DERIVATIVES IN TURBULENT FLOW IN AN ATMOSPHERIC SURFACE LAYER WITHOUT TAYLOR HYPOTHESIS

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ABSTRACT

Derivatives play an outstanding role in the dynamics of turbulence for a number of reasons. The importance of velocity derivatives became especially clear since the papers by Taylor (1938) and Kolmogorov (1941). Taylor emphasized the role of vorticity, whereas Kolmogorov stressed the importance of dissipation, and thereby of strain.[1] However, the most common method of obtaining the derivatives in the streamwise direction is the use of Taylor hypothesis (Taylor, 1935, see references in Tsinober et al., 2001), the validity of which is a widely and continuously debated issue. It is related to a more general issue, the so called random Taylor hypothesis or the sweeping decorrelation hypothesis which concerns the relation between the (Eulerian) 'components' $\mathbf{a}_l = \partial\mathbf{u}/\partial t$ and $\mathbf{a}_c = (\mathbf{u}\cdot\nabla)\mathbf{u}$ of the full (Lagrangian) acceleration (see Tsinober et al. (2001) for a discussion and numerical study of this problem). In fact the issue is even more general in the sense that it concerns the relation between the Eulerian components $\partial Q/\partial t$ and $(\mathbf{u}\cdot\nabla)Q$ of the material derivative of any quantity Q (scalar, vector or tensor) in a turbulent flow.

Using conventional hot- and cold-wire techniques it is not possible to distinguish between the temporal and the streamwise spatial derivatives thus enforcing the use of the Taylor hypothesis.

This presentation contains results obtained with a system enabling to evaluate separately the temporal and streamwise spatial derivatives, the latter being obtained without employing the Taylor hypothesis. Along with experimental we present also some numerical results clearly showing strong anti-correlation and consequently cancellation between the local, $\partial Q/\partial t$, and advective, $(\mathbf{u}\cdot\nabla)Q$, components of different quantities in turbulent flows.

THE EXPERIMENT AND FACILITY

The experiment was performed in an open field at a height of 10 m above the ground with the mean velocity at this height 7 m/s and $\mathrm{Re}_\lambda = 10^4$. More details on the experiments and the techniques are given in Kholmyansky and Tsinober (2000) and Kholmyansky et al. (2001*a*). In the experiments to be reported all three components of the velocity fluctuations vector, u_i, all nine components of the spatial velocity gradients tensor, $\partial u_i/\partial x_j$, and all the three temporal velocity derivatives, $\partial u_i/\partial t$, were measured along with corresponding data on the mean flow at $\mathrm{Re}_\lambda = 10^4$. This was done by modification of techniques used in Kholmyansky and Tsinober (2000) and Kholmyansky et al. (2001*a*). The substantial new development is a probe allowing estimating the spatial derivative in the streamwise direction independently of the time derivative, i.e. without invoking the Taylor hypothesis. This is achieved by constructing a five-array probe (see Figure 1) with the central array shifted out forward in the streamwise direction. Such a probe enables to estimate all the three velocity components at two streamwise positions simultaneously: one at the tip of the shifted array, and the other in the plane of the four other arrays via interpolation of the four values obtained from these four arrays. This probe was also equipped by five cold wires as shown in Figure 1 enabling to evaluate the temporal derivative of temperature and all its spatial derivatives.

[1] A list of other reasons is given in Tsinober (2001).

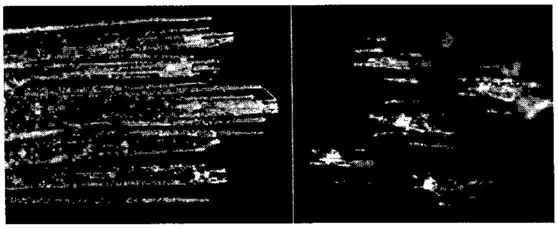

Figure 1. Two different views of the tip of the NTH (non-Taylor-hypothesis) probe with five arrays each containing four hot-wires and one cold wire. Note that the probe is quite 'empty' in spite of having many components.

