NUMERICAL SIMULATION OF TRANSONIC BUFFET OVER AN AIRFOIL

Eric Goncalves, Jean-Christophe Robinet
SINUMEF Laboratory, ENSAM-Paris
151, bd. de l'Hopital, Paris, 75013, France
Eric.Goncalves@hmg.inpg.fr, Jean-Christophe.Robinet@paris.ensam.fr

Robert Houdeville
Department of Aerodynamics and Energetics
ONERA-Toulouse
BP 4025, Toulouse, 31055 cedex, France
Robert.Houdeville@onecert.fr

ABSTRACT

The prediction of shock induced oscillations over transonic rigid airfoils is important for a better understanding of the buffeting phenomenon. The unsteady resolution of the Navier-Stokes equations is realized with various transport-equation turbulence models in which corrections are added for non-equilibrium flows. The lack of numerical efficiency due to the CFL stability condition is circumvented by the use of a wall law approach and a dual time stepping method. Comparisons are made with experimental results obtained for the supercritical RA16SC1 airfoil. They show the interest of the use of the SST correction or realizability conditions to get correct predictions of the frequency, the amplitude and the pressure fluctuations over the airfoil.

INTRODUCTION

The transonic buffet is an aerodynamic phenomenon which results in a self-sustained periodic motion of the shock over the surface of the airfoil, due to the development of instabilities caused by the boundary layer separation and the shock wave interaction. This problem is of primary importance for aeronautics applications because it can lead to the buffeting phenomenon through the mechanical response of the wing structure. The large amplitude periodic variation of lift associated with buffet limits the cruising speed of commercial aircrafts and severely degrades the manoeuvrability of combat aircrafts. A detailled description of the physical features of shock induced osillations (SIO) are given by Lee (2000).

The classical eddy-viscosity models based on the linear Boussinesq relation are known to be afflicted by numerous weaknesses, including an inability to capture the boundary layer separation and a violation of realizability at large rates of strain. Moreover, theses models are formulated following the spectral energy of Kolmogorov with an equilibrium assumption of turbulence and they are calibrated for steady flows. For unsteady flows, the presence of coherent structures can break this equilibrium and yield to a different energy distribution. An observed consequency is the over-production of eddy-viscosity which limits the unsteadiness development and modifies the flow topology. In the present study, we have investigated some improvement and correction for linear models by limiting the eddy-viscosity or the production of turbulence kinetic energy instead of testing more sophisticated models as Barakos (2000) or Wang (2000). A first study has shown the great influence of the shear stress transport (SST) Menter correction (Goncalves, 2001). Moreover, another way to improve the behaviour of models have been tested as the use of realisability constraints and a recalibration of the constants for the $k-\omega$ model.

NUMERICAL METHODS

The numerical simulations have been carried out using an implicit CFD code solving the uncoupled RANS-turbulent systems for multi-domain structured meshes. This solver is based on a cell-centered finite-volume discretization. Numerical fluxes are computed with the Jameson scheme for the mean flow and a second-order Roe scheme for the turbulence transport equations (Couaillier, 1999).
Time integration is performed through a matrix-free implicit method (Luo, 1998). The feature of this method is that the storage of the Jacobian matrix is completely eliminated. The implicit time integration procedure leads to a system which is solved with a Point Jacobi relaxation algorithm.
For steady state computations, convergence acceleration is obtained using a local time step and the full approximation storage multigrid method.
For unsteady computations, a dual time stepping method is used to overcome the lack of numerical efficiency of the global time stepping approach.

TURBULENCE MODELLING

Various two-equation turbulence models are used for the present study : the Smith $k-l$ model, the Wilcox $k-\omega$ model, the Menter $k-\omega$ model, the high Reynolds version of the Jones-Launder $k-\varepsilon$ model, the Kok $k-\omega$ model and also the one-equation Spalart-Allmaras model.
As the discretization scheme does not insure the positivity of the turbulent conservative variables, limiters are used to avoid negative values. These limiters are set equal to the corresponding imposed boundary values in the far field.
For unsteady flows, corrections are added to the standard model to limit the over-production of eddy-viscosity.

The SST limiter

The SST correction is based on the empirical Bradshaw's assumption which binds the shear stress to the turbulent kinetic energy for two-dimensional boundary layer. This correction is extended for the $k-\varepsilon$ model and the $k-l$ model.

