MODELLING OF COMPRESSIBILITY EFFECTS IN MIXING LAYER

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ABSTRACT

The ability of turbulence models to predict self-similar mixing layers is investigated. The influence of velocity ratio is well captured but no model reproduces the sensitivity of the mixing layer to density differences. A correction proposed for boundary layer flows hardly affects mixing layer predictions. A correction accounting for baroclinic effect is derived but is not satisfactory. At last, compressible turbulence effects are investigated. Without corrections, models cannot predict the spreading rate reduction. Standard corrections, based upon dilatational terms, predict too weak a reduction. The sonic eddy concept is validated whatever the turbulence model. A form suitable for Navier-Stokes codes is proposed.

INTRODUCTION

The prediction of the spreading rate of a mixing layer and of its entrainment remains a challenge for turbulence models. A simple test case is the prediction of self-similar mixing layers.

For self-similar mixing layers, the standard argument (see e.g. Dimotakis (1991)) is that, in a frame linked to the coherent structures, the spreading rate is proportional to the velocity difference across the layer, which is the only velocity scale. In the laboratory reference frame, the spreading rate is thus proportional to the ratio of the velocity difference to the advection speed of the structures. This advection speed is determined considering equilibrium of the stagnation pressure at the stagnation point between coherent structures. So, it is stated that the spreading rate depends upon three parameters

- the velocity ratio of the two streams $r = \frac{u_2}{u_1}$, where the subscripts $_1$ and $_2$ respectively refer to high- and low-speed streams,
- the density ratio $s = \frac{\rho_2}{\rho_1}$,
- the convective Mach number M_c which characterizes the compressible character of the turbulent motion. For gases of identical isentropic exponents, it reads $M_c = \frac{u_1 - u_2}{c_1 + c_2}$ where c is the speed of sound.

This paper addresses the ability of standard turbulence models to correctly reproduce the evolution of the spreading rate with each parameter. The effect of the velocity and density ratios will be addressed first, the rôle of the compressible character of the turbulent motion will be discussed later.

COMPRESSIBLE CHARACTER OF THE MEAN FLOW

Experimental references

The standard analysis yields the following expression for the mixing layer spreading rate

$$\delta' = \frac{d\delta}{dx} = C_\delta \frac{(1-r)(1+\sqrt{s})}{2(1+\sqrt{s}r)} \tag{1}$$

Various mixing layer thicknesses δ can be considered but they roughly remain proportional so that the analysis will be restricted to the vorticity thickness $\delta_\omega = (u_1 - u_2)/\left(\frac{\partial u}{\partial y}\right)_{max}$ The spreading rate and thus the constant C_δ depends upon the experimental conditions such as incoming boundary layers, noise... but low speed experiments in air suggest a value close to 0.135 (Patel (1973), Bell and Metha, (1990)). There are very few experiments for low speed mixing layers with gases of different densities, the main ones being due to Brown and Roshko (1974). These experiments support equation (1) but for a larger value of the constant C_δ so that they cannot be used to compare velocity profiles. Therefore, the validation can only be performed considering the spreading rate evolution.

Model Predictions

The ability of turbulence models to reproduce the influence of the velocity and density ratios on the mixing layer spreading rate has been investigated using a code solving self-similarity solutions. As the problem is one-dimensional, grid convergence is easily achieved.

The analysis has been performed considering that the density difference across the mixing layer can be due either to different gases on each side or to a temperature difference. The way the density difference is generated does not change the predictions.

Predictions using a simple mixing length model, in which the mixing length is assumed constant and proportional to the mixing layer thickness, are given in figures 1 and 2. The symbols correspond to relation (1). For identical densities, the linear evolution of the spreading rate versus the ratio $\frac{u_1 - u_2}{u_1 + u_2} = \frac{1-r}{1+r}$ is well reproduced (figure 1) but the sensitivity to the density ratio is underestimated, as emphasized in figure 2 which only gives the spreading rate versus the density ratio s for a velocity ratio $r = 0$, i.e. the highest spreading rate.