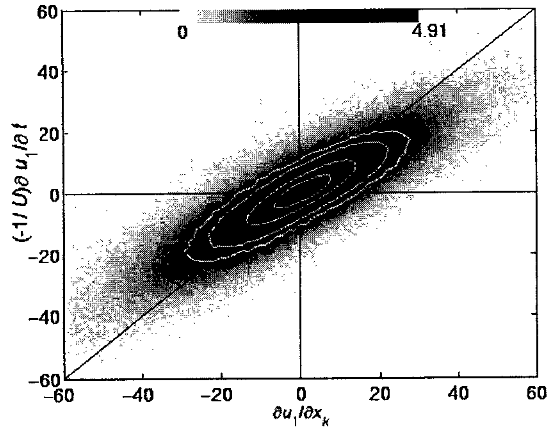

Figure 2. Joint PDF of the temporal derivative, $\partial u_1/\partial t$, and the streamwise derivative, $\partial u_1/\partial x_1$. Very similar plots were obtained for the two other velocity components, u_2, u_3.

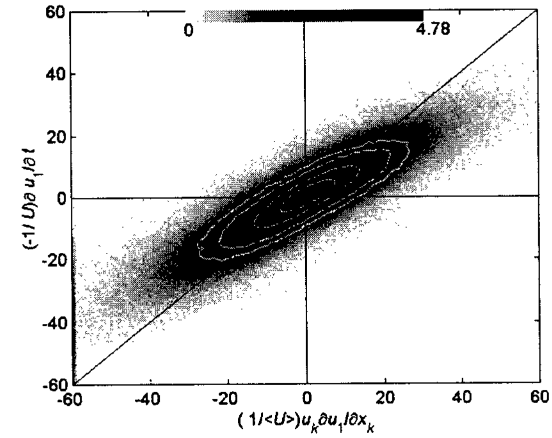

Figure 3. Joint PDF of the temporal derivative $\partial u_1/\partial t$ and the full advective derivative $u_k \partial u_1/\partial x_k$. Very similar plots were obtained for the two other velocity components u_2, u_3.

NUMERICAL SIMULATIONS

The main purpose of numerical simulations was to observe qualitative effects relevant to the issues of concern here. These effects appear quite strong and essentially are the same as observed experimentally at different geometries and at much larger values of Reynolds number. Therefore, the numerical simulations were performed in the simplest configuration: cubic box with periodical boundary conditions and several versions of forcing to maintain a statistically stationary turbulent flow as described in Galanti and Tsinober (2000).

RESULTS

The availability of the data on all the nine components of the tensor of spatial velocity gradients, $\partial u_i/\partial x_j$, and the time derivatives of all the three velocity components, $\partial u_i/\partial t$, enables to address a number of issues which are essentially beyond both DNS and phenomenology. The general issue of concern here is the study of the field of Lagrangian accelerations, $\mathbf{a} \equiv D\mathbf{u}/Dt$, and its Eulerian components: the local acceleration, $\mathbf{a}_l = \partial\mathbf{u}/\partial t$, and the advective acceleration, $\mathbf{a}_c = (\mathbf{u}\cdot\nabla)\mathbf{u}$, as it was done for low Reynolds numbers in DNS (Tsinober et al., 2001). This includes the issue called random Taylor (sweeping decorrelation) hypothesis and associated issues of geometrical statistics of accelerations, and the conventional Taylor hypothesis as a special case. Specifically, one is interested in mutual (statistical) cancellation between the local acceleration, $\mathbf{a}_l = \partial\mathbf{u}/\partial t$, and the advective acceleration, $\mathbf{a}_c = (\mathbf{u}\cdot\nabla)\mathbf{u}$. Since these quantities are vectors the degree of this mutual cancellation should be studied both in terms of their magnitude and the geometry of vector alignments. In particular this cancellation is exhibited in that the total acceleration, $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_c$, is much smaller its local and advective components, $\mathbf{a}_l$ and $\mathbf{a}_c$, high negative correlation between $\mathbf{a}_l$ and $\mathbf{a}_c$, and strong anti-alignment of $\mathbf{a}_l$ and $\mathbf{a}_c$.

