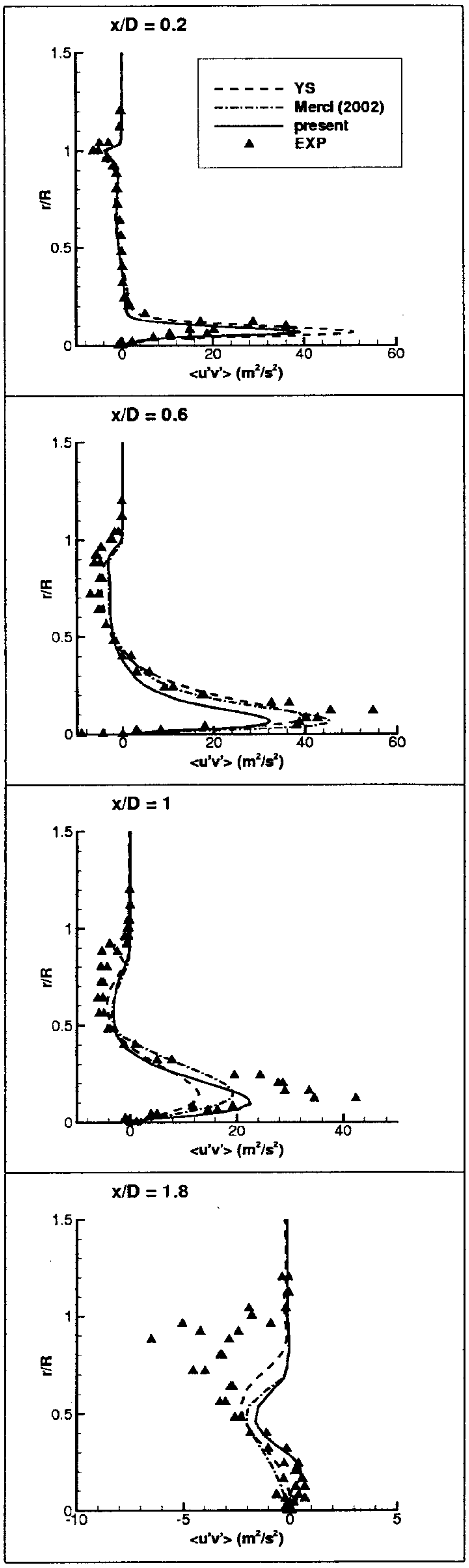

Figure 3: Turbulent shear stress profiles (inert).

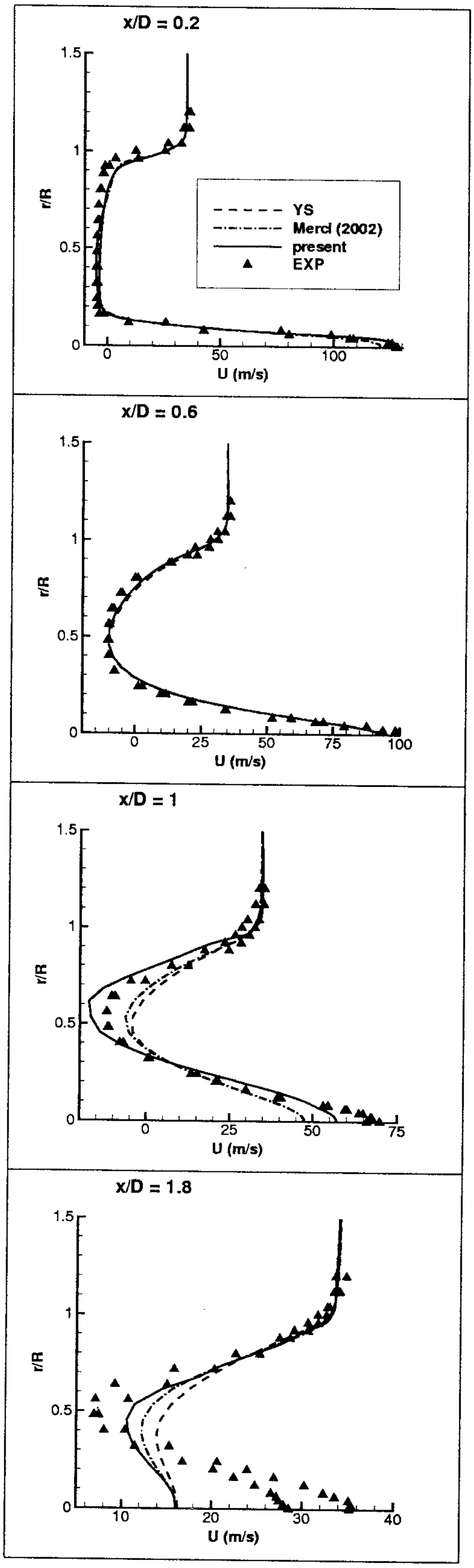

Figure 4: Mean axial velocity profiles (reacting).