Predictions using the standard $k - \varepsilon$ model by Launder and Sharma (1974) are plotted in figure 3. The prediction is good for isochoric mixing layers (i.e. $s = 1$) but the effect of the density ratio is predicted in the opposite way. Catris and Aupoix (2000) have proposed a rationale to sensitize any transport equation turbulence model to density gradient. This approach is based upon the scaling of the logarithmic region of compressible boundary layers. When this correction is applied, the model yields identical results, whatever the density ratio, as shown in figure 4. This correction improves the prediction but strongly underestimates the density ratio effect upon the mixing layer spreading rate.

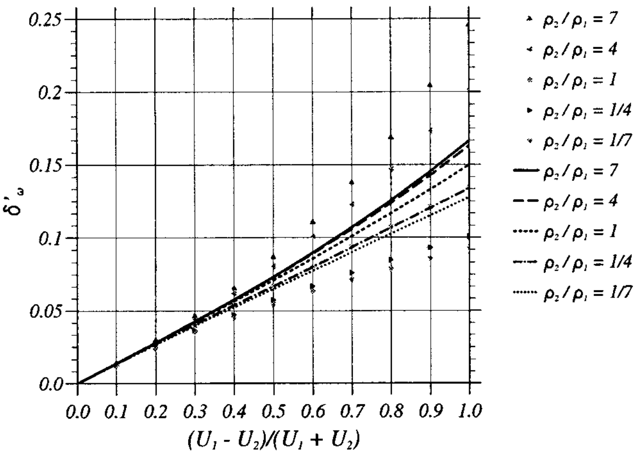

Figure 1: Predictions of the spreading rate with respect to velocity and density ratios – Mixing length model

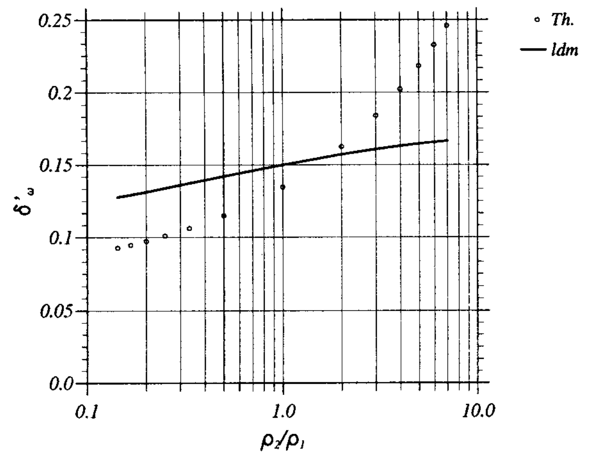

Figure 2: Predictions of the spreading rate with respect to density ratio for $r = 0$ – Mixing length model

This trend is general as can been shown in figures 5. Only the mixing layer model (noted ldm) or a model with one transport equation for the turbulent kinetic energy (noted k), the length scale being the mixing length, predict an increase of the spreading rate with the density ratio, but strongly underpredict it. Whatever the model constants in the $k-\varepsilon$ model (Launder-Sharma or Bézard (2000)), changing the length scale determining variable and e.g. using a $k-\varphi$ model (Cousteix et al., 1997) or the Spalart-Allmaras model (1993), the basic model predicts the wrong sensitivity to density ratio. The Catris and Aupoix correction the predictions of which are given in figures 5 correct the trend but the effect of density ratio is still strongly underpredicted. Even a change in the constitutive relation, to get rid of the eddy viscosity hypothesis, yields the same results as shown by the explicit algebraic Reynolds stress model (EARSM) predictions. Here, the Wallin and Johansson EARSM model (2000) has been used, coupled with an optimized $k-\varepsilon$ model. This result is not surprising as, in the mixing layer, the production to dissipation ratio remains close to unity so that the EARSM model gives levels for the Reynolds stress $-\widetilde{u''v''}$ close to the eddy viscosity relation.

