VERY-LARGE-EDDY SIMULATION SUBGRID MODELLING USING THE STATISTICAL MECHANICS RENORMALIZATION GROUP

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ABSTRACT

In this paper transport equations are constructed for subgrid turbulent kinetic energy and subgrid mean dissipation rate in Large-Eddy or Very-Large-Eddy Simulations by using a Renormalization Group approach. The resulting equations and effective viscosity are dependent on the (V)LES filter width, the main dependence coming from the timescale in the $\overline{\varepsilon}$ equation.

INTRODUCTION

In a good Large-Eddy Simulation, most of the turbulent kinetic energy should be resolved, meaning that the filter width should be comparable to inertial range lengthscales. In complex geometries one is often not sure whether this is reached, and generally many grid points are needed to fulfill this requirement, especially near walls. Very-Large-Eddy Simulations can be seen as a general class of models which do not require the filter width to be in the inertial range. Of course, this means that more complicated subgrid models should be used as more of the large, anisotropic and geometry-dependent structures have to be modelled. A class of models that are able to model integral lengthscales are the RANS models, and therefore the usual approach to VLES modelling has been to modify an existing RANS model by somehow making it filter width-dependent. The comments on these models are that they

- are mostly very ad hoc.
- contain empirically calibrated constants, coming from comparing RANS simulations with experiments and/or DNS. These calibrations are generally no longer valid in LES simulations.
- lack a physical basis.

This work is an attempt to construct a VLES model that does not suffer from these shortcomings. To that end, transport equations for subgrid quantities were constructed with the Renormalization Group. The advantage of this approach is that, within one framework, LES subgrid models as well as RANS models can be constructed. The only difference between the two is the lengthscale: in LES it is defined by the filter width and in RANS by the integral lengthscale.

GENERAL CONSIDERATIONS FOR VLES MODELLING

It is clear that transport equations for mean turbulent quantities will be necessary in a model that removes large, anisotropic, integral range structures from the turbulent field. Further it is well known that at least two of these quantities are necessary to completely define the integral turbulent length- and timescales required by the turbulent viscosity. On the other hand, when the filter width gets smaller, these subgrid quantities should change accordingly, as less is being modelled. A general approach to VLES modelling has been to start from a conventional RANS model by making the lengthscale in these models filter width dependent (the DES models belong to this class (Spalart e.a., 1997). For the two-equation model, this results in an increase of the dissipation term in the $\overline{K}$-equation. In another approach, proposed by Speziale (Speziale,1998), the turbulent viscosity, as calculated with a RANS model, is premultiplied by a filter width dependent function. The comments from the introduction apply for all these approaches, except maybe for the model of Dejoan and Schiestel (2001) in which a two-equation VLES model is constructed by means of a so called split spectrum method. The result of their approach was a filter width dependent factor in front of the destruction term in the ε equation. Their approach is most closely to the one we propose, which also results in a filter width-dependent $\overline{\varepsilon}$-equation. Some other models blend RANS-methods near the wall with LES-methods at some fixed distance from the wall. These are no genuine VLES models, and can rather be seen as hybrid approaches or as LES with wall-modelling.

RENORMALIZATION GROUP AND VLES MODELLING

The Renormalization Group has had some successes in turbulence theory and turbulence modelling. It's main accomplishments in turbulence modelling have been the confirmation and extension of previous, empirically calibrated models. The main advantage of RG for our purpose is that with this method it is possible to construct LES and RANS models within one framework. The connection between the two classes of models is made when the filter width is replaced with the integral lengthscale.

Applications of RG to the turbulence problem have a wide history. The first attempt to tackle turbulence with RG techniques was by Yakhot and Orszag (1986), who applied the dynamical RG, mixed with EDQNM renormalized perturbation theory, to calculate a variety of turbulence constants and some standard type turbulence models (like a Smagorinsky type subgrid model and $\overline{K} - \overline{\varepsilon}$ type RANS model). Many subsequent RG work in the turbulence literature consisted of remarks on, and improvements of, the original work of Yakhot and Orszag. The work of Giles (1994a,1994b) follows a different RG procedure, analogous to the original RG method used in statistical mechanics. This led to some different constants than the YO method, and was less flawed by uncontrolled approximations. For our purpose, Giles approach to the derivation of turbulence transport equations was more amenable for VLES, because Yakhot and Smith had to rely on heuristic elements for the construction of the production and destruction terms in the $\bar{\varepsilon}$-equation. When one wants to adapt the YO RANS equation to their LES-form, extra terms arise with no clear physical interpretation (De Langhe e.a., 2001).