Additional aspects are related to scale dependence of the mentioned above phenomena, and kinematical versus dynamical effects. A natural by-product is the direct check of the Taylor hypothesis using the new technique. We would like to stress that only the technique allowing to measure simultaneously and independently the spatial and temporal velocity derivatives enables one to address all these issues, in all of which it is essential that neither the random Taylor hypothesis nor the conventional Taylor hypothesis can be valid *precisely* in principle (Tsinober et al., 2001) and both are only an approximation which seems to become better as the Reynolds number increases.

Results for the velocity field (Kholmyansky et al., 2001*b*) are shown in Figures 2–4. Qualitatively similar results were obtained by Lüthi et al. (2001) and Tsinober et al. (2001) for low Reynolds numbers in three-dimensional PTV laboratory experiments and in DNS of NSE.

Table 1. Variances of local (temporal), streamwise and full advective derivatives of temperature.
$D/DT = \partial/\partial t + u_k \partial/\partial x_k$

$DT/Dt \cdot 10^{-3}$	$\partial T/\partial t \cdot 10^{-3}$	$u_k \partial T/\partial x_k \cdot 10^{-3}$
120	405	336

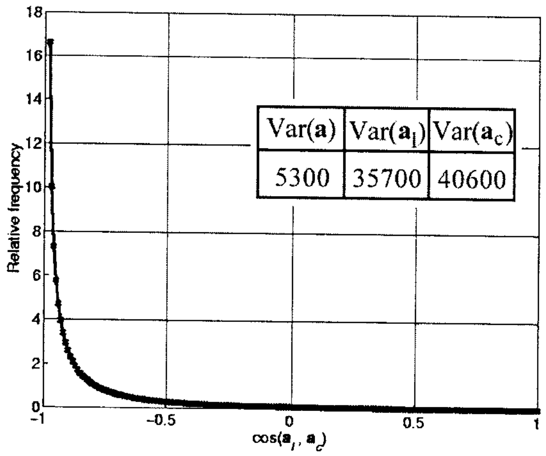

Figure 4. PDF of the cosine of the angle between $\mathbf{a}_l$ and $\mathbf{a}_c$, exhibiting the tendency for strong anti-alignment: they are strongly negatively correlated with the correlation coefficient -0.93. It is seen also from the inset that that the variance of the total acceleration is more than six times smaller than that of $\mathbf{a}_l$ and $\mathbf{a}_c$, the latter being of the same magnitude.

The joint PDF between the temporal derivative, $\partial u_1/\partial t$, and the streamwise derivative, $\partial u_1/\partial x_1$, shown in Figure 2, is quite similar to that between the temporal derivative, $\partial u_1/\partial t$, and the full advective derivative, $u_k \partial u_1/\partial x_k$, as shown in Figure 3. This is the consequence of the fact that the conventional Taylor hypothesis is just a special case of the random Taylor hypothesis. Other aspects of the latter are manifested in strong anti-alignment of the local and advective accelerations, Figure 4, and in the fact that the magnitude of the total acceleration (i.e. the sum of $\mathbf{a}_l = \partial \mathbf{u}/\partial t$ and $\mathbf{a}_c = (\mathbf{u} \cdot \nabla)\mathbf{u}$) is much smaller than $\mathbf{a}_l$ and $\mathbf{a}_c$.

As mentioned, the above measurements allow looking at the issue of the conventional and random Taylor hypotheses not only in respect of velocity but also temperature, Figure 5 and Table 1. These results were obtained in a slightly heated jet flow that was set in our calibration unit with controlled heating in the settling chamber with $Re_\lambda \cong 10^3$ at the centerline of the jet at the distance of eight diameters from the jet exit nozzle.

