

# A comparison between DNS derived and measured streamwise oscillating-cylinder wake

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## ABSTRACT

The wake of a streamwise oscillating cylinder is presently investigated. The Reynolds number, based the cylinder diameter  $d$ , investigated is 300. The cylinder oscillates at an amplitude of  $0.5d$  and a frequency  $f_e/f_s$ , where  $f_e$  is the cylinder oscillating frequency and  $f_s$  is the natural vortex shedding frequency of a stationary cylinder. Under these conditions the flow is essentially two dimensional. A two-dimensional direct numerical simulation (DNS) scheme has been developed to calculate the flow. The DNS results display a street of vortices symmetrical about the centreline, each containing two counter-rotating structures, which is in excellent agreement with measurements. The drag and lift on the cylinder have also been examined.

## 1. INTRODUCTION

The wake of an oscillating structure is frequently seen in engineering, such as flow around cable-stayed bridges, power lines, offshore structures and skyscrapers. It is therefore of both fundamental and practical significance to investigate how the wake of an oscillating cylinder behaves. Previous studies mostly focused on the transverse oscillation of one or more cylinders, perhaps because the fluctuating lift force on a structure is in many situations, say in an isolated cylinder case, one order of magnitude larger than the drag force. Subsequently the lateral structural oscillation prevails against that in the streamwise direction. However, the streamwise force can be significant. Structural failure may result from synchronization between the fluid excitation force and the system natural frequency in the streamwise direction. One example is the damage of piling during the construction of an oil terminal on the Humber estuary of England in 1960s (Griffin and Ramberg 1976). The problem of streamwise oscillation could be particularly severe when a lightly damped cylindrical structure is used in water.

There have been a limited number of studies involving a streamwise oscillating cylinder in a cross-flow. These studies have uncovered many important aspects of physics associated with a streamwise oscillating cylinder wake. For example, It is now established that the flow structure is dependent on the combination of  $A/d$  and  $f_e/f_s$  (Karniadakis & Triantafyllou 1989), where  $f_e$  is the excitation frequency and  $f_s$  is the natural vortex shedding frequency of a stationary cylinder,  $A$  and  $d$  are the oscillation amplitude and the diameter of cylinder, respectively. Ongoren & Rockwell (1988) classified the flow structure ( $A/d = 0.13 \sim 0.3$ ,  $f_e/f_s = 0.5 \sim 4.0$ ) into 4 modes, i.e. S mode for the symmetric vortex formation and A-I, III, IV modes for the anti-symmetric vortex formation. However, the cylinder oscillation amplitude previously investigated has been relatively small ( $A/d \leq 0.3$ ). In engineering, the structural oscillation amplitude could be in the order of one cylinder diameter or even significantly larger. Xu *et al.* (2002) examined experimentally the flow structure behind a streamwise oscillating cylinder at a relatively large  $A/d$  ( $= 0.5 \sim 0.67$ ) for  $f_e/f_s = 0 \sim 3.1$  and observed a new flow structure, consisting of a street of symmetric binary vortices. However, due to the limitation of experiments, many aspects of the physics for this new flow structure remain to be clarified. For example, there is no information on the pressure field of the flow; the drag and lift forces associated with the flow structure is not available. The present work aims to conduct a numerical investigation on this flow and complement the experimental investigation.

## 2. Numerical formulation

In the present numerical simulation an incompressible flow past a streamwise oscillating circular cylinder is considered. The Reynolds number,  $Re$  ( $= U_\infty d/\nu$ , where  $U_\infty$  is the free-stream

velocity and  $\nu$  is the kinematic viscosity), investigated is 300. At this  $Re$ , experimental data indicated a predominantly two-dimensional (2-D) flow. Therefore, a 2-D direct numerical simulation (DNS) is considered to be adequate and subsequently used for the calculation of the flow. The cylinder is allowed to oscillate, as specified by  $x_t/d = A/d \sin(2\pi f_e t)$ ,  $y_t = 0$ , where  $t$  is time,  $x_t$  and  $y_t$  denote the cylinder position.  $A/d$  and  $f_e/f_s$  are 0.5 and 1.8, respectively, following the experimental conditions (Xu et al. 2002).