THE RG METHOD

In this section a very short sketch of the statistical mechanical RG as applied to turbulence will be given. Only the general principle will be explained. In the statistical mechanical RG approach to turbulence (Giles,1994a,1994b) one starts with the NS equation, forced by a stochastic force (with know probability distribution). The exact PDF of the velocity field can then be constructed from that of the force as $\mathcal{P}(\mathbf{u}) = \frac{\mathcal{D}\mathbf{f}}{\mathcal{D}\mathbf{u}}\mathcal{P}(\mathbf{f})$ with $\frac{\mathcal{D}\mathbf{f}}{\mathcal{D}\mathbf{u}}$ the functional Jacobian determinant of the transformation from the force field to the velocity field (as given by the NS equation). One then proceeds by defining an initial PDF $\mathcal{P}_0$ and initial value of any functional of $\mathbf{u}$, $K_0(\hat{k},\mathbf{u})$, $\hat{k} = (\mathbf{k},\omega)$ by integrating out all the wavenumbers larger than a UV cutoff Λ_0 (e.g. the Kolmogorov wavenumber):

$$\mathcal{P}_0 = \int_{k>\Lambda_0} \mathcal{P}\mathcal{D}\mathbf{u}$$
$$K_0 = \int_{k>\Lambda_0} K\mathcal{D}\mathbf{u}$$

with the condition that the average of the functional remains invariant:

$$\int K_0\mathcal{P}_0\mathcal{D}\mathbf{u} = \int K\mathcal{P}\mathcal{D}\mathbf{u}$$

Starting from $\mathcal{P}_0$ and K_0 a sequence $\mathcal{P}_1, \mathcal{P}_2, \cdots$ and $K_1, K_2, \cdots$ are generated by successive integrating out wavenumber-bands $b^{-1}\Lambda_0 < k < \Lambda_0$ $(b > 1)$ and rescaling of variables such that the UV cutoff is kept fixed (i.e. $\hat{k}_{n+1} = b\hat{k}_n = (b\mathbf{k}_n, a_n\omega_n)$ and $\mathbf{u}_{n+1} = \xi_n\mathbf{u}_n)$[1].

$$\mathcal{P}_{n+1} = \left.\int_{b^{-1}\Lambda_0<k<\Lambda_0} \mathcal{P}_n(\mathbf{u}_n)\mathcal{D}\mathbf{u}_n\right|_{\mathbf{u}_n(\hat{k}_n)\rightarrow\xi_n\mathbf{u}_n(\hat{k}_{n+1})}$$

idem for K_{n+1} and with the condition that the average of K_{n+1} over $\mathcal{P}_{n+1}$ is, to within a rescaling factor ψ_n equal to the average of K_n over $\mathcal{P}_n$:

$$\begin{aligned}\int K_n(\hat{k}_n,\mathbf{u}_n)\mathcal{P}_n\mathcal{D}\mathbf{u}_n &= \\ &\psi_n \int K_{n+1}(\hat{k}_{n+1},\mathbf{u}_{n+1})\mathcal{P}_{n+1}\mathcal{D}\mathbf{u}_{n+1}.\end{aligned}$$

This iteration proceeds until $\mathcal{P}_{n+1} = \mathcal{P}_n = \mathcal{P}^*$ and $K_{n+1} = K_n = K^*$, i.e. when a fixed point of the RG iteration is reached; this fixed point denotes inertial range scaling behaviour.

When the rescaling is undone, the result of the RG procedure is the inertial range form of K, which can be applied to the terms occurring in the transport equations for LES subgrid quantities and the effective viscosity. When the cutoff wavenumber occurring in these inertial range expression is taken to be equal to the integral wavenumber, RANS transport equations are obtained.

[1]This transformation generally results in the generation of infinitely many new terms (that occur in a series expansion in $\mathbf{u}_n$), and one has to rely on perturbation theory to find recursion relations between the series coefficients of K_{n+1} ad K_n. All these and other technicalities can be found in the papers of Giles (1994a,1994b) and the review of Barber (1977).