Model Improvements

As the transport equation for the turbulent kinetic energy requires little modelling and has already been improved using Catris and Aupoix approach, the only ways to improve

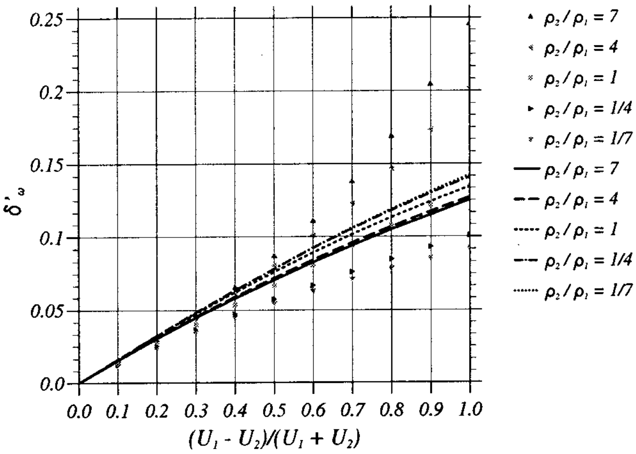

Figure 3: Predictions of the spreading rate with respect to velocity and density ratios – Launder and Sharma $k-\varepsilon$ model

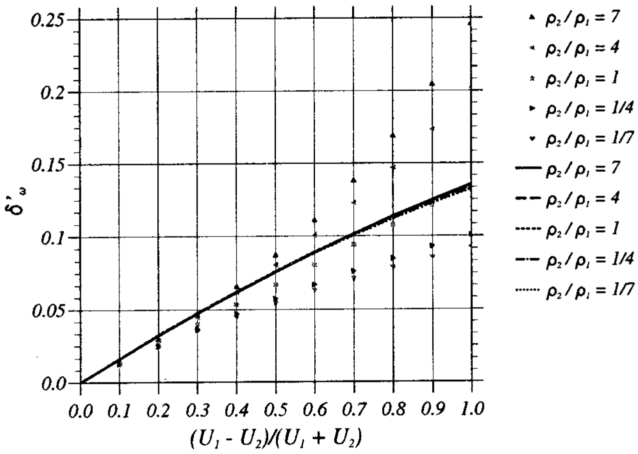

Figure 4: Predictions of the spreading rate with respect to velocity and density ratios – Launder and Sharma $k-\varepsilon$ model with Catris and Aupoix correction

the model are through either the length scale equation or the constitutive relation. The use of an EARSM formulation has already shown that this last point is quite hopeless.

In the exact transport equation for the dissipation rate, there is a term due to the mean flow divergence $-\frac{4}{3}\rho\varepsilon\,\mathrm{div}\underline{u}$ but accounting for this term does not significantly affect the predictions, as expected.

A natural way to account for mean density gradient effects in the length scale equation is to consider that the dissipation rate can be approximated as $\varepsilon \approx \nu\langle\underline{\omega}'\cdot\underline{\omega}'\rangle$, and to look at the baroclinic effect which appears in the vorticity equation.

$$\frac{D\underline{\omega}}{Dt} = \underline{\underline{\mathrm{grad}}}\,\underline{u}\cdot\underline{\omega} - \underline{\omega}\,\mathrm{div}\,\underline{u} + \frac{1}{\rho^2}\,\underline{\mathrm{grad}}\,\rho \wedge \underline{\mathrm{grad}}\,p + \underline{\mathrm{rot}}\left(\frac{1}{\rho}\,\underline{\mathrm{div}}\,\underline{\underline{\tau}}\right) \qquad (2)$$

The baroclinic term $\frac{1}{\rho^2}\,\underline{\mathrm{grad}}\,\rho \wedge \underline{\mathrm{grad}}\,p$ yields two main contributions in the fluctuating vorticity equation: $\frac{1}{\rho^2}\,\underline{\mathrm{grad}}\,\rho' \wedge \underline{\mathrm{grad}}\,p + \frac{1}{\rho^2}\,\underline{\mathrm{grad}}\,\rho \wedge \underline{\mathrm{grad}}\,p'$. The first one is null as the mean pressure is constant. The second term can be modelled considering that $-\underline{\mathrm{grad}}\,p' \approx \rho\,\underline{\underline{\mathrm{grad}}}\,\underline{u}\cdot\underline{u}'$. This leads in the dissipation rate transport equation to a term $-\nu\langle\left[\frac{1}{\rho^2}\,\underline{\mathrm{grad}}\,\rho \wedge \left(\rho\,\underline{\underline{\mathrm{grad}}}\,\underline{u}\cdot\underline{u}'\right)\right]\cdot\underline{\omega}'\rangle$ which reduces, for two-dimensional mean flows, to $\frac{\nu}{\rho}\langle v'\omega'_z\rangle\frac{\partial\rho}{\partial y}\frac{\partial u}{\partial y}$ and is modelled