The non-equilibrium correction of Smith

The non-equilibrium correction of Smith (1997), developped for the $k-l$ model, consists in modifying the computation of the eddy viscosity by introducing a function σ :

$$\mu_t = \sigma \mu_{t_{eq}} \; ; \; \sigma = \frac{\alpha - 0.25\alpha^{1/2} + 0.875}{\alpha^{3/2} + 0.625} \; ; \; \alpha = \frac{\min\left(P_{k_{eq}}, 0\right)}{\varepsilon}$$

where the subscript *eq* denotes the equilibrium value. The non-equilibrium function have been chosen to limit the eddy-viscosity when production is greater than dissipation and to increase the viscosity above the equilibrium model value in the contrary case.

Durbin correction - link with realizability

Based on the realizability principle a minimal correction was derived for two-equation turbulence models and was shown to cure the stagnation-point anomaly (Durbin, 1996). The condition to ensure realizability in a three-dimensional flow is :

$$C_\mu \le \frac{1}{s\sqrt{3}} \quad ; \quad s = \frac{k}{\varepsilon} S \quad ; \quad S^2 = 2 S_{ij} S_{ij} - \frac{2}{3} S_{kk}^2$$

It allows to obtain a weakly non-linear model with a C_μ coefficient function of the dimensionless mean strain rate :

$$C_\mu = \min\left(C_\mu^o, \frac{c}{s\sqrt{3}}\right) \quad \text{with} \quad c \le 1$$

where C_μ^o is set to the constant value 0.09 and the constant c is fixed to 0.5.
For the $k-\varepsilon$ model, we have the relation :

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \quad ; \quad C_\mu = \min\left(C_\mu^o, \frac{0.3}{s}\right)$$

And for the $k-\omega$ model :

$$\mu_t = \rho C_\mu \frac{k}{\omega} \quad ; \quad C_\mu = \min\left(1, \frac{0.3}{C_\mu^o s}\right)$$

This model has been successfully tested on shock wave/boundary layer interactions with the Wilcox $k-\omega$ model (Thivet, 2002).

Recalibration of the constants for the Kok model

When regarding unsteady results of the Kok model (2000), it appears that the calibration of the constant has a great influence. This model has been built in order to resolve the dependence on freestream values of ω. The turbulence transport equation of the model are given by :

$$\begin{aligned}
\frac{\partial \rho k}{\partial t} + \operatorname{div}\left[\rho k \mathbf{V} - (\mu + \sigma_k \mu_t) \operatorname{grad} k\right] &= P_k - \beta^* \rho k \omega \\
\frac{\partial \rho \omega}{\partial t} + \operatorname{div}\left[\rho \omega \mathbf{V} - (\mu + \sigma_\omega \mu_t) \operatorname{grad} \omega\right] &= P_\omega - \beta \rho \omega^2 \\
+ \sigma_d \frac{\rho}{\omega} \operatorname{grad} k . \operatorname{grad} \omega &
\end{aligned}$$

Kok obtained additional constraints for the constants :

$$\sigma_\omega - \sigma_k + \sigma_d > 0$$
$$\sigma_k - \sigma_d > 0$$

The choice of Kok is :

$$\sigma_\omega = 0.5 \quad ; \quad \sigma_k = 2/3 \quad ; \quad \sigma_d = 0.5$$

We have change the constant values, following all constraints, to show the sensitivity of the model to the cross-diffusion term $\operatorname{grad} k . \operatorname{grad} \omega$ in the ω equation for these unsteady computations.

test 1 : $\sigma_\omega = 0.5 \; ; \; \sigma_k = 2/3 \; ; \; \sigma_d = 0.65$
test 2 : $\sigma_\omega = 0.5 \; ; \; \sigma_k = 1 \; ; \; \sigma_d = 0.85$

WALL LAW APPROACH

At the wall, a no-slip condition is used coupled to a wall law treatment. It consists in imposing the diffusive flux densities, required for the integration process, in adjacent cells to a wall. The shear stress and the heat flux are obtained from an analytical velocity profile. As concerns the turbulent quantities, the turbulent kinetic energy is set equal to zero at the wall and its production is computed from the velocity profile. The second turbulent variable is deduced from an analytical relation and is imposed in adjacent cells to a wall (Goncalves, 2001).
To use the wall law approach with the multi-grid algorithm, the wall law boundary condition is applied on the fine grid and the classical no-slip condition is applied on the coarse grids.