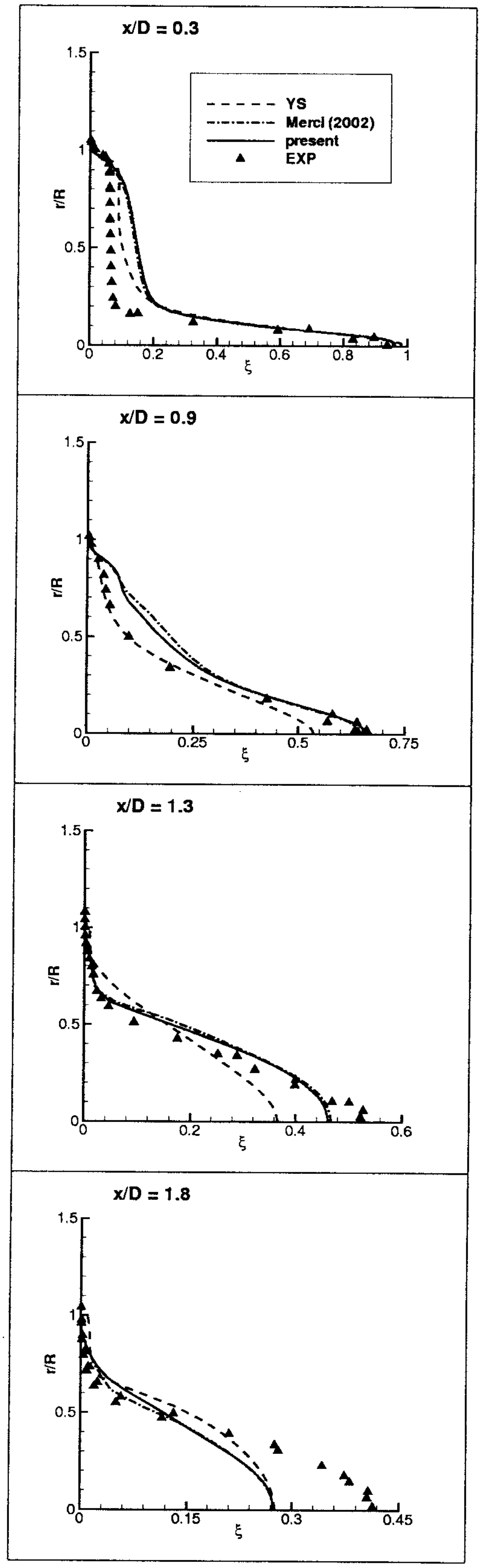

Figure 5: Mean mixture fraction profiles (reacting).

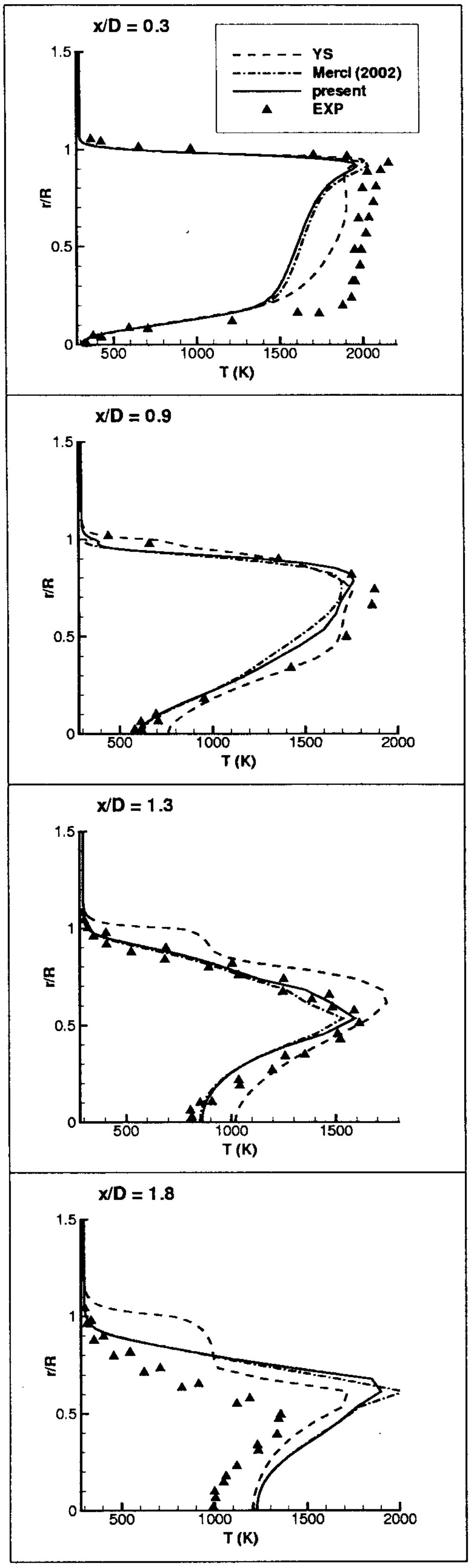

Figure 6: Mean temperature profiles (reacting).

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