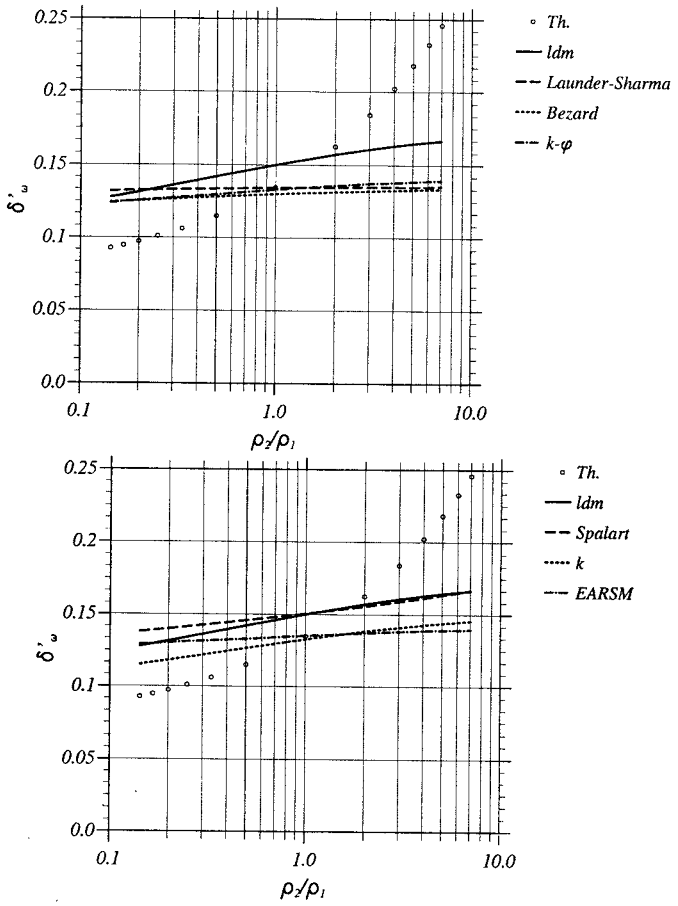

Figure 5: Predictions of the spreading rate with respect to density ratio for $r = 0$

as:

$$Ck^{3/2}\frac{\partial \rho}{\partial y}\frac{\partial u}{\partial y} \quad (3)$$

where C must be positive to give the correct influence. Figures 6 and 7 show that this correction improves the prediction but that the curvature of the evolution of the spreading rate with the velocity ratio is the same whatever the density ratio, which is not consistent with equation (1). Moreover, different values of C are required to have a good agreement when the low speed flow is the lightest or the more dense as shown in figure 8. Thus, this correction is not satisfactory and should, moreover, degrade compressible boundary layer predictions.

COMPRESSIBLE CHARACTER OF THE TURBULENT MOTION

References

Slessor et al. (2000) pointed out that, if the temperature of the two streams are strongly different, and hence the speeds of sound c, the convective Mach number M_c is ruled by the hottest flow, which usually is the slowest, while it should mainly be ruled by the high speed flow where compressibility effects are larger. Pointing out that the Mach number is the ratio of the kinetic to internal energy since $\frac{u^2}{2h} = \frac{\gamma-1}{2}M^2$, they introduce another Mach number:

$$\Pi_c = \max_i \left[\frac{\sqrt{\gamma_i - 1}}{c_i}\right] \Delta u \quad (4)$$

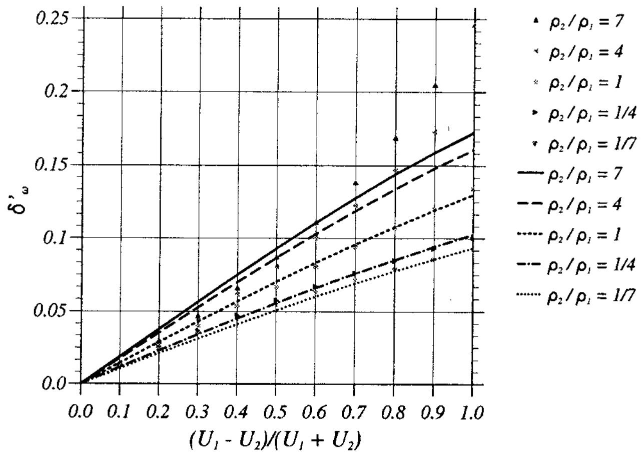

Figure 6: Predictions of the mixing layer spreading rate with respect to velocity and density ratios – Bézard $k - \varepsilon$ model with baroclinic correction - C = 0.05