RESULTS

Experimental conditions

The experimental study has been conducted in the S3MA ONERA wind tunnel with the RA16SC1 airfoil (Benoit, 1987). It is a supercritical airfoil with a relative thickness equal to 16% and a chord length c equal to $180mm$. The flow conditions are : $M_\infty = 0.732$, $T_i = 283K$, $Re_c = 4.2\,10^6$ and the angle of attack α varies from 0 to 4.5°. Transition is fixed near the leading edge at $x/c = 7.5\%$ on both sides of the airfoil.

Computational conditions and meshes

For the computations, experimental corrections are used due to the lateral wall effects. The Mach number is decreased by 0.09 and the angle of attack is decreased by 1° at all incidences with respect to experiment. The grid is a C-type topology. It contains 321x81 nodes, 241 of which are on the airfoil (an enlargement of the mesh around the airfoil is plotted in figure 1. This mesh has been obtained from a fine mesh, with y^+ values of order of unity near the wall, by removing 16 lines near the wall. The y^+ values of the coarse mesh, at the center of the first cell, are presented in figure 2 for a steady computation at $\alpha = 4°$.

Comparison of turbulence models

The frequency f and the amplitude of the lift coefficient ΔC_L are reported on the table 1 for all turbulence models and for three angles of attack $\alpha = 3, 4$ and $5°$. These values correspond respectively to the buffet onset, the established phenomenon and the buffet exit i.e. the back to a steady state.

First, we examinate the capacity of turbulence models to restitute the natural unsteadiness of the flow without

and with any correction.
The Spalart-Allmaras model is able to reproduce the buffet phenomenon, the frequency being underestimated with respect to the experimental values.
The Smith $k-l$ model need a correction to obtain unsteady results. The Smith correction does not seem to be efficient for these unsteady computations. Yet, the SST corrections make the model able to simulate the shock induced oscillations.
The Jones-Launder $k-\varepsilon$ model is able to provide unsteady solutions without correction. Yet, the lift amplitude is largely underestimated for $\alpha = 4^o$ and the model completely damps the natural unsteadiness for the onset. The shock induced oscillations appear to an angle of attack of 3.7^o rather than 3^o for the experimental value. The add of the SST correction allows to obtain a larger amplitude of the lift coefficient but the buffet onset is not predicted. The realizability constraints of Durbin make the model able to get oscillations at $\alpha = 3^o$ but the amplitude is largely underpredicted. For the established phenomenon, the amplitude is closer to the experimental value when using the Durbin correction in comparison with the use of the SST correction.
The Wilcox and Menter $k-\omega$ models fail to compute this application, results obtained being completely steady. The add of the SST correction to the Menter model allows to predict self-sustained oscillations with a very good agreement with respect to the experimental data.
The Kok $k-\omega$ is able to compute natural unsteadiness for the established phenomenon but the buffet onset and the buffet exit are not predicted. It seems that the SST corrections and the realizability constraints do not modify the behaviour of the model.
The recalibration of the constant of the Kok model has been tested for the three angles of attack. The frequency and the amplitude of the lift coefficient are reported on the table 2. The increasing of the σ_d coefficient induces an increasing of the amplitude of the lift coefficient for all angles of attack and, especially, allows to predict the entrance in the SIO domain. Yet, there is no buffet exit at $\alpha = 5^o$.

When comparing all turbulence models, the best results are clearly obtained with the SST Menter model, for the three angles of attack. The amplitude of the lift coefficient if remarkably predicted and the buffet exit is only predicted when using this model. All these results show the interest of the use of a correction for this unsteady application.

The RMS values of the pressure fluctuations over the airfoil are compared in figure 3 with experimental results at the angle of attack $\alpha = 3^o$. The pressure side is represented by the negative values of the abscisse. The SST Menter model clearly provides the best result. Over the pressure side the computed pressure fluctuation is in very close agreement with the measured values. The peak on the upper side, corresponding to the shock movment, is well located but is underestimated by 15%. The results obtained by the other turbulence models are very far from the experimental data, pressure fluctutations over the airfoil are largely underestimated.
In figure 4 are presented the RMS pressure fluctuations over the airfoil obtained with the modified Kok $k-\omega$ models. For the two tests, the peak over the upper side is at a downstream location in comparison with experiment. Both model under-estimate the maximum value on the upper side, especially the test 1 modified model, and the amplitude of the shock displacement. Over the pressure side, the test 1 Kok model under-predicts the level of pressure fluctuations.