THE RG VLES MODEL

Model equations

The RG procedure leads to the following effective viscosity $\nu(\Lambda_c)$ and transport equations for the subgrid turbulent kinetic energy $\overline{K}$ and mean rate of dissipation $\bar{\varepsilon}$

$$\begin{aligned}\nu(\Lambda_c) &= \nu_0\left(1+\frac{0.1\bar{\varepsilon}}{\nu_0^3}\mathcal{H}\left(\Lambda_c^{-4}-\Lambda_0^{-4}\right)\right)^{1/3} \\ \frac{D\overline{K}}{Dt} &= P_k - \bar{\varepsilon} + \alpha\frac{\partial}{\partial x_i}(\nu(\Lambda_c)\frac{\partial\overline{K}}{\partial x_i}) \\ \frac{D\bar{\varepsilon}}{Dt} &= \frac{4}{3}\nu(\Lambda_c)\Lambda_c^2 P_k - 2\nu(\Lambda_c)\Lambda_c^2\bar{\varepsilon} \\ &\quad +\alpha\frac{\partial}{\partial x_i}(\nu(\Lambda_c)\frac{\partial\bar{\varepsilon}}{\partial x_i})\end{aligned}$$

with ν_0 the molecular kinematic viscosity, $\mathcal{H}(x) = max(x,0)$ and Λ_c is the wavenumber corresponding to the filter width, $\alpha = 1.39$ and $P_k = -\overline{u_i u_j}S_{ij}$ is the production of turbulent kinetic energy. Λ_0 is a wavenumber corresponding to the viscous lengthscales where the RG iteration procedure is started from. One notices that the timescale in the $\bar{\varepsilon}$-equation is now dependent on the filter width. This dependence results in a shorter timescale on finer grids, corresponding to a quicker response of $\bar{\varepsilon}$ on changes in $\overline{K}$. This is physically consistent, as on a finer grid their are less wavenumbers between the filter-wavenumber and the dissipation wavenumber that have to be passed in order to transmit information to the dissipation range.

Limiting behaviour

The DNS-limit follows if $\Lambda_c \geq \Lambda_0$, with $\Lambda_0 = 0.2\left(\frac{\bar{\varepsilon}}{\nu_0^3}\right)^{1/4}$ the Kolmogorov dissipation wavenumber (in that case the step function turns zero). The RANS limit follows if $\Lambda_c \leq \Lambda_e$, where Λ_e is the wavenumber corresponding to the peak of the energy spectrum, estimated in the computation as $\Lambda_e = \left(\frac{3}{2}C_K\right)^{3/2}\frac{\varepsilon_t}{K_t^{3/2}}$ with C_K the Kolmogorov constant (calculated by RG to be equal to 1.44). The ensemble-averaged values K_t and ε_t are the sum of their resolved and modelled parts. Substitution of Λ_c with Λ_e leads to the following RANS model (with the t-subscript left out)

$$\nu = \nu_0\left(1+\mathcal{H}\left(\left(\frac{C_\mu\overline{K}^2}{\nu_0\bar{\varepsilon}}\right)^3 - C\right)\right)^{1/3} \quad (1)$$

$$\frac{D\overline{K}}{Dt} = P_{\overline{K}} - \bar{\varepsilon} + \frac{\partial}{\partial x_i}(\alpha\nu\frac{\partial\overline{K}}{\partial x_i}) \quad (2)$$

$$\frac{D\bar{\varepsilon}}{Dt} = C_{\varepsilon 1}\frac{\bar{\varepsilon}}{\overline{K}}P_{\overline{K}} - C_{\varepsilon 2}\frac{\bar{\varepsilon}^2}{\overline{K}} + \frac{\partial}{\partial x_i}(\alpha\nu\frac{\partial\bar{\varepsilon}}{\partial x_i}) \quad (3)$$

with

$$\begin{aligned}&C_\mu = 0.1 \qquad \alpha = 1.39 \qquad C_{\varepsilon 1} = 1.33 \\ &C_{\varepsilon 2} = 2.0 \qquad C = \mathcal{O}(100).\end{aligned} \quad (4)$$

Near-wall behaviour

The near-wall behaviour is regularized through the Heaviside function in the eddy-viscosity formulation: near the wall the effective viscosity reduces to the molecular viscosity. In LES-form this happens when $\Lambda_c > \Lambda_0 \Rightarrow \nu_t = \nu_0$ and in RANS-mode when $\overline{K}^2/\nu_0\overline{\varepsilon} < 50$. In order to get a correct logarithmic velocity profile in channel flow however, the sharp jump in the Heaviside function had to be smoothened, which can be done using the identity

$$\mathcal{H}(x) \equiv \lim_{n\to\infty} \frac{1}{2}[1 + \tanh(nx)] \tag{5}$$

where smaller n leads to a smoother Heaviside function. The value of n can be obtained empirically, and the final, low-Reynolds form of the eddy viscosity is (where we also simplified the exponential form)