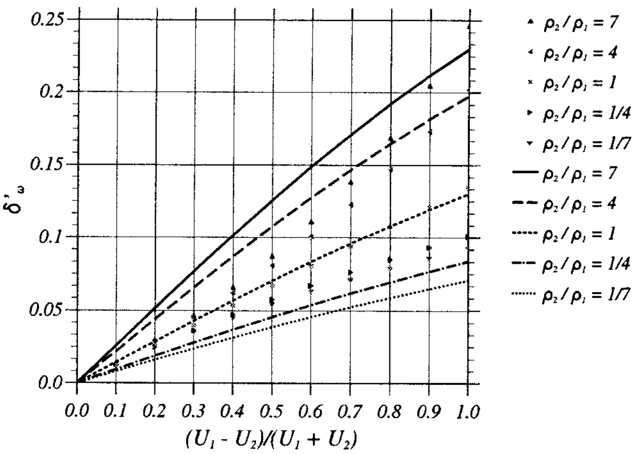

Figure 7: Predictions of the spreading rate with respect to velocity and density ratios – Bézard $k - \varepsilon$ model with baroclinic correction - C = 0.10

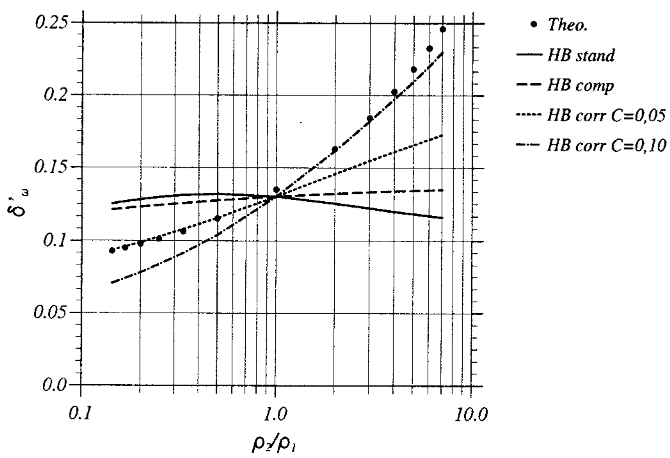

Figure 8: Predictions of the spreading rate with respect to density ratio for $r = 0$ – Bézard $k - \varepsilon$ model with baroclinic correction

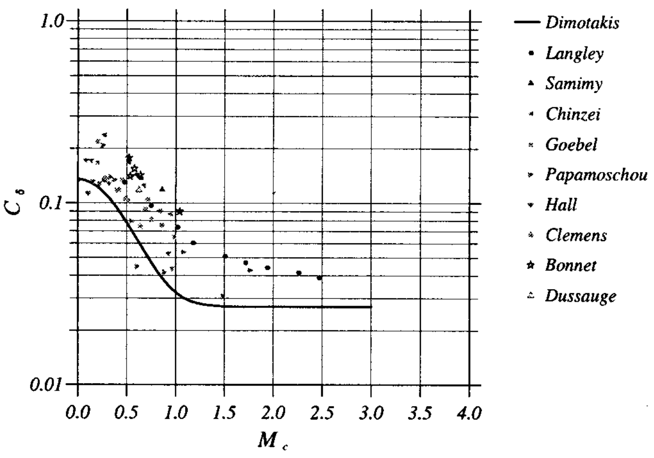

Figure 9: Experimental spreading rates coefficients versus convective Mach number