The RMS values of the pressure fluctuations over the airfoil are plotted in figure 5 for $\alpha = 4^o$ and for the $k-\varepsilon$ models. We can observe the great influence of the SST correction and the realizability constraints. Without any correction, the pressure fluctuations are largely under-estimated on the pressure side and on the trailing edge of the upper side. The amplitude of the shock displacement are too weak in comparison with the experimental values and the peak is not well located on the upper side. With corrections, the pressure fluctuations on the trailing edge of the upper side are close to the experimental data. The amplitude of the shock and the peak location are in better agreement with the experiment. Yet, the maximum value on the upper side if over-predicted and, on the pressure side, the fluctuations level is over-estimated. Perhaps the change of the value of the constant c in the realizability constraints should improve the results.

The RMS pressure fluctuations over the airfoil obtained with the Kok $k-\omega$ models are the same and are not plotted together. In figure 6 are plotted the RMS pressure fluctuations for all generic turbulence models. Over the pressure side, the $k-\varepsilon$ with the Durbin correction over-predicts the pressure fluctuation and all other models give good results. Over the upper side, the peak is well located except for the Kok model. The maximum value is under-estimated by the SST Menter model. Downstream the shock location, at the trailing edge, a large discrepancy with experimental values, which can reach 50%, is observed for some models.

Finnaly, the RMS values of the pressure fluctuations over the airfoil are presented in figure 7 for the modified Kok $k-\omega$ models. When the constant is increased, the displacement of the shock over the upper side is extended and the pressure levels becomes more important.

CONCLUSION

The unsteady two-dimensional computations of the transonic buffet over a supercritical airfoil are performed with an implicit solver which reveals the great sensitivity to the turbulence modelling. Usual turbulence models fail in correctly predicting shock induced oscillations and the introduction of a weakly non-linear correction in the definition of the eddy-viscosity yields better results. Two different corrections are tested, the use of the empirical Bradshaw's assumption through the SST correction and the enforcement of the realizability principle. Another approach consists in recalibrating the constant of the model for unsteady flows. For the Kok $k-\omega$ model, by increasing the constant of the cross-diffusion term, results are improved and the buffet onset can be predicted.

	$\alpha = 3^o$		$\alpha = 4^o$		$\alpha = 5^o$	
model	f (Hz)	ΔC_L	f (Hz)	ΔC_L	f (Hz)	ΔC_L
experiment	**88**	**0.11**	**100**	**0.308**	**probably steady state**	
Spalart-Allmaras	82	0.0146	92	0.325	100	0.55
$k-l$	-	-	steady state		-	-
$k-l$ corrected	-	-	steady state		-	-
$k-l$ SST	79.5	0.0084	97.6	0.296	101.8	0.53
$k-\varepsilon$	steady state		95.6	0.17	97.6	0.43
$k-\varepsilon$ SST	steady state		95.6	0.48	101.8	0.67
$k-\varepsilon$ Durbin	85.2	0.012	93.7	0.437	101.8	0.67
$k-\omega$ Wilcox	-	-	steady state		-	-
$k-\omega$ Menter	-	-	steady state		-	-
$k-\omega$ SST Menter	90	0.11	96.6	0.33	steady state	
$k-\omega$ Kok	steady state		94.6	0.26	95.6	0.48
$k-\omega$ Kok SST	steady state		94.6	0.26	96.6	0.445
$k-\omega$ Kok Durbin	steady state		94.6	0.26	96.6	0.45

Table 1: Frequency and amplitude of the lift coefficient

	$\alpha = 3^o$		$\alpha = 4^o$		$\alpha = 5^o$	
model	f (Hz)	ΔC_L	f (Hz)	ΔC_L	f (Hz)	ΔC_L
experiment	**88**	**0.11**	**100**	**0.308**	**steady state**	
$k-\omega$ Kok	steady state		94.6	0.26	95.6	0.48
$k-\omega$ Kok - test 1	91	0.051	93.7	0.318	95.6	0.55
$k-\omega$ Kok - test 2	87.6	0.084	91	0.46	91	0.735