$$\nu = \nu_0 + f(0.1\overline{\varepsilon}\Lambda_c^{-4})^{1/3} \tag{6}$$

with

$$f = \frac{1}{2}\left[1 + \tanh\left(n\left(\frac{0.1\overline{\varepsilon}\Lambda_c^{-4}}{\nu_0} - C\right)\right)\right] \tag{7}$$

where $n = 0.02$ and $C = 125$, as determined by comparison with DNS data. An additional production term in the $\overline{\varepsilon}$-equation was also included. This term corresponds to a, due to RG, neglected term in the high-Reynolds derivation of the model. We did not calculate this term with RG, but instead used the form as used in the Yang-Shih model (Yang and Shih, 1993), i.e. we add the term

$$\nu_0\nu\left(\frac{\partial^2 v_i}{\partial x_j \partial x_k}\right)^2 \tag{8}$$

to the rhs of the $\overline{\varepsilon}$-equation. Finally, singularities in the $\overline{\varepsilon}$-equation resulting from terms $\sim \overline{\varepsilon}/\overline{K} \equiv 1/\tau$ are prevented by adding the Kolmogorov timescale $\sqrt{\frac{\nu_0}{\varepsilon_t}}$ to $\tau = \overline{K}/\overline{\varepsilon}$.

RESULTS

We show results of a simulation of turbulent flow over a backward-facing step at a $Re = 5100$, based on the step height. When h denotes the step-height, an entry section of length $10h$ was put before the step, and the section behind the step measures $20h$. The spanwise direction is $4h$ wide. The dimensionless distance from the wall for the nearest grid cell is approximately 1, based on the friction velocity at the end of the region behind the step. The dimensionless streamwise spacing ranges between 1 and 125. The uniform spanwise spacing (Δz^+) is around 20. The total mesh consisted of approximately 480000 cells. To show the LES-behaviour of the model, the coherent structures behind the step are shown in Fig.5. More specifically, an isosurface of a positive value of the second invariant of the rate of strain-tensor, $Q = 1/2(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$ is shown here, which is known to depict well regions of low pressure. In the plot we can see how the vortex rolls that are shed behind the step are broken up by streamwise streaks, a typical feature of this flow. In Fig.2 the skin-friction coefficient behind the step is shown in comparison with the DNS results of Le et al. (1997). One sees that the reattachment length is well predicted. The obtained length of 6.4 is within 2% of the DNS value. The Strouhal-frequency $S = \frac{fh}{U_0}$ of the vortex shedding behind the step is approximately 0.073, which is also well within the experimentaly obtained range (Eaton and Johnston(1980)) found that the spectral peak occured

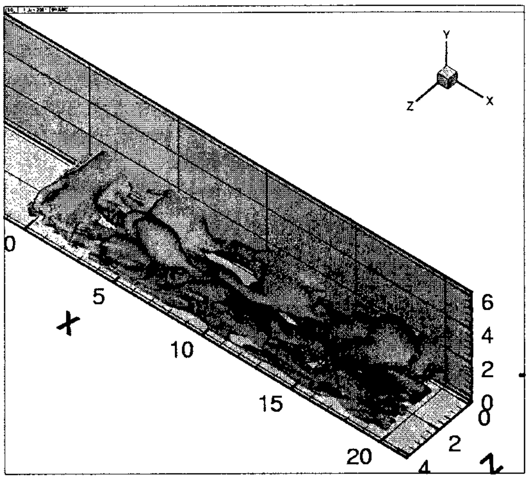

Figure 1: Isosurfaces of $Q = 11$ show the coherent flow structures behind the step. The flow comes from the left and is shed at $x = 0$.

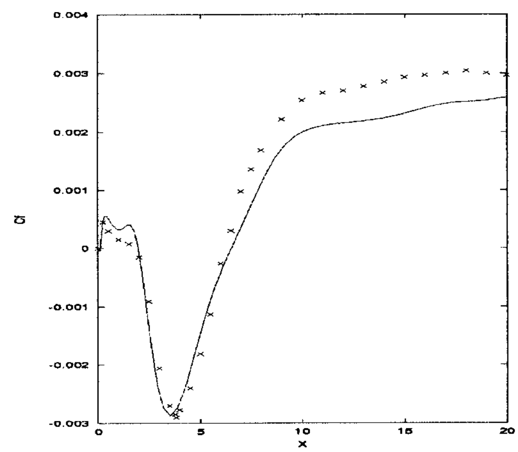

Figure 2: Skin-friction coefficient behind the backward facing step (Re=5100 based on the step height). Line: VLES computation, symbols: DNS.

in the Strouhal number range $0.066 < S < 0.08$). The amplitudes of the skin-friction coefficient behind the recirculation zone is not completely satisfying, and this problem is still under investigation.