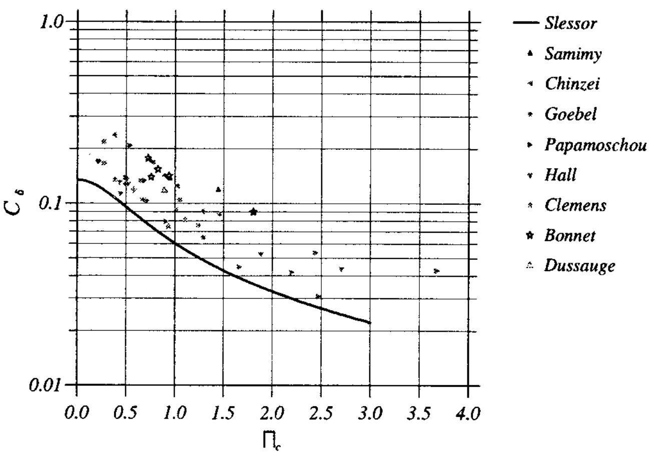

Figure 10: Experimental spreading rates coefficients versus Slessor et al. Mach number

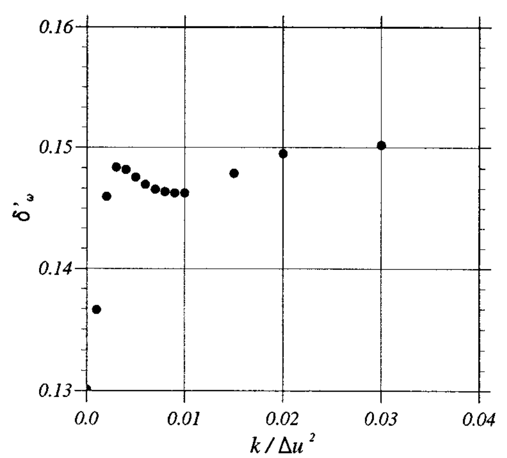

Figure 11: Influence of the external turbulence level on the spreading rate ($\Delta u = u_1 - u_2$)

A large set of compressible mixing layer experiments has been analyzed. It includes experiments by Samimy et al. (1990, 1992), Chinzei et al. (1986), Goebel and Dutton (1991), Papamoschou and Roshko (1988), Hall et al. (1993), Clemens and Mungal (1995), Bonnet and Debisschop (1992) and Barre et al. (1994, labelled Dussauge in the figures). The values of the coefficient C_δ are plotted in figures 9 and 10. A semi-logarithmic plot is used to compare experiments as a change in C_δ from one experiment to another is just a vertical shift in semi-logarithmic plot. Experiments are compared to Dimotakis' (1991) curve, the "Langley curve" (dots in figure 9) and Slessor et al. (2000) curve, all three plotted for a value of C_δ of 0.135. A detailed analysis shows that each experiment gives a spreading rate reduction versus Mach number in good agreement with Dimotakis' and Slessor's curves, not with the Langley curve. However the reference level for low speed flow is higher, corresponding to values of C_δ dependent upon the experiment and ranging between 0.18 and 0.31.

This is consistent with computations performed increasing the turbulence level outside of the mixing layer, in which the spreading rate strongly increases, as shown in figure 11. Moreover, the analysis of self-similarity equations shows that the turbulent kinetic energy has to be referred to the velocity difference as $\frac{k}{(u_1-u_2)^2}$ so that, for the above considered experiments, small levels of turbulence in the supersonic stream result in high levels of reduced turbulent kinetic energy and important increases of the spreading rate.

This has a drastic consequence: comparison of computations with experiments requires a good knowledge of the turbulence in both streams outside the mixing layer, which is usually not the case. Therefore, the validation will only deal with the ability of models to reproduce the Dimotakis' and Slessor's curves.

At last, it can be noticed that the use of Slessor parameter Π_c does not significantly reduce the scatter, even when each experiment is considered solely.

Standard models

As models poorly predict mean density effects, computations have been performed for various velocity and density ratios and a large range of convective Mach numbers. One test case for a Mach number $\Pi_c = 0.01$ serves as the "incompressible" reference.

There is no information about the speed of sound in the self-similarity equations. Therefore, whatever the turbulence model used, no reduction of the spreading rate is observed as the convective Mach number is increased, as shown in figure 12. Various corrections, to model the dilatational dissipation and the pressure-dilatation correlation have been proposed by Sarkar (1991) and Zeman (1990). Predictions using Sarkar's model are given in figure 13. Predictions with Zeman's model are similar. They are close to Langley's curve and underestimate the spreading rate reduction.