Table 2: Frequency and amplitude of the lift coefficient - modified Kok models

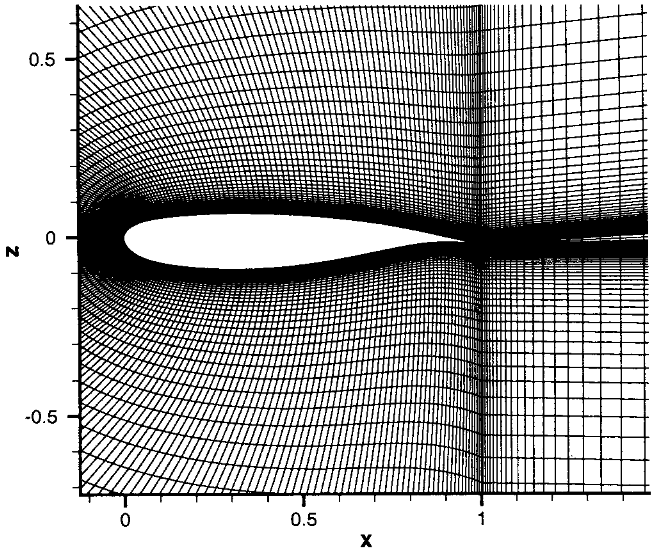

Figure 1: Enlargement of the mesh around the airfoil

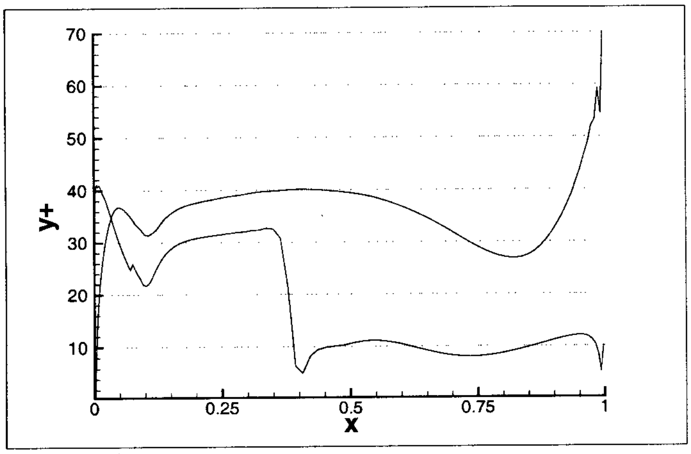

Figure 2: y^+ values of the first cell - 4° angle of attack - Steady computations