Also shown are the results for turbulent flow over a periodic hill. Shown in Fig.5 are the stream streaks for the mean velocity of turbulent flow at a Reynolds number of 10595 (based on the hill height h and the bulk velocity above the top of the hill). The recirculation length is slightly larger, but still in good agreement with the results obtained from highly resolved LES, in which about 20 times more grid-points were used (Temmerman and Leschziner,2001). Also shown are streamwise velocity profiles, uv-stresses and turbulent kinetic energy profiles at the locations $x/h = 2$ and $x/h = 6$. The total turbulent kinetic energy depicted in these figures is the sum of the resolved and modelled parts. From the modelled part of the turbulent kinetic energy we see that against the wall, a large part of the kinetic energy is modelled, while in the center of the flow, most of the kinetic energy is resolved.

Figure 3: Stream streaks for turbulent flow over a periodic hill at Re=10595.

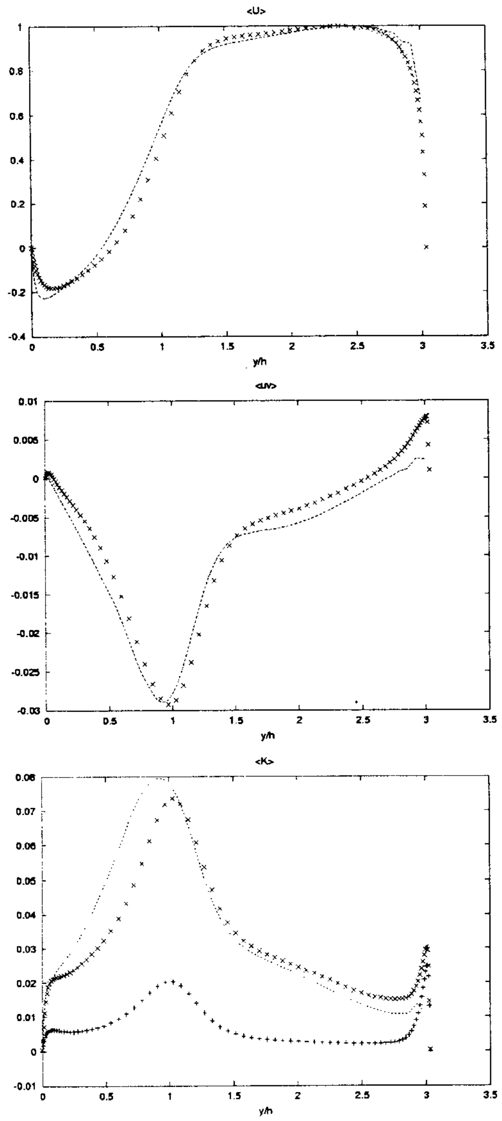

Figure 4: Mean streamwise velocity profiles, uv-stresses and turbulent kinetic energy profiles at $x/h = 2$. Lines: benchmark LES, $\times$: our computation, $+$: modelled part of turbulent kinetic energy.