Model improvement

Direct numerical simulations by Freund et al. (2000) and Pantano and Sarkar (2002) show that the rôle of the dilatational terms is negligible and that the main effect of compressibility is via a reduction of the redistribution due to pressure fluctuations. The energy transfer from $\widetilde{u''^2}$ to $\widetilde{v''^2}$ is reduced, and therefore the production of the shear stress $-\widetilde{u''v''}$ is lowered and hence the turbulence production and

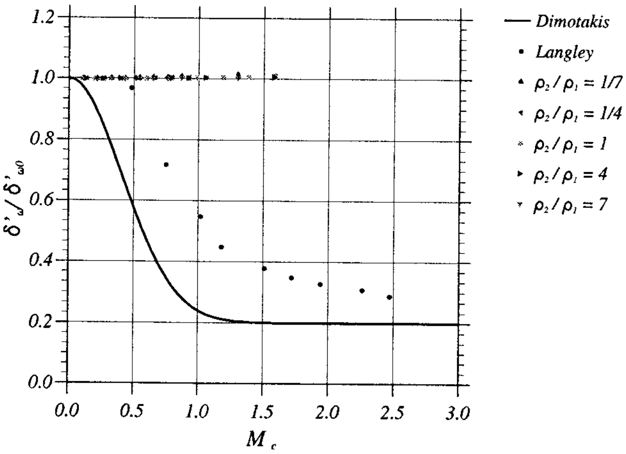

Figure 12: Prediction of the spreading rate reduction with a $k-\varepsilon$ model

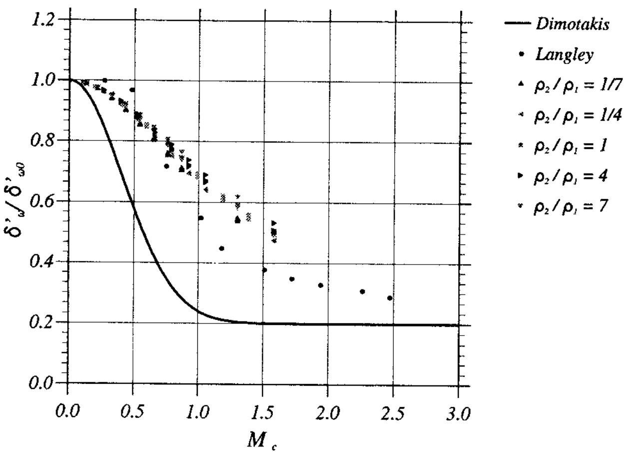

Figure 13: Prediction of the spreading rate reduction with a $k-\varepsilon$ model + dilatational dissipation model

the spreading rate.

This reduction of communication within a turbulent structure can be reproduced through the sonic eddy concept introduced by Breidenthal (1992) and adapted to the mixing length model by Kim (1990). No communication can exist between two points the velocity difference is supersonic. Thus, a turbulent eddy cannot extend beyond points the velocity difference is sonic. In Kim's model, instead of being equal to the mixing layer thickness, the length scale is limited to the distance between points under and above the considered point such that the velocity difference with the considered point is sonic (i.e. points where the velocity is $u \pm c$).

This kind of limitation of the length scale has been tested for various turbulence models, i.e. mixing length model, one-equation model and $k-\varepsilon$ model. Whatever the model, the predictions are similar. Kim's formulation gives predictions close to the Langley's curve, as shown in figure 14. This is due to an overestimation of the turbulent structure size as the total velocity difference within a turbulent structure can be twice the speed of sound. If the maximum structure size is really reduced to a velocity difference equal to the speed of sound, i.e. reducing the length scale to points where the velocity is $u \pm \frac{c}{2}$ the prediction is in good agreement with Dimotakis' curve as shown in figure 15.

This kind of model requires to identify the mixing layer and is therefore not usable in general Navier-Stokes codes.