The governing equations for the flow are given by continuity and Navier-Stokes equations. With the frame of reference fixed to the oscillating cylinder, Navier-Stokes equations may be written in terms of curvilinear coordinates, viz.

$$\begin{aligned} \frac{\partial u_i}{\partial \tau} + \frac{1}{J} [(u - x_\tau)y_\eta - (v - y_\tau)x_\eta] \frac{\partial u_i}{\partial \xi} + \frac{1}{J} [(u - x_\tau)y_\xi + (v - y_\tau)x_\xi] \frac{\partial u_i}{\partial \eta} \\ = \frac{1}{J} \left[ -\frac{1}{\rho} \left( \frac{\partial p}{\partial \xi} y_\eta - \frac{\partial p}{\partial \eta} y_\xi \right) \right] + \frac{\nu}{J^2} \left( \frac{\partial^2 u_i}{\partial \xi^2} + \frac{\partial^2 u_i}{\partial \eta^2} \right) \end{aligned}$$

where  $\xi = \xi(x, y, t)$ ,  $\eta = \eta(x, y, t)$ ,  $\tau = \tau(x, y, t)$ , and  $x_\tau$  represents the grid speed in the in-line direction. The flow governing equations can be further written in the integral form:

$$\frac{d}{dt} \int \rho \phi dV + \int [\rho(V - V_b)\phi] \cdot dS = \int S_\phi dV,$$

where  $\phi = 1$ ,  $V$  denotes the continuity and momentum equations, respectively,  $V_b$  and  $S_\phi$  represent the grid velocity vector and the corresponding source terms, respectively. No-slip boundary condition is applied to the cylinder, i.e. the fluid velocity on the cylinder surface equals to the cylinder oscillation velocity. At the exit of the computation domain, zero convection is assumed to suppress boundary disturbances, i.e.

$$d\psi/dt + u_m d\psi/dx = 0,$$

where  $\psi$  can be an arbitrary transport quantity and  $u_m$  represents the average  $u$ -velocity at the exit boundary. The governing equations are discretized based on finite volume method formulation.

The cylindrical-polar coordinate system is used to generate the grid, providing good body-fit and accuracy. The circular computational domain is  $30d$ , divided into inner, outer and transition regions (Figure 1 a-b) in view of the cylinder oscillation. The inner region ( $r < 1.5$ ) is fixed to the oscillating cylinder, whose coordinate system is therefore non-inertial. At the boundaries of the transition region ( $1.5d < r < 10d$ ), grid lines have been designed to ensure smoothness in terms of both grid spacing and grid velocity. This region is also in non-inertial frame with a decaying grid speed as  $r$  increases. Total mesh number is about  $250 \times 120$  ( $\theta$  by  $r$ ). Note that the

grid density is not uniform (Figure 1c). A computational domain of  $50d \times 30d$  was tested with HO-hybrid grid system (Figure 1d). The effect of the grid system on the flow structure is expectedly insignificant (not shown). The non-dimensional time step used in the calculation is  $(1/f_e)/128$ .

### 3. PRESENTATION OF RESULTS

The structural oscillation and vortex shedding may be synchronized or may not. The present investigation focused on the synchronization state. In order to compare with the experimental data, calculation was conducted for  $f_e/f_s = 1.8$ ,  $A/d = 0.5$  and  $Re = 300$ , which were close to the experimental conditions.