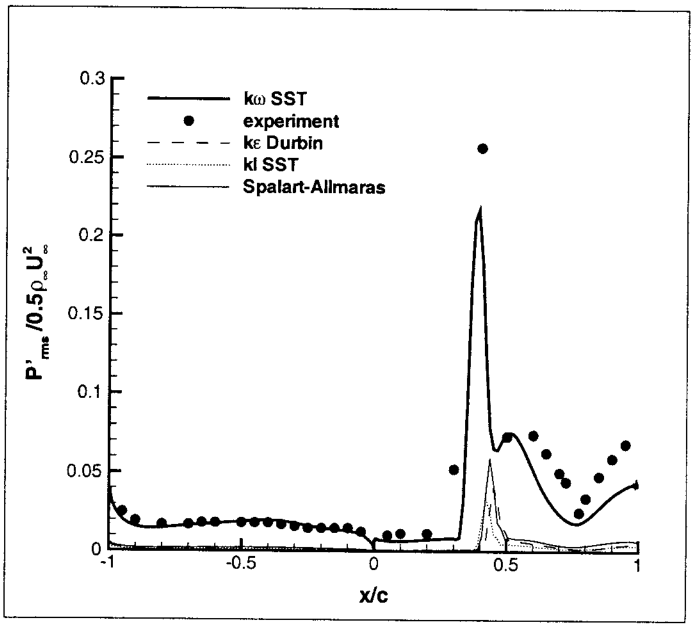

Figure 3: RMS pressure fluctuations over the airfoil - $\alpha = 3^o$

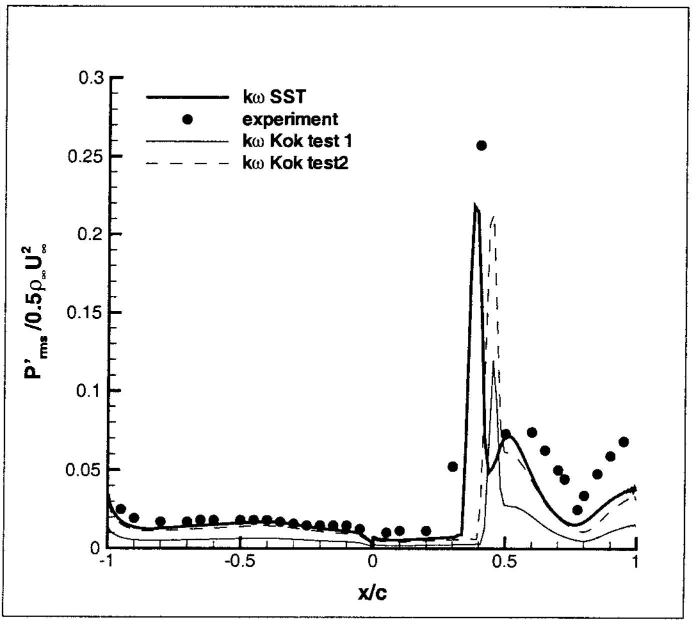

Figure 4: RMS pressure fluctuations over the airfoil - $\alpha = 3^o$ - modified $k-\omega$ Kok models

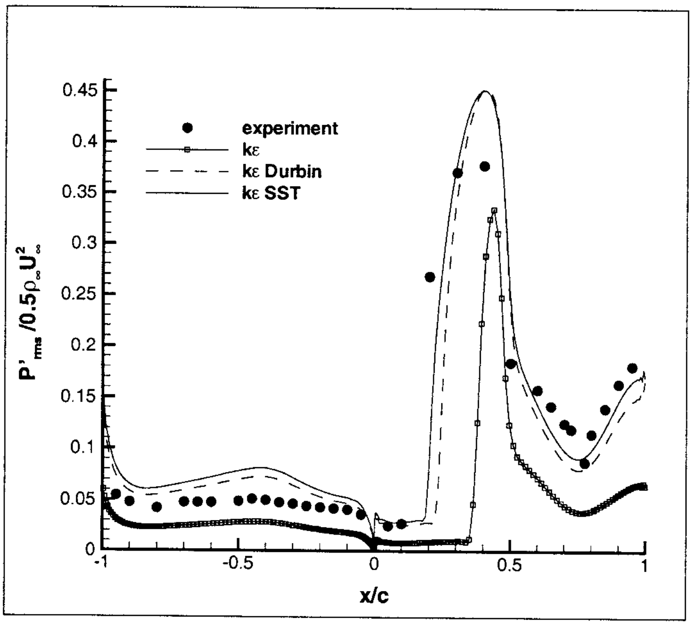

Figure 5: Rms pressure fluctuations over the airfoil - $\alpha = 4^o$ - $k-\varepsilon$ models