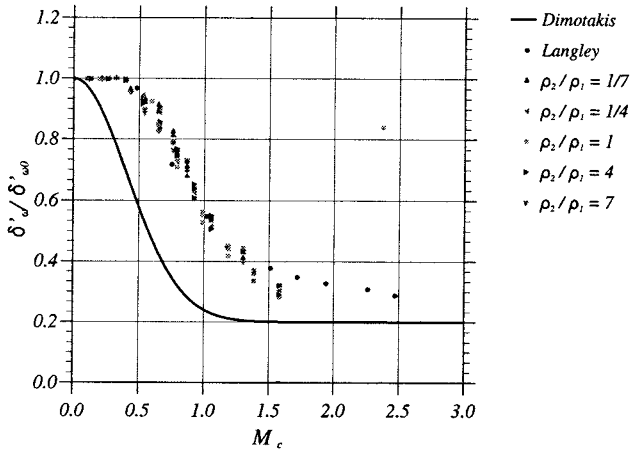

Figure 14: Prediction of the spreading rate reduction with Kim's model

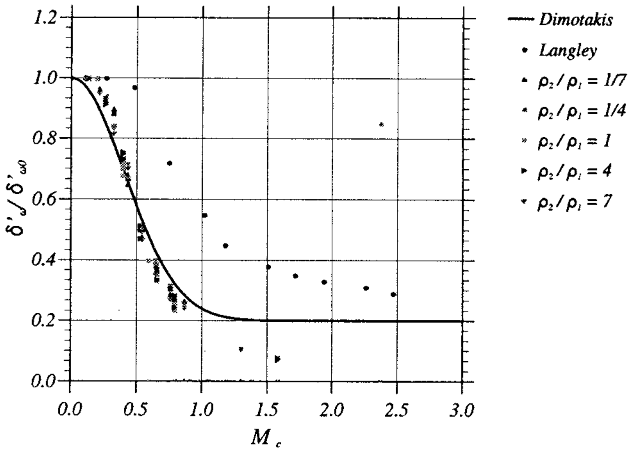

Figure 15: Prediction of the spreading rate reduction with proposed revision of Kim's model

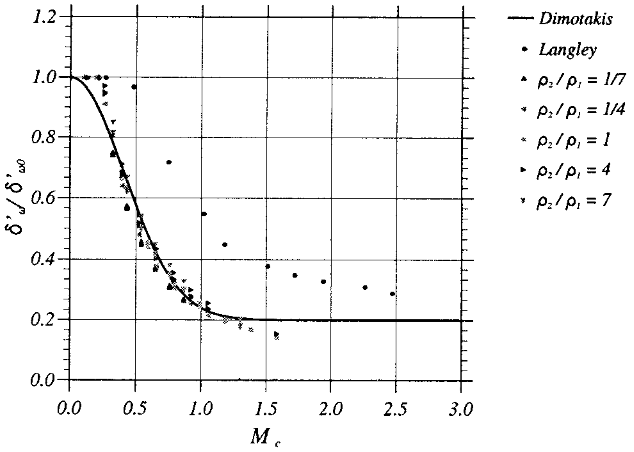

Figure 16: Prediction of the spreading rate reduction with proposed revision of $k-\varepsilon$ model

For this purpose, a more general and non-local modification of the turbulence model is required. If the dissipation equation is used, a term of the form

$$\rho \frac{C}{\tau} \max \left(0; 1 - \frac{l_0}{l} \right) \varepsilon \tag{5}$$

allows to limit the length scale to $l_0 \propto \frac{c}{|\omega|}$. For thin shear layers $\frac{l_0}{l} \propto \frac{1}{M_g}$ where M_g is the gradient Mach number introduced by Sarkar (1995) so that this correction is inactive in boundary layer flows. The final model form reads:

$$\rho C_1 \max \left(0; 1 - C_2 \frac{c\varepsilon}{k^{3/2}|\omega|} \right) \frac{\varepsilon^2}{k} \qquad C_1 = 0.9; C_2 = 0.32 \tag{6}$$

This model gives fair agreement with Dimotakis' curve, as shown in figure 16.

CONCLUSION

Standard turbulence models easily predict isochoric, low-speed mixing layer. But no model correctly account for the density gradient through the mixing layer. A correction based upon boundary layer scaling hardly improves the prediction. A correction based upon baroclinic effect is proposed but is not satisfactory.

Dimotakis' or Slessor's curve are good estimates of the spreading rate reduction for high speed mixing layers, not the Langley curve. Slessor et al. approach does not reduce the scatter. Direct comparison with experiments seems difficult as the mixing layer behaviour is strongly affected by the external flow turbulence which is usually unknown. Standard models predict no reduction, corrections based upon dilatational terms underpredict the reduction. Kim's model has been improved and a new model, based upon the sonic eddy concept, is proposed and validated.

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