Just like in case of velocity one observes a strong cancellation between the local (temporal), $\partial T/\partial t$, and the streamwise, $\partial T/\partial x_1$, or the full advective, $u_k \partial T/\partial x_k$, derivatives (Tsinober and Galanti, 2001; Yeung and Sawford, 2002).

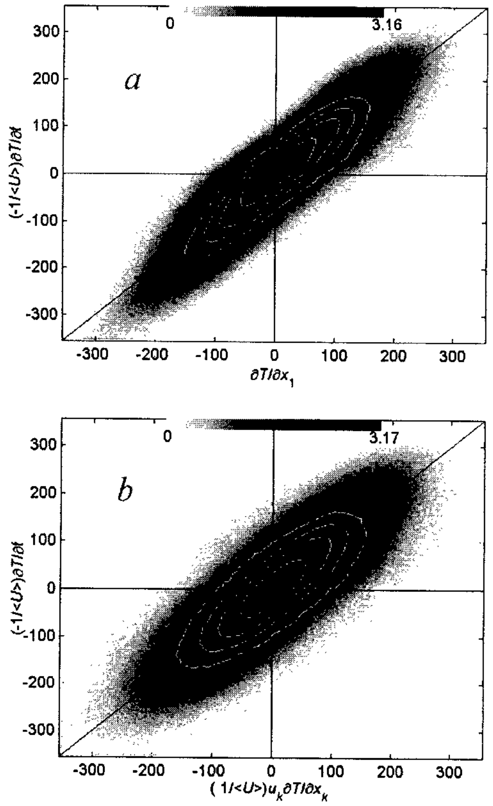

Figure 5. a) Joint PDF of the temporal derivative $\partial T/\partial t$ and the streamwise derivative $\partial T/\partial x_1$, the corresponding correlation coefficient is equal 0.89; b) joint PDF of the temporal derivative $\partial T/\partial t$ and the full convective derivative $u_k \partial T/\partial x_k$, the corresponding correlation coefficient is equal 0.83.

Our experimental resolution is not sufficient (so far) to check analogous behavior of quantities involving derivatives such as vorticity and temperature gradient. This was done via DNS of NSE. Here we checked only the relation between the temporal and full advective derivative, since the flow does not possess any mean velocity. Some of the results are shown in Figures 6–7 and in Table 2.

Just as in case of velocity (see Tsinober et al., 2001), when the quantity Q is a vector, the PDFs of the cosine of the angle between $\partial Q/\partial t$ and $u_k \partial Q/\partial x_k$ exhibit strong anti-alignment for vorticity, $\boldsymbol{\omega}$, gradient of passive scalar, $\mathbf{G} = \nabla T$, and solenoidal passive vector, $\mathbf{B}$. This is seen from Figure 6.

Similarly the joint PDFs of the temporal derivatives, $\partial Q/\partial t$, and the full advective derivative, $u_k \partial Q/\partial x_k$, exhibit essentially the same behavior for various quantities such as velocity, vorticity, passive scalar and its gradient and passive solenoidal vector, see Figure 7.