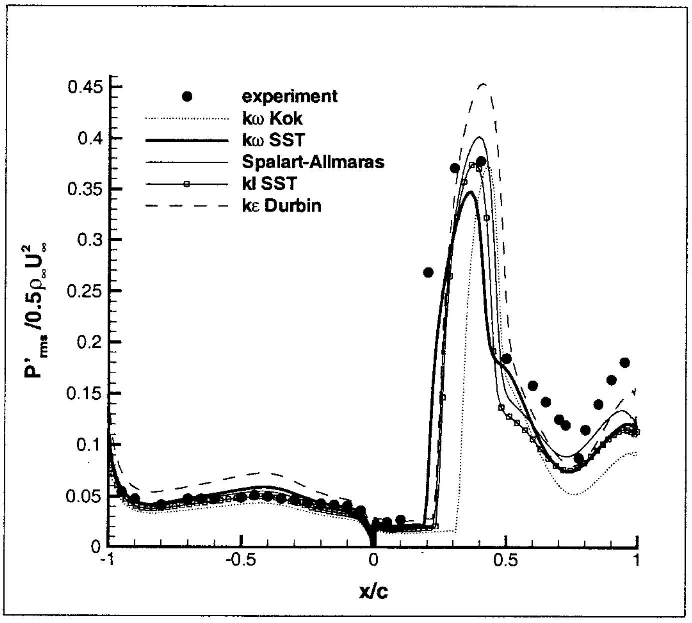

Figure 6: Rms pressure fluctuations over the airfoil - $\alpha = 4^o$

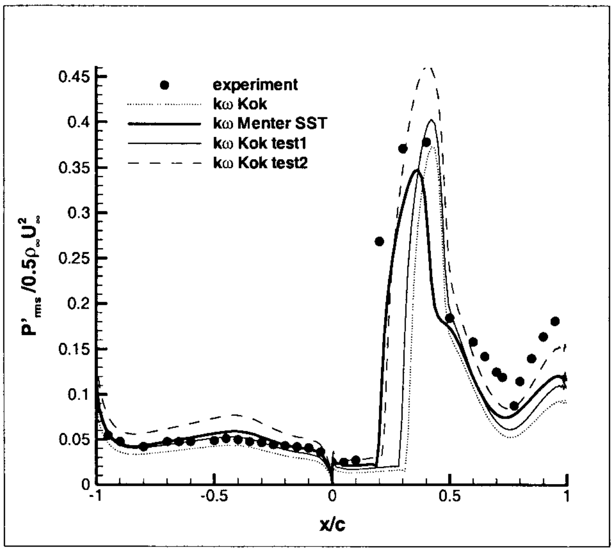

Figure 7: Rms pressure fluctuations over the airfoil - $\alpha = 4^o$ - modified $k - \omega$ Kok models

REFERENCES

Barakos, G., and Drikakis, D., 2000, "Numerical Simulation of Transonic Buffet Flows Using Various Turbulence Closures", *International Journal of Heat and Fluid Flow*, 21, pp. 620-626.

Benoit, B., and Legrain, I., 1987, "Buffeting Prediction for Transport Aircraft Applications Based on Unsteady Pressure Measurements", *Proceedings, AIAA 5th Applied Aerodynamics Conference*, Montery, CA.

Couaillier, V., 1999, "Numerical Simulation of Separated Turbulent Flows Based on the Solution of RANS/Low Reynolds Two-Equation Model", *Proceedings, AIAA Paper 99-0154*.

Durbin, P.A., 1996, "On the $k - \varepsilon$ stagnation point anomaly", *International Journal of Heat and Fluid Flow*, Vol. 17(1), pp. 89-90.

Goncalves, E., and Houdeville, R., 2001, "Reassessment of the wall functions approach for RANS computations", *Aerospace Science and Technology*, Vol. 5, pp. 1-14.

Goncalves, E., and Houdeville, R., 2001, "Numerical Simulation of Shock Oscillations over Airfoil using a Wall Law Approach", *Proceedings, AIAA 2001-2857, 31st AIAA Fluid Dynamics Conference*, Anaheim, California.

Kok, J.C., 2000, "Resolving the Dependence on Freestream Values for the $k - \omega$ turbulence model", *AIAA Journal*,Vol. 38(7), pp. 1292-1295.

Lee, B.H.K., 2001, "Self-Sustained Shock Oscillations on airfoils at transonic speeds", *Progress in Aerospace Sciences*, Vol. 37, pp. 147-196.

Luo, H., Baum, J.D., and Lohner, R., 1998, "A fast, matrix-free implicit method for compressible flows on unstructured grids", *Journal of Computational Physics*, Vol. 146, pp. 664-690.

Smith, B.R., 1997, "A non-equilibrium Turbulent Viscosity Function for the $k - l$ Two Equation Turbulence Model", *Proceedings, AIAA 97-1959, 28th Fluid Dynamics Conference*, Snowmass Village, Colorado.

Thivet, F., 2002, "Lessons Learned from RANS Simulations of Shock-Wave/Boundary Layer Interactions", *Proceedings, AIAA 2002-0583, 40th AIAA Aerospace Sciences Meeting & Exhibit*, Reno, Nevada.

Wang, D., Wallin, S., Berggren, M., and Eliasson, P., 2000, "A computationnal study of unsteady turbulent buffet aerodynamics", *Proceedings, AIAA 2000-2657*, Denver, Colorado.