The calculated vorticity contours display a symmetrically formed binary vortex street (Figure 2a). Each binary vortex encloses two counter-rotating vortices. The flow structure appears the same as that (Figure 2b) shown by streaklines measured using the laser-induced fluorescence (LIF) technique (Xu et al. 2002). The contours (not shown) at different time indicate that, when the cylinder moves oppositely to the flow direction, one clockwise rotating vortex above the centreline forms due to the natural vortex shedding. As the cylinder moves in the flow direction, the fluid near the cylinder wall goes along with the cylinder under the viscosity effect, but the fluid further away now moves oppositely (right to left) relative to the cylinder. Therefore, a vortex of the anti-clockwise sense begins forming. Eventually, the structure containing a pair of counter-rotating vortices separates from the cylinder. As they evolve downstream, the two vortices interact, experience the cancellation of oppositely signed vorticity and eventually the naturally shed vortex overpowers the other. At the same time, the counter-rotating vortices form another binary vortex and separate from the lower side of the cylinder. The observation from the calculated data shows extraordinary similarity to that observed in experimental data (Xu et al. 2002). The good agreement provides a validation for the present DNS scheme. Note that the measured vortex street appears laminar at  $Re = 500$ , which is different from the DNS data ( $Re = 300$ ). This should not invalidate the comparison; Zdravkovich (1997) suggested that, as the oscillation of a cylinder exceeds a threshold amplitude, the oscillation amplitude and frequency may become the governing parameter of the flow regime within a certain range of the free-stream velocity, instead of  $Re$ .

In order to calculate the drag and lift coefficients, the pressure around the cylinder is obtained through the integration of the following equation,

$$\frac{\partial p}{\partial n} \Big|_{body} = -\rho \bar{a} \cdot \bar{n} + \mu V^2 \bar{u} \cdot \bar{n},$$

where  $\bar{a}$  is the acceleration of the oscillating cylinder,

and  $\bar{n}$  is the unit normal vector of the cylinder surface. This approach may better account for the non-inertial and curvature effects on the pressure around the cylinder. The first order approximation would be  $\partial p / \partial n|_{body} = 0$ . The lift and drag are

obtained by integrating the pressure around the cylinder. The calculated fluctuating lift coefficient  $C_L'$  (Figure 3) is extremely small, consistent with the flow structure symmetrical about the centreline. However, the fluctuating drag coefficient  $C_D'$  is quite large, its maximum exceeding 5, significantly larger than that on a stationary cylinder. The analysis of the pressure field is under way. The effect of a further increasing  $A/d$  and  $f_e/f_s$  on the flow structure will be examined. DNS will also be conducted at a higher  $Re$  so that the flow structure in a turbulent regime could be investigated.

#### 4. CONCLUSIONS

The investigation leads to the following conclusions. Firstly, the presently developed DNS scheme can be used to calculate reliably the wake of a cylinder oscillating at a relatively large amplitude and frequency ratio. Secondly, the experimental finding of a symmetrical binary street at  $A/d = 0.5$  and  $f_e/f_s = 1.74$  by Xu et al. (2002) has been reconfirmed numerically. The numerical data further indicate that, while the drag coefficient on the cylinder has been increased appreciably compared with a stationary cylinder, the lift coefficient approaches zero because of the flow structure symmetrical about the centreline.