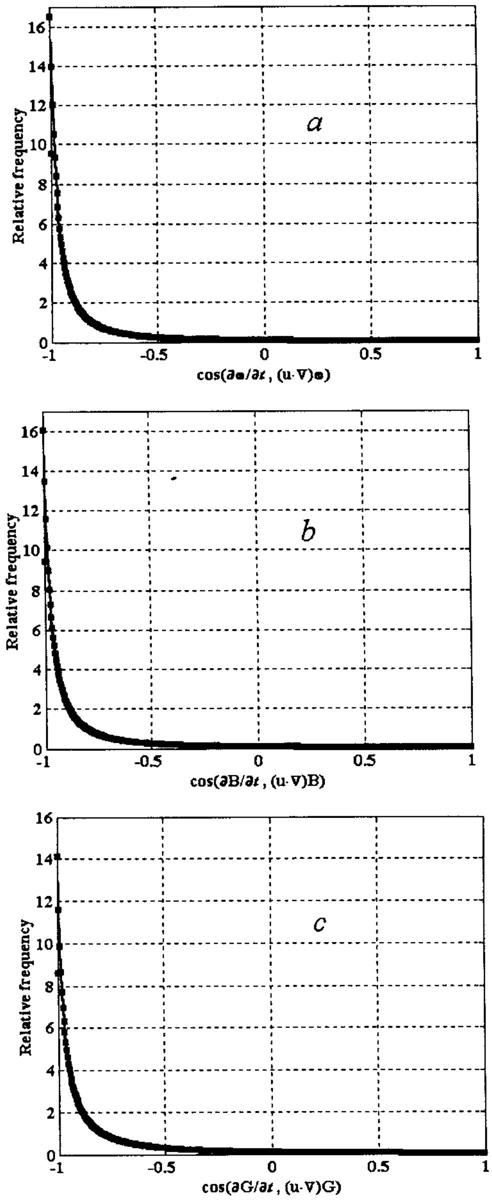

Figure 6. PDFs of the cosine of the angle between: a) $\partial\omega/\partial t$ and $(\mathbf{u}\cdot\nabla)\omega$; b) $\partial\mathbf{B}/\partial t$ and $(\mathbf{u}\cdot\nabla)\mathbf{B}$; c) $\partial\mathbf{G}/\partial t$ and $(\mathbf{u}\cdot\nabla)\mathbf{G}$. All these PDFs exhibit the tendency for strong anti-alignment. Correspondingly they are strongly negatively correlated with the correlation coefficient below -0.9.

Table 2. Variances of local (temporal), streamwise and full advective derivatives of various quantities.

Quantity Q	DQ/Dt	$\partial Q/\partial t$	$u_k\partial Q/\partial x_k$
T, $\times 10^6$	41	480	520
u_i, $\times 10^3$	27.1	81.3	108
ω_i	17.3	107.5	121.4
G_i, $\times 10^3$	136	623	740
B_i, $\times 10^{-3}$	15.2	98.2	85.3

DISCUSSION AND CONCLUDING REMARKS

Our main technical result is that it is possible to measure all the temporal and spatial velocity and temperature derivatives without invoking the Taylor hypothesis. In order to make precise quantitative evaluation of the Taylor hypothesis it is necessary to improve the quality of the data. This includes mainly an improvement of various aspects of the calibration procedure and better accounting for variations of ambient temperature.

From the physical point the main result is that the Taylor hypothesis is a special case of the so called random Taylor hypothesis (or sweeping decorrelation hypothesis) as proposed by Tennekes (1975), for later references see Tsinober et al. (2001) and Xu et al. (2001). This hypothesis appears to be valid not only for the velocity field but for a variety of other quantities such as velocity derivatives (vorticity and strain), temperature and its gradient, solenoidal passive vectors and apparently for any physically meaningful quantity. In other words random Taylor hypothesis has a universal nature, which is manifested in strong tendency for cancellation between the local temporal derivative $\partial Q/\partial t$ and the full advective derivative $u_k\partial Q/\partial x_k$ of whatever quantity. Thus the full material (Lagrangian) derivative $DQ/Dt = \partial Q/\partial t + u_k\partial Q/\partial x_k$ is much smaller (at least an order of magnitude) than its Eulerian components. This result is true in any inertial system of reference due to Galilean invariance. An important aspect is that the observed trends are qualitatively the same at $Re_\lambda \cong 10^2$, $Re_\lambda \cong 10^3$ and $Re_\lambda \cong 10^4$ and become stronger at larger Reynolds numbers. An interesting question is whether the smallness of $DQ/Dt = \partial Q/\partial t + u_k\partial Q/\partial x_k$ as compared to both $\partial Q/\partial t$ and $u_k\partial Q/\partial x_k$ can be seen as an indication of the existence of a small parameter in turbulence.