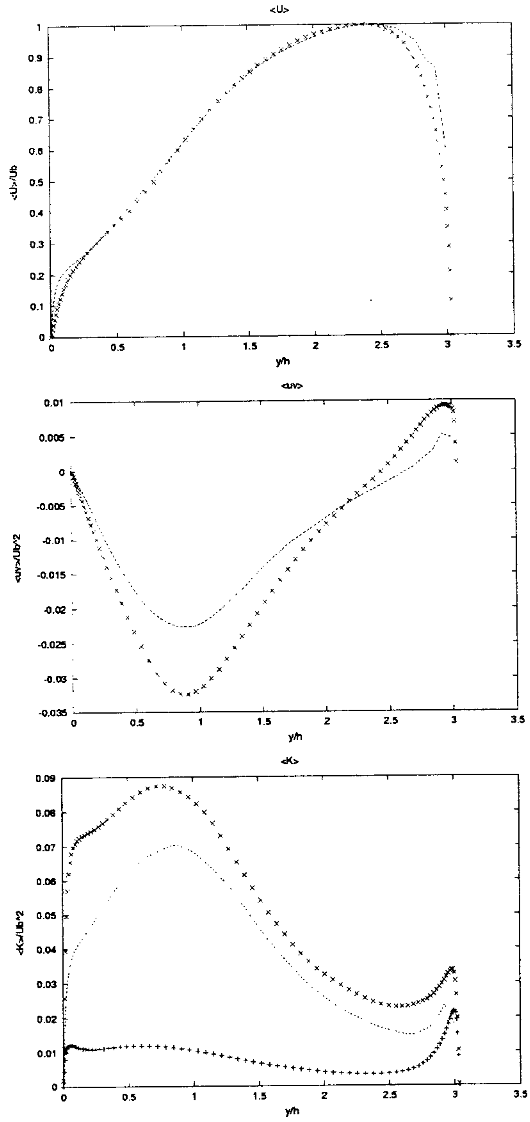

Figure 5: Mean streamwise velocity profiles, uv-stresses and turbulent kinetic energy profiles at $x/h = 6$. Lines: benchmark LES, $\times$: our computation, $+$: modelled part of turbulent kinetic energy.

CONCLUSION

A RG $\overline{K} - \overline{\varepsilon}$ model was made dependent of a cut-off wavelength Λ_c. This form makes the model appropriate for Very Large Eddy Simulations, in which a turbulence model is desired that can go continuously from a LES-regime to a RANS-regime. This adaptive behaviour occurs naturally in the present approach, without having to rely on ad-hoc assumptions.

ACKNOWLEDGMENTS

The first author thanks F. Magagnato and his colleagues at the university of Karlsruhe for making available their CFD-code 'SPARC' in which the above described model was implemented.

The second author works as Postdoctoral Fellow of the Fund for Scientific Research - Flanders (Belgium) (F.W.O.-Vlaanderen).

REFERENCES

Barber, M.N., 1977, "An introduction to the fundamentals of the renormalization group in critical phenomena", *Phys. Rep.*, 29C(3):2–84.

De Langhe, C., Merci, B., and Dick, E., 2001, "On the construction of vles subgrid models with the renormalization group" In *ECCOMAS Computational Fluid Dynamics 2001 Conference, Swansea, September 2001.*

Dejoan, A. and Schiestel, R., 2001, "Large-eddy simulations of non-equilibrium pulsed turbulent flow using transport equations subgrid scale model", In *Turbulence and Shear Flow Phenomena, Second International Symposium,* KTH, Stockholm, June 2001.

Eaton, J.K. and Johnston, J.P., 1980, "Turbulent flow reattachment: An experimental study of the flow and structure behind a backward facing step", Technical Report MD39, STANF U DEP MEC.

Giles, M.J., 1994a, "Statistical mechanics renormalization group calculations for inhomogeneous turbulence", *Phys. Fluids*, 6(11):3750–3764.

Giles, M.J., 1994b, "Turbulence renormalization group calculations using statistical mechanics methods", *Phys. Fluids*, 6(2):595–604.

Le, H., Moin, P., and Kim, J., 1997, "Direct numerical simulation of turbulent flow over a backward-facing step", *J. Fluid. Mech.*, 330:349–374.

Spalart, P.R., Jou, W.-H., Strelets M., and Allmaras S.R., 1997, "Comments on the feasibility of LES for wings, and on a hybrid RANS/LES approach", In *Advances in DNS/LES, 1st AFOSR Int. Conf. on DNS/LES, Aug 4-9, 1997*, Columbus Oh. Greyden Press.

Speziale, C.G., 1998, "Turbulence modelling for time-dependent RANS and VLES: A review", *AIAA J.*, 36(2):173–184.

Strelets, M., 2001, "Detached eddy simulation of massively separated flows", In *39th AIAA Aerospace Sciences Meeting and Exhibit*, Reno, NV, January 2001. AIAA.

Temmerman, L. and Leschziner M.A., 2001, "Large-eddy simulation of separated flow in a streamwise periodic channel constriction", *Proceedings Turbulence and Shear Flow Phenomena, Second International Symposium*, KTH, Stockholm, June 2001.

Yakhot, V. and Orszag, S.A., 1986, "Renormalization group analysis of turbulence", *J. Sci. Comput.*, 1:3–51.

Yang, Z.Y. and Shih, T.H., 1993, "A new time scale based $k - \epsilon$ model for near-wall turbulence", *AIAA Journal*, 31(7):1191–1198.