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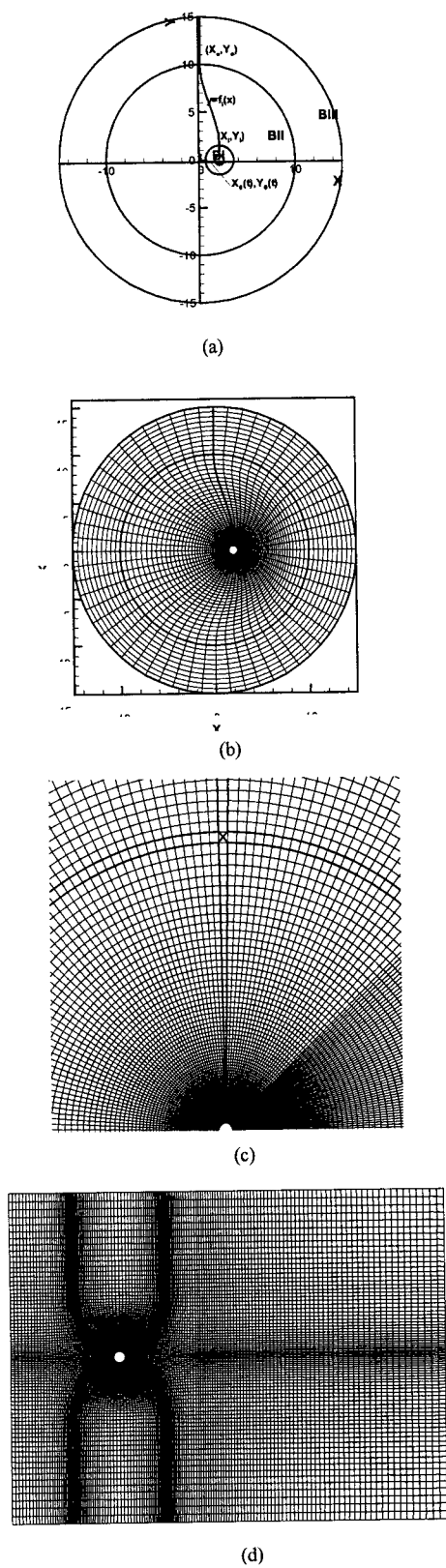
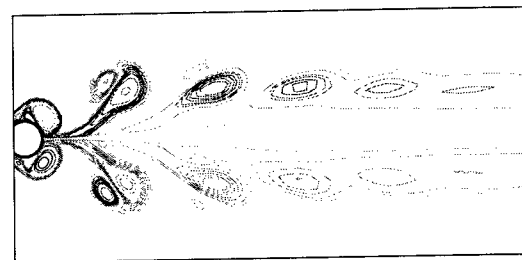
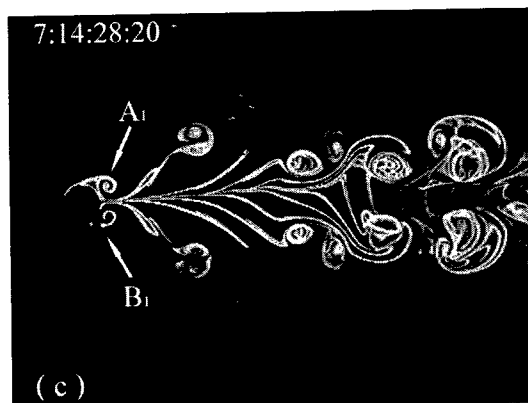


Figure 1 (a)-(b) Three regions of the computational domain. (c) O-type computational grid. (d) HO-hybrid grid.



(a)



(b)

Figure 2 Comparison between DNS result and measurements. (a) Vorticity contours from DNS,  $Re = 300$ ,  $f_e/f_s = 1.8$ ; (b) measured streaklines (Xu et al. 2002),  $Re = 500$ ,  $A/D = 0.5$ ,  $f_e/f_s = 1.74$ . Flow is left to right.

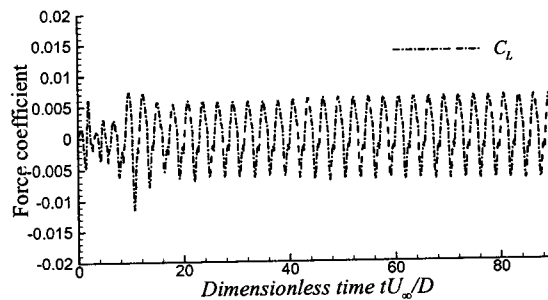
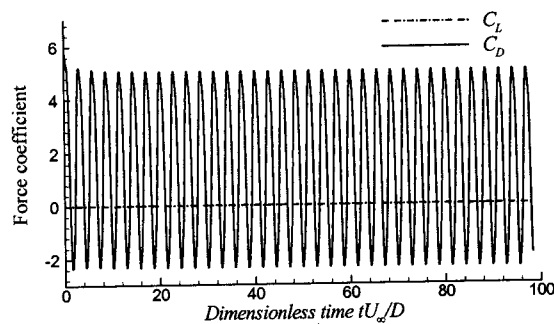


Figure 3 Time histories of drag (upper) and lift (lower) coefficients.