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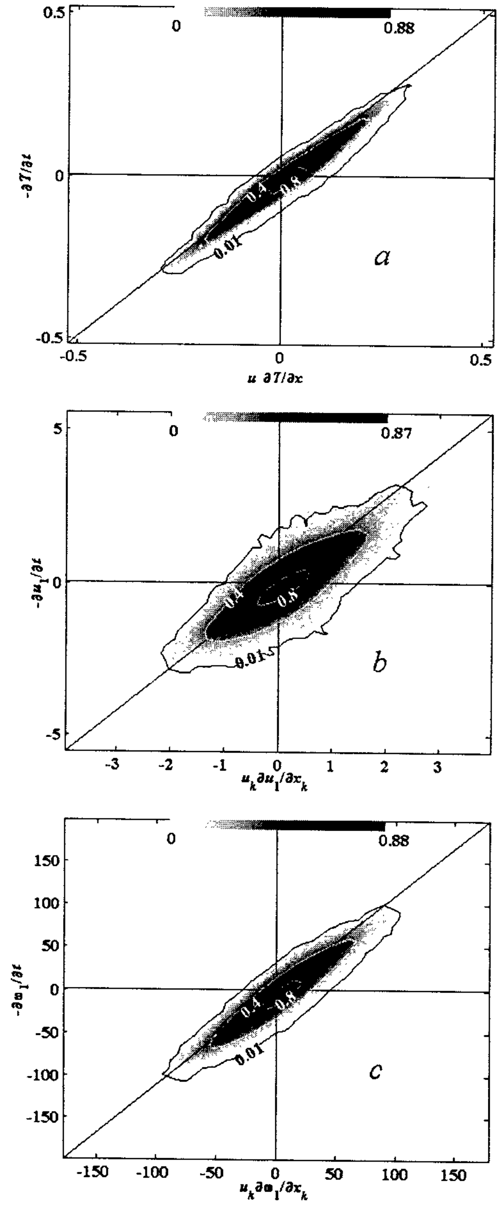

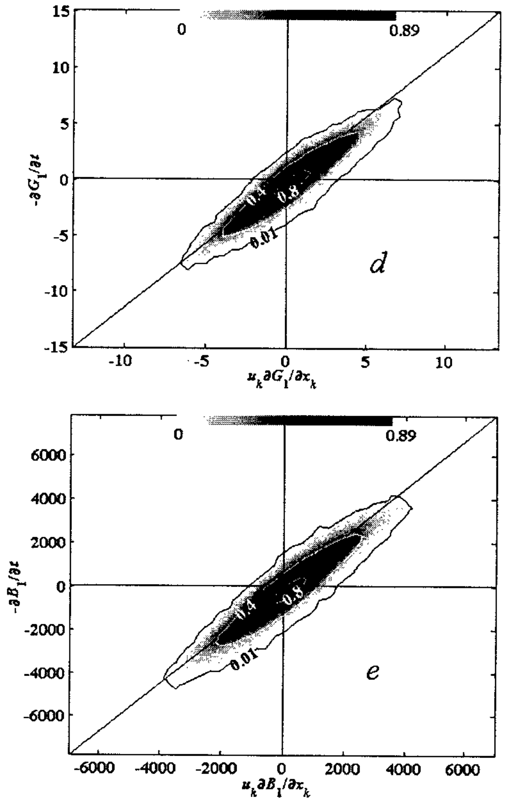

Figure 7. Joint PDFs of the temporal derivatives $\partial Q/\partial t$ and the full advective derivatives $u_k \partial Q/\partial x_k$ as obtained from DNS of NSE: a) $Q = T$, T – is the temperature; b) $Q = u_1$, u_i – is the velocity vector; c) $Q = \omega_1$, ω_i – is the vorticity vector; d) $Q = G_1$, $G_i = \nabla T$; e) $Q = B_1$, B_i – is solenoidal passive vector. Very similar plots are observed for other components of the above quantities. In all the above cases the corresponding correlation coefficient is about –0.9

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